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MASS SPECTRUM OF THE LIGHT SCALAR TETRAQUARK NONET WITH GLOZMAN-RISKA HYPERFINE INTERACTION

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Abstract. In this paper we perform a schematic study (two quark interaction) of the masses of the light scalar $qq\bar{q}\bar{q}$ tetraquarks in the SU(3) flavor representations: 8_{MS}, 8_{MA}, 1_{MS} and 1_{MA}. We use the flavor-spin Glozman-Riska interaction Hamiltonian with SU(3) flavor symmetry breaking to determine the masses of these tetraquarks. The whole method for calculation of GR hyperfine interaction is presented in detail.

1. INTRODUCTION

Already in the seventies, Jaffe (1977) suggested the tetraquark structure of the scalar nonet. Their quark content is given e.g. in Jaffe (1977) and in Brito et al. (2005). Alford & Jaffe (2000) studied the possibility that the light scalar mesons are $q^2\bar{q}^2$ states rather than $q\bar{q}$. Close & Törnqvist (2002) showed that scalar meson states above 1 GeV form a conventional $q\bar{q}$ nonet, and the scalar meson states below 1 GeV also form a nonet, but of a more complicated nature. The strong decays of the lightest scalar mesons σ, κ, f_0, a_0 are studied in 't Hooft et al. (2008); they concluded that the light scalar mesons below 1 GeV are predominantly tetraquark states, while the heavier ones (around 1.5 GeV) are predominantly $q\bar{q}$ states. The spectroscopy and many of the decay properties of the scalar mesons can be understood under the hypothesis of the mixing between two- and four-quark components (Vijande et al. 2005). It is shown (Brito et al. 2005) that these mesons fit well in the tetraquark scheme. We considered them as four-quark states and calculated their masses.

Mass spectra of light scalar tetraquark nonet is studied using Glozman-Riska hyperfine interaction (GR HFI) (about this two-particle interaction see e.g. Borka Jovanović 2007, 2008 and references therein). This nonet has the form $q_1q_2\bar{q}_3\bar{q}_4$, where $q = u$, d, s. In the flavor SU(3) group, according to product $3 \otimes 3 \otimes \bar{3} \otimes \bar{3} = (27 + 10 + 8) + (8 + 1) + (\overline{10} + 8) + (8 + 1)$, there are total 81 $qq\bar{q}\bar{q}$ tetraquark states. We discuss nonet, which consists of one singlet and one octet. These nine states are: $\sigma(500)$, $f_0(980)$, $\kappa^+(800)$, $\kappa^0(800)$, $\bar{\kappa}^0(800)$, $\kappa^-(800)$ $a_0^+(980)$, $a_0^0(980)$ and $a_0^-(980)$.

We calculated the mass spectrum based on the flavor-spin interactions following Glozman and Riska (1996). Masses of light scalar tetraquarks are considered in the nonrelativistic quark model as a sum of constituent quark masses and contributions of pair hyperfine interactions between quarks.

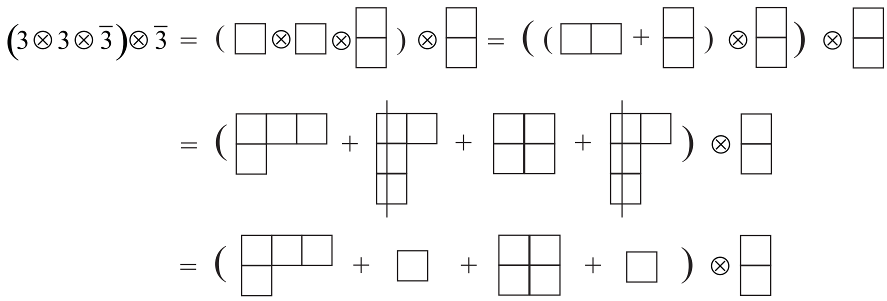

Figure 1: Young diagrams for $SU(3)_F$ multiplets according to $(3 \otimes 3 \otimes \bar{3}) \otimes \bar{3} = (15 + 3 + 3 + 6) \otimes \bar{3}$. From the second and third line of it can be noticed that Young tables which differ by left-aligned columns of 3 boxes (crossed out) depict the same irreducible representation.

