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NONSTATIONARY EXPONENTIAL DISTRIBUTIONS OF THE BREAKDOWN VOLTAGES IN RAMP BREAKDOWN EXPERIMENTS

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Abstract. The nonstationary exponential distribution of dynamic breakdown voltages U_b is derived based on the binomial distribution for the occurrence of initiating electrons. The experimental distributions in neon obtained by applying linearly rising (ramp) pulses are described by derived distribution with the voltage (time) varying electron yield Y (number of generated electrons in the interelectrode gap per second) and the breakdown probability P. It was found that the exponential form of $Y(U)$ dependence is needed to fit the experimental data. The linear variation of Y as a first approximation of the exponential dependence is also applied to the experimental distributions, as well as Weibull distribution function for the sake of comparison.

1. INTRODUCTION

The time that elapses from the moment of application of voltage greater than the static breakdown voltage U_s to the breakdown is the breakdown time delay t_d. It consists of the statistical time delay t_s (from the application of voltage to the appearance of free electron(s) initiating breakdown) and the formative time delay t_f (from this moment to the collapse of the applied voltage and occurrence of a self-sustained current) (Morgan 1978). The stochastic character of t_s was experimentally proved by Zuber (1925), while its exponential statistical distribution was theoretically derived by von Laue (1925). The cumulative distribution function of the statistical time delay (for step pulses) $F(t_s) = 1 - \exp(-t_s / \overline{t_s})$, where the distribution parameter $\overline{t_s}$ is the mean value of the statistical time delay, was derived on strictly mathematical grounds by Kiselev (1965). When the electron yield and the breakdown probability do not change (step pulses), the process of electron occurrence is a stationary Poisson process, but in the case of ramp pulses, Y and P are voltage dependent and the electrical breakdown is nonstationary Poisson process (Marković et al 2005). The breakdown voltages are described by the nonstationary exponential distribution and it is derived in this paper. The experimental distributions of U_b are presented and fitted by the derived distribution function with the exponential and linear $Y(U)$ dependence. The distributions are also fitted by the Weibull distributions for the sake of comparison.

2. THE EXPERIMENTAL DETAILS

The measurements were performed on the gas tube made of borosilicate glass (8245, Schott technical glass) filled with research grade neon at the pressure of $13.3 mbar$. The electrodes were cylindrical made of copper (diameter $D = 6mm$, gap $d = 6mm$), with a hard galvanic layer of gold. The static breakdown voltage was $U_s = 197V$. The measurements were carried out by applying the sequence of linearly rising voltage pulses to the gas discharge tube at glow currents $I_g = 100\mu A$ and the rate of voltage rise $k = 800V/s$. More details about experimental procedure and the electronic automatic system can be found in Marković et al. (2005, 2006b).

3. DERIVATION OF NONSTATIONARY EXPONENTIAL DISTRIBUTION FUNCTION

The statistical time delay as a random variable is determined by the probability of occurrence at least one electron in the interelectrode space for the time interval t_s. When Y and P are constant (step pulses), the process of electron appearance is a stationary Poisson process and the probability of occurrence of l electrons is given by the stationary Poisson distribution (Kiselev 1965, Marković et al. 2006a):

$$w(l) = e^{-YPt_s} \frac{(YPt_s)^l}{l!}. \quad (1)$$

This discrete distribution passes into a homogeneous continuous one for waiting time t_s until the first effective electron occurs $w(l = 0) = \exp(-YPt_s)$, equivalent with Laue representation $w \equiv n/N = \exp(-YPt_s)$. When Y and P are constant, the t_s distribution is the stationary one and it is linear in Laue representation.

In the case of electrical breakdown by ramp pulses, when Y and P are voltage dependent variables, the statistics of the electrical breakdowns is a nonstationary one, and the process of electron occurrence is a nonstationary Poisson process.

The nonstationary distribution of the dynamic breakdown voltages can be derived in the following way. Let us define the cumulative electron yield for the nonstationary Poisson process when the electron yield $Y(t)$ and the breakdown probability $P(t)$ are time dependent variables, as $\beta(t) = \int_0^{t_s} Y(t)P(t)dt$. If we divide the time interval t_s to a large number m of the subintervals $\Delta t = t_s / m$, than each subinterval Δt will be infinitely small. Moreover, m represents a maximum number of occurred electrons, in order to obtain at most one accident in each sub-interval (independent accidents). Therefore, the probability of occurrence of an effective electron in the time interval Δt is given by $\beta(t)/m$ and the probability of non-occurrence by $1-\beta(t)/m$. The breakdown will happen if one or more electrons appear during t_s i.e. the probability of breakdown W in the time interval t_s is equal to the sum of the probabilities of occurrence of l electrons in the interelectrode space, and l can take all the whole number values from $[j-m]$:

