Publ. Astron. Obs. Belgrade No. 89 (2010), 221-224 *Contributed Paper*

INFLUENCE OF EXPERIMENTAL ERRORS ON DETERMINATION OF A REAL PROFILE OF SPECTRAL LINES BY REGULARIZATION METHOD

SLAVOLJUB MIJOVIĆ and MIRA VUČELJIĆ*
Faculty of Natural Sciences and Mathematics,
University of Montenegro, Montenegro
**E-mail: mirav@rc.pmf.ac.me*

Abstract. Correlating experimental data with the parameters of theoretical model is always a delicate task. Experimental data are always influenced by statistical fluctuations. These determine the signal to noise ratio and thus finally limit the accuracy of the measurements. In most cases it is only possible to improve this ratio by extreme expenditures of experimental set up. On the other hand, consistent applications of statistical methods such as a regularization method allows us to increase the accuracy of the result for given experimental data. It is most important in this context to know something about the relation between the precision of regularization method and experimental errors. In this work we analysed how an estimation of experimental errors influence the accuracy of the result obtained by regularization method. Usually in a real experiment the exact error is never known i.e. we either overestimated or underestimated it. To investigate this dependence we generated model function with added artificial noise. Specially, we were focused on the determination of a Stark profile from the measured Voigt profile of a spectral line. There were no significant shifts of the spectral lines obtained by regularization, but the lines are broadened. Finally conclusion is that is better to overestimate the experimental errors than underestimate because of the later case the results start to oscillate.

1. TEST AND RESULTS

Many problems in plasma diagnostics (Vuceljic and Mijovic 1996, 1998) can be formulated as a linear inverse problem i.e., the problem that requires determination of unknown plasma parameters from known experimental data. Computer-supported techniques play an important role in the evaluation of experimental data but even only discretization of inverse problems generally gives rise to very ill-conditioned linear system of algebraic equations. The ill-posed problem means that little non-avoidable errors in the measured values can lead to significant changes in the solution.

Typically, the linear systems obtained have to be regularized to make the computation of a meaningful approximate solution possible. This means that the systems must be replaced with nearby systems that is less sensitive to perturbations. Tikhonov regularization is one of the oldest and most popular regularization method. He found an effective way to regulate an ill-posed problem by using the minimum *a priori information,* as an estimation error of experimental data. In this work the regularization method FORTRAN-subroutine has been adapted for different experimental plasma diagnostics applications whose model can be given in the operator form:

$$Az = u, \tag{1}$$

where $z -$ the function we are looking for, $u -$ data recorded in an experiment and $A -$ a linear operator in the most cases. Tikhonov's method stabilizes least squares deviation of Az from u in Eq. 1, using the functional $M^{\alpha}[z]$

$$M^{\alpha}[z] = \|A_{\sigma}z - u_{\delta}\|^2 + \alpha\|z\|^2 \tag{2}$$

where σ and δ are the errors of the operator and the right part of the Eq. 1, respectively and α is the regularization parameter. The parameter $\alpha(\sigma, \delta)$ is determined by applying generalized discrepancy principle to an iterative method for which the functional $M^{\alpha}[z]$ attains its minimal (Tikhonov and Goncharski 1987).

Extensive numerical experiments are reported to demostrate the practical perfomance of the presented algorithm. As an example, we tested the problem of solving the Fredholm integral equation of the first kind

$$\int_a^b K(x,s)z(s)ds = u_0(x) + \delta(x), \qquad c \le x \le d. \tag{3}$$

where $u = u_0(x) + \delta(x)$, i.e., $\delta(x)$ is separated stationary chance process with average valuee equal to zero and $u_0(x)$ is the exact distribution.

Eq. 3 decribes many plasma diagnostics methods such as, obtaining real profiles of spectral lines in dense plasmas (Vuceljic and Mijovic 1996), electron energy distribution function from the probe measurements (Vuceljic and Mijovic 1998) etc. It is known that the problem (3) is ill-posed.