2. METHOD

The flavor SU(3) decomposition of the 81 possible $qq\bar{q}\bar{q}$ combinations is given in Figures 1 and 2 by the Young diagrams. In the Fig. 1, the numbers 15, 3, 3 and 6 are dimensions of Young diagrams $3 \otimes 3 \otimes \bar{3}$ and they designate the number of particles in the same group i.e. SU(3) flavor multiplet. Particles belonging to the same multiplet have the same baryonic number, spin and intrinsic parity. They also have similar

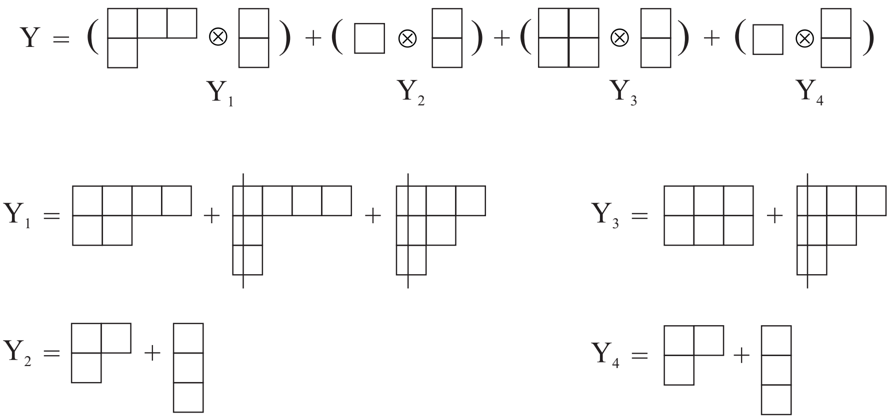

Figure 2: Young diagrams for $SU(3)_F$ multiplets according to $3 \otimes 3 \otimes \bar{3} \otimes \bar{3} = (15 \otimes \bar{3}) + (3 \otimes \bar{3}) + (3 \otimes \bar{3}) + (\bar{6} \otimes \bar{3}) = (27 + 10 + 8) + (8 + 1) + (\overline{10} + 8) + (8 + 1)$. Tetraquarks with quark content $qq\bar{q}\bar{q}$ form the following multiplets: one 27-plet, decuplet, anti-decuplet, four octets and two singlets.

masses. Then, in Fig. 2 it is showed how the whole product $Y = (15 + 3 + 3 + \bar{6}) \otimes \bar{3}$ divides on its addends: $Y_1 = 27 + 10 + 8$, $Y_2 = 8 + 1$, $Y_3 = \overline{10} + 8$ and $Y_4 = 8 + 1$. Weight diagram for light scalar multiplet 8 + 1 (nonet) is presented in Fig 3.