$$W = \sum_{l=j}^{m} C_m^l \left(\frac{\beta(t)}{m}\right)^l \left(1-\frac{\beta(t)}{m}\right)^{m-l} \tag{2}$$

where C_m^l are the binomial coefficients. When $m \to \infty$ and at least one electron occurs in the interelectrode space ($j=1$), the sum (2) goes into a continuous time distribution and terms under the sign of summation go into Poisson distribution:

$$W = \sum_{l=1}^{\infty} \frac{[\beta(t)]^l}{l!} e^{-\beta(t)} = e^{-\beta(t)} \left[\frac{\beta(t)}{1!} + \frac{\beta^2(t)}{2!} + \frac{\beta^3(t)}{3!} + \cdots\right]. \tag{3}$$

After the summation the cumulative nonstationary distribution of t_s is obtained:

$$F_E(t_s) = 1-e^{-\beta(t)} = 1-\exp\left[-\int_o^{t_s} Y(t)P(t)dt\right]. \tag{4}$$

In the case of electrical breakdown by linearly rising voltage pulses with the rates of voltage rise $k = dU/dt \approx (U_b - U_s)/t_s$ the relation (4) can be rewritten as:

$$F_E(U_b) = 1-e^{-\beta(U)} = 1-\exp\left[-\int_{U_s}^{U_b} (Y(U)P(U)/k)dU\right]. \tag{5}$$

Therefore, the distribution of the breakdown voltages in ramp experiments when Y and P are voltage dependent, is the nonstationary exponential distribution.

The derived distribution is used to describe the experimental data obtained at $k = 800V/s$. The probability P in the Townsend model is given as $P = 1-1/q$ for $q>1$, and $P=0$ for $q<1$ (Wijsman 1949), where $q = \gamma[\exp(\alpha d)-1]$, α is the electron ionization coefficient and γ is the secondary electron yield (Marković et al 2007). In our experimental conditions $\gamma \approx \gamma_s$, where γ_s is the secondary electron yield corresponding to U_s. The voltage dependence of the electron yield Y is given by relation:

$$Y = Y_0 \exp(\lambda \Delta U), \tag{6}$$

where Y_0 is the initial electron yield and λ is the voltage growth constant. The fits of the experimental distributions are shown in Fig. 1 (solid lines), with the parameters $Y_0 = 4 s^{-1}$ and $\lambda = 0.0105 V^{-1}$. The linear approximation of $Y(U)$ dependence is represented by relation:

$$Y = Y_0 (1+\lambda \Delta U), \tag{7}$$

and it is also used for fitting of experimental distributions (Fig 1, dashed lines), but unsuccessfuly.

For the sake of comparison, the two-parameter Weibull distribution is also applied to the experimental distributions in Fig. 1. The Weibull probability density function is given by relation:

$$f(U) = \left(\frac{\kappa}{\eta k}\right)\left(\frac{\Delta U}{k}\right)^{\kappa-1} \exp\left[-\left(\frac{\Delta U}{k}\right)^{\kappa} \frac{1}{\eta}\right] \tag{8}$$

where κ is the shape parameter and η is the scale parameter. The best fits of the experimental data are presented in Fig. 1 (dotted lines) with parameters $\kappa = 2.53$ and $\eta = 0.011$, but they do not have a physical meaning.

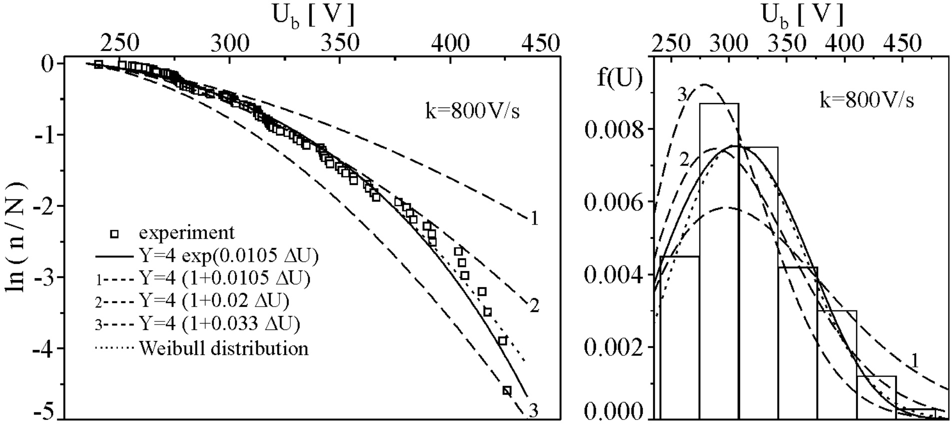

Figure 1: The fits of the experimental distributions by the nonstationary distribution with exponential and linear $Y(U)$ dependence and with Weibull distribution.

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