REGULAR AND CHAOTIC MOTIONS OF PROTONS IN THE HÉNON-HEILES POTENTIAL WELL

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Abstract. In this paper we report on the regular and chaotic motions of 10 MeV protons in a <111> channel of a Si crystal. The study is devoted to the hyperchanneled protons. The continuum proton-crystal interaction potential is described using the Hénon-Heiles potential function. We investigate the Poincaré maps corresponding to a large number of proton trajectories. The analysis shows that there are two different regions in the impact parameter plane, which correspond to the regular and chaotic proton motions.

1. INTRODUCTION

Chaotic systems have been analyzed for more than two hundred years. This report is related to the chaotic behavior of deterministic systems (Cvitanović et al. 2005). Studying such behavior began with Hénon and Heiles (1964), who analyzed the motion of stars in galaxies. The potential function they introduced has been used afterwards in many studies of galactic dynamics (Barrio et al. 2008).

2. METHOD

The system we investigate is a proton of the kinetic energy of 10 MeV moving along a <111> channel of a Si crystal. The z axis coincides with the lower subchannel axis and the origin lies in the entrance plane of the crystal. We treat the interaction of the proton and crystal classically and apply the continuum approximation (Gemmell 1974). The continuum proton-crystal interaction potential is described using the Hénon-Heiles potential function. The thermal vibrations of the crystal's atoms and the collisions of the proton with the crystal's electrons are neglected (Gemmell 1974). The components of the normalized proton impact parameter vector, x_0 and y_0, are chosen within the Hénon-Heiles triangle, being the longest closed equipotential line in the potential well under consideration. The initial proton velocity vectors are taken to be parallel to the subchannel axis.

We calculate the proton trajectories and determine the corresponding Poincaré maps (Cvitanović et al. 2005). This is done by solving numerically the proton equations of motion in the transverse position plane (Petrović et al. 2000). Such a map is a set of points in the yv_y plane that correspond to the crossings of the phase space trajectory with the y axis (where $x = 0$), at which $v_x \geq 0$; x and y are the normalized transverse components of the proton position vector, and v_x and v_y the normalized transverse components of the proton velocity vector, respectively. If the map contains only the closed well-defined lines, the proton motion is regular. If this is not the case, the proton motion is chaotic.

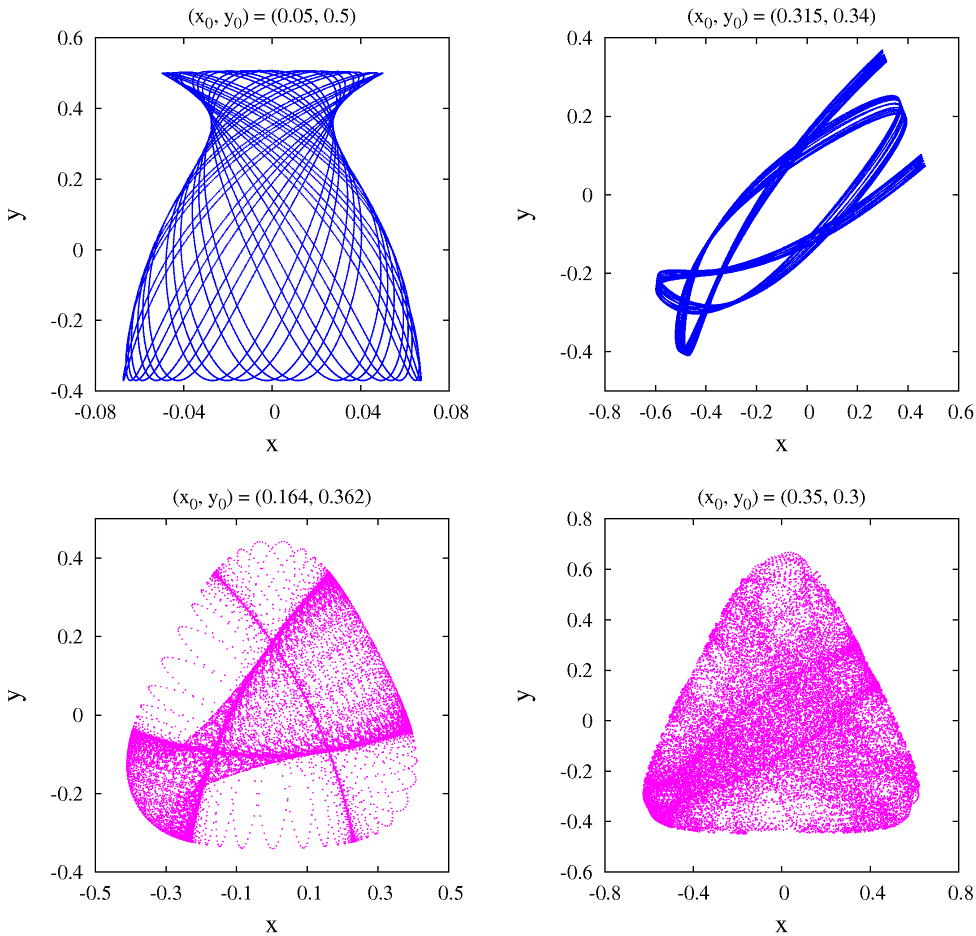

Figure 1: The proton trajectories in the transverse position plane obtained for (x_0, y_0) = (0.05, 0.5) and (0.315, 0.34) (top), and for (x_0, y_0) = (0.164, 0.362) and (0.35, 0.3) (bottom).

3. RESULTS AND DISCUSSION

Fig. 1 gives two proton trajectories in the transverse position plane when the proton motion is regular and two trajectories when the motion is chaotic. The corresponding Poincaré maps are shown in Fig. 2.