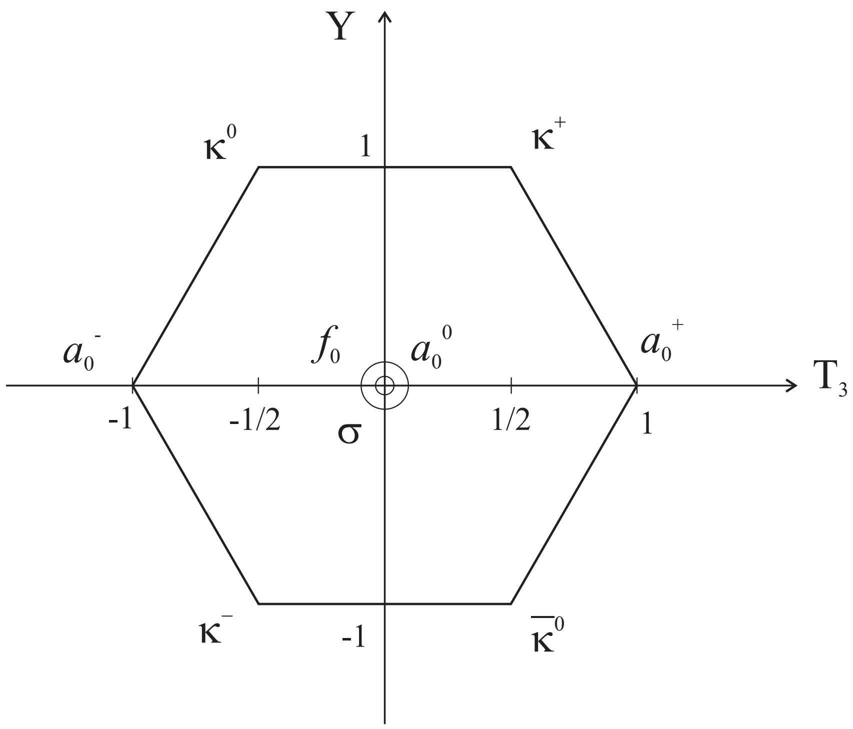

Figure 3: Weight diagram for scalar meson nonet. The ordinate shows hypercharge Y and abscissa 3-component T_3 of isotopic spin magnitude.

To calculate masses of the given $qq\bar{q}\bar{q}$ nonet, with GR HFI contribution included, we use formulas (4) and (5) that are given in Borka Jovanović (2007). To apply these equations, here we first show how to obtain product of Pauli and Gell-Mann matrices.

3. RESULTS AND DISCUSSION

The spin part of the wave function can be written as $|S_i\rangle = [(12)_{S12}(34)_{S34}]_S$, where the spin of the two quarks is coupled to S_{12} and that of the antiquarks to S_{34} (Vijande et al. 2004). Scalar tetraquarks have total spin $S = 0$. For symmetric spin wave function, it implies $S_{12} = 1$ and $S_{34} = 1$, so for product of Pauli spin matrices it follows: $\vec{\sigma}_1\vec{\sigma}_2 = \vec{\sigma}_3\vec{\sigma}_4 = 1$ and $\vec{\sigma}_1\vec{\sigma}_3 = \vec{\sigma}_1\vec{\sigma}_4 = \vec{\sigma}_2\vec{\sigma}_3 = \vec{\sigma}_2\vec{\sigma}_4 = -2$. For antisymmetric spin wave function, it implies $S_{12} = 0$ and $S_{34} = 0$, so it follows: $\vec{\sigma}_1\vec{\sigma}_2 = \vec{\sigma}_3\vec{\sigma}_4 = -3$ and $\vec{\sigma}_1\vec{\sigma}_3 = \vec{\sigma}_1\vec{\sigma}_4 = \vec{\sigma}_2\vec{\sigma}_3 = \vec{\sigma}_2\vec{\sigma}_4 = 0$.

There are two nonets which consist of octet and singlet and they have mixed symmetry which is the permutation symmetry just of the first quark pair q_1q_2. Due to mutual orthogonality, one octet is mixed symmetric (MS) and the other one is mixed antisymmetric (MA).

First we will show how quark-quark, quark-antiquark and antiquark-antiquark pairs distribute into $SU(3)_F$ multiplets:

1. two quarks, according to $3 \otimes 3 = \bar{3} + 6$, may be members of multiplets $\bar{3}$ or 6
   members of $\bar{3}$ are: $\frac{1}{\sqrt{2}}(ud - du)$, $\frac{1}{\sqrt{2}}(ds - sd)$, $\frac{1}{\sqrt{2}}(su - us)$
   members of 6 are: $\frac{1}{\sqrt{2}}(ud + du)$, $\frac{1}{\sqrt{2}}(ds + sd)$, $\frac{1}{\sqrt{2}}(su + us)$, uu, dd, ss
2. quark and antiquark, according to $3 \otimes \bar{3} = 1 + 8$, may be members of multiplets 1 or 8
   member of 1 is: $\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$
   members of 8 are: $\frac{1}{\sqrt{2}}(u\bar{d} + \bar{d}u)$, $\frac{1}{\sqrt{2}}(d\bar{s} + \bar{s}d)$, $\frac{1}{\sqrt{2}}(s\bar{u} + \bar{u}s)$, $\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$, $\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$, $\frac{1}{\sqrt{2}}(d\bar{u} + \bar{u}d)$, $\frac{1}{\sqrt{2}}(s\bar{d} + \bar{d}s)$, $\frac{1}{\sqrt{2}}(u\bar{s} + \bar{s}u)$,
3. two antiquarks, according to $\bar{3} \otimes \bar{3} = 3 + \bar{6}$, may be members of multiplets 3 or $\bar{6}$
   members of 3 are: $\frac{1}{\sqrt{2}}(\bar{u}\bar{d} - \bar{d}\bar{u})$, $\frac{1}{\sqrt{2}}(\bar{d}\bar{s} - \bar{s}\bar{d})$, $\frac{1}{\sqrt{2}}(\bar{s}\bar{u} - \bar{u}\bar{s})$
   members of $\bar{6}$ are: $\frac{1}{\sqrt{2}}(\bar{u}\bar{d} + \bar{d}\bar{u})$, $\frac{1}{\sqrt{2}}(\bar{d}\bar{s} + \bar{s}\bar{d})$, $\frac{1}{\sqrt{2}}(\bar{s}\bar{u} + \bar{u}\bar{s})$, $\bar{u}\bar{u}$, $\bar{d}\bar{d}$, $\bar{s}\bar{s}$