In this work special attention was devoted to the problem of recovering a real profil of a spectral line from the experimental data. The procedures of the test and the regularization algorithm were following: the $z(s)$ and $K(x,s)$ in (3) were chosen and $u_0(x)$ were calculated according to Eq. 3. i.e., the forward problem was solved. Then, artificial random noises $\delta(x)$ were added to $u_0(x)$, to simulate a "measured" data with experimental errors. Finally, Tikhonov's regularization method was used to solve the inverse problem, i.e., from such "measured" data, the

$z(s)$ was determined and compared with the given (exact) one. In this test *z(s)* is chosen to be Lorentzian profile and *K(x,s)* is Gussian profile both with equal full width at half maximum (FWHM). In that case convolution function $u_0(x)$ is a Voigt profile. Experimental errors were simulated on two different ways. In first case in every point of a Voigt profile was added a value:

$$\eta_i = 2\% u_0(x_i) rand(j) \tag{4}$$

and in the second case to $u_0(x)$ was added

$$\eta_i = 1\% u_{0\max} rand(j) \tag{5}$$

- where $rand(j)$ is a random number within the range(-1,1) and $u_{0\max}$ is the maximum value of the Voigt profile. Estimation of added error in regularization procedure was done according to parameter $\delta^2 = \sum_i \eta_i \cdot h_s$ - (h_s is a step of the uniform nets $\{s_i\}_{i=1,n}$). Since experimental errors are never known exactly, we tried to investigate accuracy of regularization procedure in the cases when the values δ^2, calculated according Eq. 4 and Eq. 5, are overestimated and underestimated. The results are presented on Fig. 1 and Fig. 2. FWHM values obtained by regularization are presented as its ratio with the exact FWHM of the model function r. It is obvious that FWHM obtained by regularization is wider than exact. As we can see from this figures regularization method is more sensitive in case when artificial noise was underestimated and the results start to oscillate. There were no significant shifts of the spectral lines obtained by regularization.

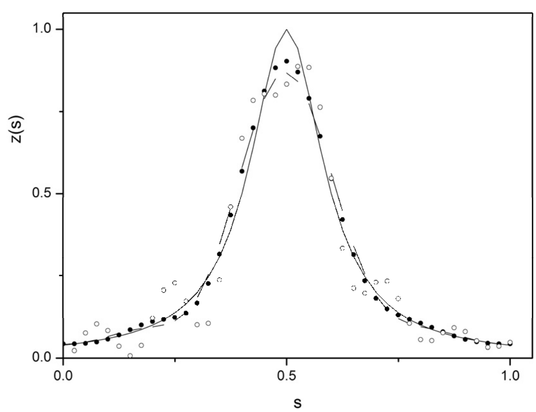

Figure 1: The result obtained by regularization method, in a case when artificial noise was added according to Eq. 4: (solid line)-the exact model profile z(s); (black circle)-correct estimation of error $\delta^2 = 5\cdot10^{-7}$, $r = 1.12$; (empty circle)-underestimate error $\delta^2 = 2.6\cdot10^{-7}$, $r = 1.21$; (dash line)- overestimate error $\delta^2 = 5\cdot10^{-6}$, $r = 1.28$.

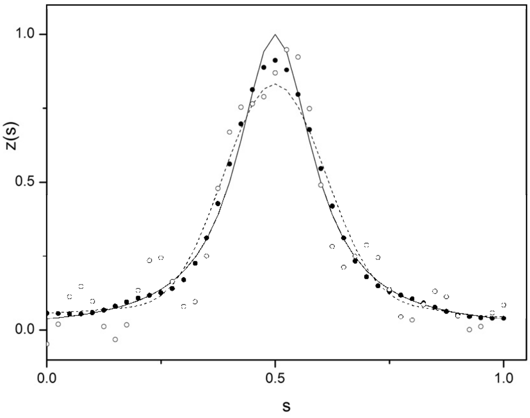

Figure 2: The result obtained by regularization method, in case when artificial noise was added according to Eq. 5: (solid line)-exact model profile z(s); (black circle)-correct estimation of error $\delta^2 = 7 \cdot 10^{-7}$, $r = 1.217$; (empty circle)-underestimate error $\delta^2 = 4.5 \cdot 10^{-7}$, $r = 1.17$; (dash line)- overestimate error $\delta^2 = 7 \cdot 10^{-6}$, $r = 1.4$.

References

Tikhonov, A. N. and Goncharski, A. V.: 1987, *Ill-posed Problems*, (Moscow University Press, Moscow).

Vuceljic, M. and Mijovic, S.: 1996, *J. Quant. Spectrosc. Radiat. Transfer*, **56**, No 4, 617-621.

Vuceljic, M. and Mijovic, S.: 1998, *Journal of Applied Physics*, **84**, No. 9, 4731.