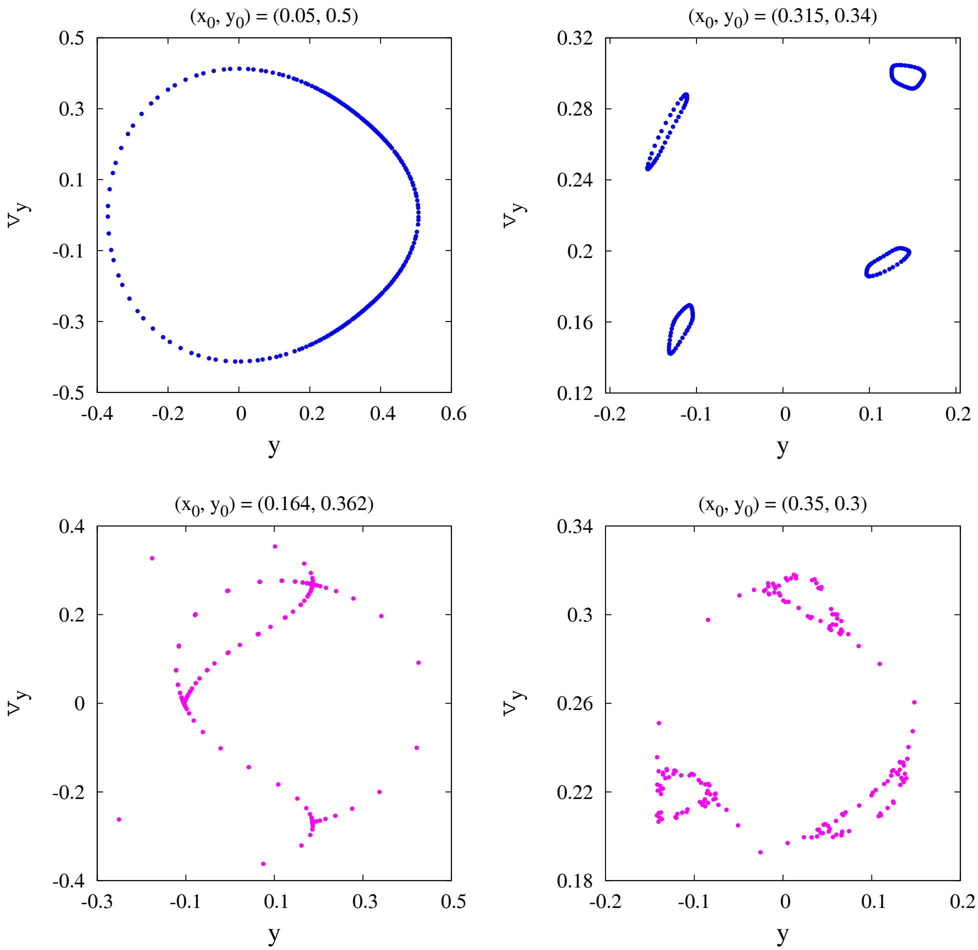

Figure 2: The Poincaré maps obtained for $(x_0, y_0) = (0.05, 0.5)$ and $(0.315, 0.34)$, showing that the corresponding proton motions are regular (top), and for $(x_0, y_0) = (0.164, 0.362)$ and $(0.35, 0.3)$, showing that the motions are chaotic (bottom).

Fig. 3 (left) gives the equipotential lines in the potential well under consideration. The value of the Hénon-Heiles potential function corresponding to the triangle is 1/6. Our analysis shows that there are two different regions in the impact parameter plane, within the triangle. When point (x_0, y_0) is in the central part of the triangle, the motion is regular, and when (x_0, y_0) is in its peripheral region, the motion is chaotic. These two regions are shown in Fig. 3 (right). It should be noted that all the points on the three symmetry axes of the triangle belong to the former region. One can see that the two regions are separated by a rugged line.

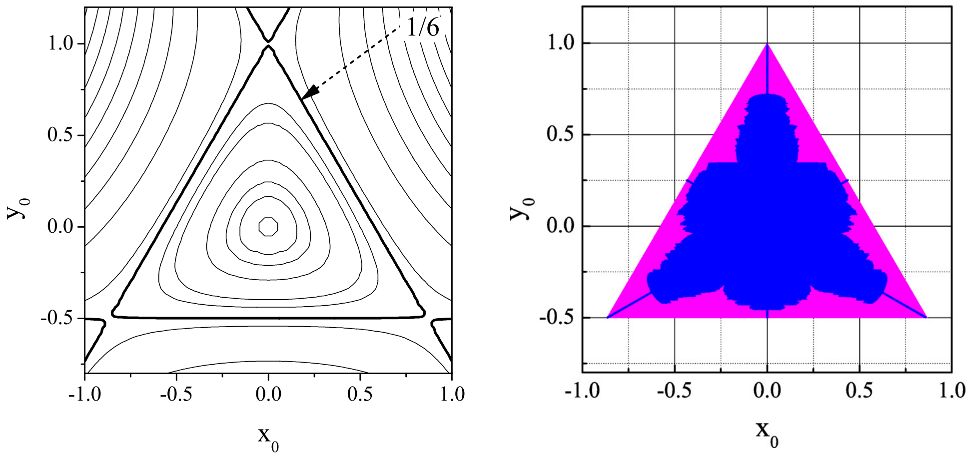

Figure 3: The equipotential lines in the potential well under consideration (left). The central and peripheral regions within the Hénon-Heiles triangle, in which the proton motions are regular and chaotic, respectively (right).

Acknowledgements

This research has been done within the project *Physics and Chemistry with Ion Beams*, No. 151005B, funded by the Ministry of Science and Technological Development of Serbia.

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