Just notice that the coefficient for a given quark state, when squared, denotes the probability that a certain quark pair has that exact combination. For instance, the state $\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$ from multiplet 8 has the probability $\frac{1}{6}$ to be $u\bar{u}$, $\frac{1}{6}$ to be $d\bar{d}$ and $\frac{2}{3}$ to be $s\bar{s}$ quark-antiquark pair (and thus giving total probability equal to 1 for the whole state).

Let us write down the flavor wave functions (see in Jaffe (1977) or in Brito et al. (2005)) with aim to separate their addends by quark pairs: q_1q_2, $q_i\bar{q}_j$ and $\bar{q}_3\bar{q}_4$, as shown in Table 1:

$|\sigma\rangle = |ud\bar{u}\bar{d}\rangle$

$|f_0\rangle = \frac{1}{\sqrt{2}}|us\bar{u}\bar{s} + ds\bar{d}\bar{s}\rangle$

$|a_0^+\rangle = |us\bar{d}\bar{s}\rangle$, $|a_0^0\rangle = \frac{1}{\sqrt{2}}|us\bar{u}\bar{s} - ds\bar{d}\bar{s}\rangle$, $|a_0^-\rangle = |ds\bar{u}\bar{s}\rangle$

$|\kappa^+\rangle = |ud\bar{d}\bar{s}\rangle$, $|\kappa^0\rangle = |ud\bar{u}\bar{s}\rangle$, $|\bar{\kappa}^0\rangle = |us\bar{u}\bar{d}\rangle$, $|\kappa^-\rangle = |ds\bar{u}\bar{d}\rangle$

Then we compare addends of the flavor functions in Table 1 for light nonet with addends of the flavor functions for multiplets $\bar{3}$, 6, 1, 8, 3 and $\bar{6}$. In that way it can be determined which q_iq_j combination corresponds to certain representation. On that basis we calculate product $\langle R|\lambda_1\lambda_2|R\rangle$, where R is representation.

For example, for f_0 particle regarding q_1q_2 pair, one can see that addends us and ds stand in both multiplets of $3 \otimes 3$ product. This result is not surprising because this particle has its MS and MA part. Then we conclude that MS part belongs to 6, and MA part belongs to $\bar{3}$. So, when calculating $\lambda_1\lambda_2$ product, for MS part we calculate matrix element $\langle 6|\lambda_1\lambda_2|6\rangle$ and for MA we put $\langle\bar{3}|\lambda_1\lambda_2|\bar{3}\rangle$.

Table 1: Parts of the flavor wave functions for light scalar nonet $q_1q_2\bar{q}_3\bar{q}_4$, according to quark-pair combinations q_iq_j: two quarks, quark - antiquark and two antiquarks.

q_iq_j	σ	f_0	a_0^+	a_0^0	a_0^-	κ^+	κ^0	$\bar{\kappa}^0$	κ^-
q_1q_2	ud	$\frac{1}{\sqrt{2}}(us+ds)$	us	$\frac{1}{\sqrt{2}}(us-ds)$	ds	ud	ud	us	ds
$q_1\bar{q}_3$	$u\bar{u}$	$\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$	$u\bar{d}$	$\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})$	$d\bar{u}$	$u\bar{d}$	$u\bar{u}$	$u\bar{u}$	$d\bar{u}$
$q_1\bar{q}_4$	$u\bar{d}$	$\frac{1}{\sqrt{2}}(u\bar{s}+d\bar{s})$	$u\bar{s}$	$\frac{1}{\sqrt{2}}(u\bar{s}-d\bar{s})$	$d\bar{s}$	$u\bar{d}$	$u\bar{s}$	$u\bar{d}$	$d\bar{d}$
$q_2\bar{q}_3$	$d\bar{u}$	$\frac{1}{\sqrt{2}}(s\bar{u}+s\bar{d})$	$s\bar{d}$	$\frac{1}{\sqrt{2}}(s\bar{u}-s\bar{d})$	$s\bar{u}$	$d\bar{d}$	$d\bar{u}$	$s\bar{u}$	$s\bar{u}$
$q_2\bar{q}_4$	$d\bar{d}$	$\frac{1}{\sqrt{2}}(s\bar{s}+s\bar{s})$	$s\bar{s}$	$\frac{1}{\sqrt{2}}(s\bar{s}-s\bar{s})$	$s\bar{s}$	$d\bar{s}$	$u\bar{s}$	$s\bar{d}$	$s\bar{d}$
$\bar{q}_3\bar{q}_4$	$\bar{u}\bar{d}$	$\frac{1}{\sqrt{2}}(\bar{u}\bar{s}+\bar{d}\bar{s})$	$\bar{d}\bar{s}$	$\frac{1}{\sqrt{2}}(\bar{u}\bar{s}-\bar{d}\bar{s})$	$\bar{u}\bar{s}$	$\bar{d}\bar{s}$	$\bar{u}\bar{s}$	$\bar{u}\bar{d}$	$\bar{u}\bar{d}$

Thus, for the quark-quark, quark-antiquark and antiquark-antiquark pairs, we obtained the following $\lambda_i\lambda_j$ products:

1. quark-quark:
   MS: $\lambda_1\lambda_2 = \langle 6\,|\lambda_1\lambda_2|\,6\rangle = \frac{4}{3}$,
   MA: $\lambda_1\lambda_2 = \langle \bar{3}\,|\lambda_1\lambda_2|\,\bar{3}\rangle = -\frac{8}{3}$
2. quark-antiquark:
   MS: $\lambda_1\lambda_3 = \lambda_1\lambda_4 = \lambda_2\lambda_3 = \lambda_2\lambda_4 = \langle 8\,|\lambda_1\lambda_3|\,8\rangle = \frac{2}{3}$,
   MA: $\lambda_1\lambda_3 = \lambda_1\lambda_4 = \lambda_2\lambda_3 = \lambda_2\lambda_4 = \langle 8\,|\lambda_1\lambda_3|\,8\rangle = \frac{2}{3}$
3. antiquark-antiquark:
   MS: $\lambda_3\lambda_4 = \left\langle \bar{6}\,\middle|\bar{\lambda}_3\bar{\lambda}_4\middle|\,\bar{6}\right\rangle = \frac{4}{3}$,
   MA: $\lambda_3\lambda_4 = \left\langle 3\,\middle|\bar{\lambda}_3\bar{\lambda}_4\middle|\,3\right\rangle = -\frac{8}{3}$

The products $\lambda_i\lambda_j$ and $\sigma_i\sigma_j$ for MS and MA multiplets are given in Table 2.

We calculate masses by formulas (4) and (5) from Borka Jovanović (2007). Thus, GR HFI contribution to light tetraquark masses is:

$$\begin{aligned} m_{\nu,GR} = \quad & - \quad C_\chi \langle \nu \uparrow| \frac{\vec{\sigma}_1\vec{\sigma}_2}{m_1m_2}(\lambda_1\lambda_2) - \vec{\sigma}_1\vec{\sigma}_3(\lambda_1\lambda_3)\left(\frac{1}{m_1m_3}+\frac{1}{m_2m_3}+\frac{1}{m_1m_4}+\frac{1}{m_2m_4}\right) \\ & + \quad \frac{\vec{\sigma}_3\vec{\sigma}_4}{m_3m_4}(\lambda_3\lambda_4)\,|\nu \uparrow\rangle, \end{aligned} \tag{1}$$

where we apply ν using equality: $|\nu \uparrow\rangle = \frac{1}{\sqrt{2}}\,|\nu_{MA} + \nu_{MS}\rangle$.

Table 2: The products $\lambda_i\lambda_j$ (Gell-Mann matrices for flavor SU(3)) and $\sigma_i\sigma_j$ (Pauli spin matrices) for MS and MA multiplets: 8_{MS}, 8_{MA} and 1_{MS}, 1_{MA}.

symmetry	q_iq_j	$q_i\bar{q}_j$	$\bar{q}_i\bar{q}_j$
MS	$\lambda_1\lambda_2 = \frac{4}{3}$	$\lambda_1\lambda_3 = \lambda_1\lambda_4 = \lambda_2\lambda_3 = \lambda_2\lambda_4 = \frac{2}{3}$	$\lambda_3\lambda_4 = \frac{4}{3}$
	$\vec{\sigma}_1\vec{\sigma}_2 = 1$	$\vec{\sigma}_1\vec{\sigma}_3 = \vec{\sigma}_1\vec{\sigma}_4 = \vec{\sigma}_2\vec{\sigma}_3 = \vec{\sigma}_2\vec{\sigma}_4 = -2$	$\vec{\sigma}_3\vec{\sigma}_4 = 1$
MA	$\lambda_1\lambda_2 = -\frac{8}{3}$	$\lambda_1\lambda_3 = \lambda_1\lambda_4 = \lambda_2\lambda_3 = \lambda_2\lambda_4 = \frac{2}{3}$	$\lambda_3\lambda_4 = -\frac{8}{3}$
	$\vec{\sigma}_1\vec{\sigma}_2 = -3$	$\vec{\sigma}_1\vec{\sigma}_3 = \vec{\sigma}_1\vec{\sigma}_4 = \vec{\sigma}_2\vec{\sigma}_3 = \vec{\sigma}_2\vec{\sigma}_4 = 0$	$\vec{\sigma}_3\vec{\sigma}_4 = -3$

When we put values for $\sigma_i\sigma_j$ and $\lambda_i\lambda_j$ (see Table 2) into mass equation (1), we get the following GR HFI contributions for MS and MA multiplets:

$$
\begin{aligned}
m_{\nu,GR,MS} &= -\frac{4}{3}C_\chi \langle\nu_{MS}| \frac{1}{m_1m_2} + \left(\frac{1}{m_1m_3} + \frac{1}{m_2m_3} + \frac{1}{m_1m_4} + \frac{1}{m_2m_4}\right) + \frac{1}{m_3m_4} |\nu_{MS}\rangle, \\
m_{\nu,GR,MA} &= -8C_\chi \langle\nu_{MA}| \frac{1}{m_1m_2} + \frac{1}{m_3m_4} |\nu_{MA}\rangle.
\end{aligned} \tag{2}
$$

The total GR HFI contribution has the following form:

$$
m_{\nu,GR} = \frac{1}{2}\left(m_{\nu,GR,MS} + m_{\nu,GR,MA}\right), \tag{3}
$$

and thus, from eqs. (2) and (3) we derive:

$$
\begin{aligned}
m_{\nu,GR} = -2C_\chi \Bigg(\frac{1}{3}\langle\nu_{MS}| \frac{1}{m_1m_2} + \frac{1}{m_1m_3} + \frac{1}{m_2m_3} + \frac{1}{m_1m_4} + \frac{1}{m_2m_4} + \frac{1}{m_3m_4} |\nu_{MS}\rangle \\
+ 2\langle\nu_{MA}| \frac{1}{m_1m_2} + \frac{1}{m_3m_4} |\nu_{MA}\rangle\Bigg).
\end{aligned} \tag{4}
$$

Taking into account flavor wave functions ν (see e.g. Jaffe (1977) and Brito et al. (2005)), we finally obtain:

$$
m_{\nu,GR} = -\frac{2}{3}C_\chi \left(\frac{7}{m_1m_2} + \frac{1}{m_1m_3} + \frac{1}{m_2m_3} + \frac{1}{m_1m_4} + \frac{1}{m_2m_4} + \frac{7}{m_3m_4}\right). \tag{5}
$$

Here we gave complete method how to calculate theoretical masses. The values of these masses are given in Borka Jovanović (2007):

$$m_\sigma = 4m_u - 12C_\chi \frac{1}{m_u^2}$$
$$m_{f0} = 2m_u + 2m_s - \frac{2}{3}C_\chi \left(\frac{1}{m_u^2} + \frac{1}{m_s^2} + \frac{16}{m_u m_s} \right)$$
$$m_{a0} = m_{f0}$$
$$m_\kappa = 3m_u + m_s - 6C_\chi \left(\frac{1}{m_u^2} + \frac{1}{m_u m_s} \right)$$

Using values from Table VI in Borka Jovanović (2007), we show the spectrum in Fig. 4. We used the constituent quark masses: m_u = 311 MeV, m_s = 487 MeV and the constant C_χ = 6.0 $\times 10^6$ MeV^3.

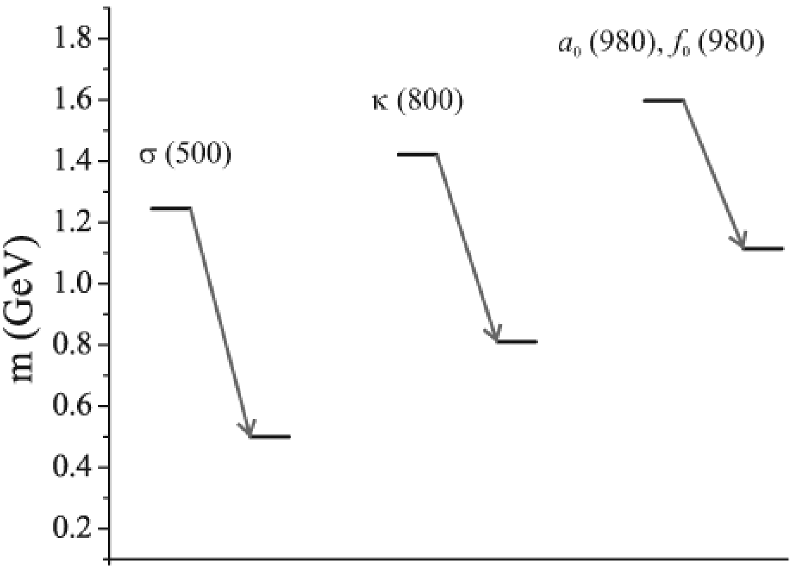

Figure 4: Light tetraquark mass spectrum without (left column) and with (right column) GR HFI, both with SU_F symmetry breaking.

4. CONCLUSIONS

For light tetraquark nonet, which form an SU(3) flavor nonet, we explained whole procedure for finding product of Pauli and Gell-Mann matrices. Also, we showed how to obtain GR HFI contribution to the masses of MS and MA multiplets, and after that how to obtain total theoretical masses. We study mass spectra of light scalar tetraquarks using GR HFI in schematic approximation (two-particle interaction). There are uncertainties in calculating masses because we use the model with schematic interaction.

We considered the lightest scalars σ (500) (one state), f_0 (980) (one state), κ (800) (four states) and the a_0 (980) (three states) as four quark states $qq\bar{q}\bar{q}$ and showed detailed method for calculation of their theoretical masses with GR HFI contribution.

For their values see Borka Jovanović (2007). We showed that these mesons fit well in the tetraquark scheme. From the Fig. 4 we can see that GR interaction significantly reduces tetraquark masses.

Tetraquark masses are calculated using values of constituent quarks obtained from equations of meson masses. As can be seen from Table VI in Borka Jovanović (2007), GR HFI significantly reduces the theoretical masses of the light scalar tetraquarks and brings them closer to their experimental masses. This fact confirms the conclusion from Brito et al. (2005) about the tetraquark nature of these light scalars.

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