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STATUS OF THE MINIMAL SUPERSYMMETRIC SO(10)

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Abstract. We investigate status of the minimal supersymmetric SO(10) model in a low and split supersymmetry regime. To demonstrate viability of the theory we present a good fit of the fermion masses and their mixing parameters. The solution needs a strongly split supersymmetry breaking scenario with gauginos and higgsinos around 10^2 TeV, sfermions close to 10^{14} GeV and a low GUT scale of around 6×10^{15} GeV. It predicts fast proton decay rates through SO(10) type of $d = 6$ operators, hierarchical neutrino masses and large leptonic mixing angle $\sin\theta_{13} \approx 0.1$.

1. MINIMAL SO(10) THEORY

The supersymmetric renormalizable SO(10) with three generations of matter 16_F and the Higgs representations of 210_H, 126_H, $\overline{126}_H$ and 10_H(Clark et al. 1982, Aulakh & Mohapatra, 1982) has only 26 free parameters on top of the usual soft supersymmetry (SUSY) breaking terms (Aulakh et al., 2003) when all renormalizable terms allowed by symmetry are taken into account. It can thus be considered the minimal renormalizable grand unified theory (GUT) model and as such has been exposed to unprecedented level of scrutiny. (For a review of various theoretical and numerical studies of the model see Aulakh, 2005; Aulakh & Garg, 2005). Its superpotential $W(= W_Y + W_H)$ reads

$$
\begin{aligned}
W_H &= \frac{m}{4!}210_H^2 + \frac{\lambda}{4!}210_H^3 + \frac{M}{5!}126_H\overline{126}_H + \frac{\eta}{5!}126_H 210_H \overline{126}_H \\
&\quad + m_H {10_H}^2 + \frac{1}{4!}210_H 10_H(\alpha 126_H + \overline{\alpha}\,\overline{126}_H)\,, \qquad (1)\\
W_Y &= 16_F(Y_{10}10_H + Y_{\overline{126}}\overline{126}_H)16_F\,, \qquad (2)
\end{aligned}
$$

where Y_{10} and $Y_{\overline{126}}$ are the two complex symmetric Yukawa matrices. W_H contains terms responsible for the breaking of SO(10) all the way down to the SM gauge group whereas W_Y generates masses and mixing parameters of the Standard Model (SM) matter fields.

The exact mass spectrum of all fields that is needed for the consistent renormalization group equation running of gauge and Yukawa coupling constants has been laboriously established over the years. In what follows we will use notation advocated in Ref. (Bajc et al., 2006). After a required fine-tuning, it depends only on nine real parameters: m, α, $\overline{\alpha}$, $|\lambda|$, $|\eta|$, $\phi = \arg(\lambda) = -\arg(\eta)$, $x = \mathrm{Re}(x) + i\mathrm{Im}(x)$ and g_{GUT}. Here, g_{GUT} is the gauge coupling constant at the GUT scale while m is the only relevant mass parameter in the superpotential of the theory. The other two mass parameters in W_H, i.e., M and m_H, get traded for m and an additional complex parameter x. The exact connection between x and m, on one side, and M and m_H, on the other side, is specified in Ref. (Bajc et al., 2006). In addition to these parameters there are two more parameters M_{SUSY}—the scale of supersymmetry breaking—and $\tan\beta$—the ratio of up-type and down-type vacuum expectation values of the minimal supersymmetric standard model (MSSM)-like Higgs fields—that enter the running of both Yukawa and gauge couplings. All in all, there are eleven parameters that govern mass spectrum of the theory. In our numerical analysis of gauge coupling unification we treat them all as free parameters. The only simplification we make is to set $\phi = \arg(\lambda) = -\arg(\eta)$ to zero for simplicity.

The exact fermion mass relations that stem from the form of W_Y have also been extracted over the years. These are needed for the fitting of the light fermion masses and mixing parameters at the GUT scale. To illustrate how predictive this theory is with respect to the fermion masses and mixing parameters it is sufficient to go into a basis where one of the two symmetric Yukawa matrices is diagonal and real (three parameters). Then, the other Yukawa matrix will be most general complex symmetric matrix (twelve parameters). There are thus all in all fifteen parameters in the Yukawa sector that must successfully describe all the masses and mixing parameters of the SM fermions that are measured or are to be measured if the theory is to be viable. In other words, the theory must generate seventeen postdictions and five predictions for fermion masses and their mixing parameters based on only fifteen parameters in W_Y.

The last feature can be expressed more formally as follows. There exist two matrix sum-rules (Bajc et al., 2006) in the minimal supersymmetric SO(10)

$$M_u = \frac{N_u}{N_d}\tan\beta[(1+\xi(x))M_d - \xi(x)M_e], \tag{3}$$

$$M_n = \frac{v}{m}\frac{\sin^2\beta}{\cos\beta}\alpha\sqrt{\frac{|\lambda|}{|\eta|}}\frac{N_u^2}{N_d}\left[m_I\, f_I(x) + m_{II}\, f_{II}(x)\right], \tag{4}$$

where $M_{u\,(d)}$ and $M_{e\,(n)}$ are 3×3 complex symmetric mass matrices of up (down) quarks and charged (neutral) leptons, respectively. N_u and N_d as well as all the ratios of polinomials of x, i.e., f_I, f_{II} and ξ, are spelled out in Ref. (Bajc et al., 2006). $v(=174\,\mathrm{GeV}$ at the M_Z scale) is the scale of SU(2) breaking. $m_I = M_e(M_d - M_e)^{-1}M_e - 6\xi M_e + 9\xi^2(M_d - M_e)$ $(m_{II} = M_d - M_e)$ is proportional to the type I (type II) seesaw contribution to the light neutrino mass matrix.

As we said, numerous efforts have been employed in trying to fit the fermion masses in the minimal SO(10) framework. It has been finally suggested that either the neutrino mass scale came out too small or the gauge coupling constants entered the non-perturbative regime (Aulakh, 2005; Aulakh and Garg, 2006; Bertolini et al., 2006). The former issue can be easily understood. Namely, the only mass parameter of the theory m is constrained from below by the proton decay lifetime limits. But, as can be seen from Eq. (4), neutrino mass is directly proportional to m^{-1}. Thus, the larger m is the slower a proton decays and the smaller neutrino mass is. But, neutrino mass cannot be arbitrarily small since the current experimental limit on the heaviest neutrino is $m_3 \geq 0.05\,\mathrm{eV}$. So, what would usually happen was that in order to have sufficiently large proton decay lifetime to be in agreement with experimental observations one would generate too small neutrino mass scale and vice versa. The latter issue, on the other hand, is due to the fact that the minimal SO(10) theory employs very large representations which, even for small threshold corrections, would force gauge coupling to become non-perturbative at sufficiently high scale.

We have been prompted to reinvestigate the status of the minimal SO(10) (Bajc et al., 2008) in view of the following facts. Firstly, in all previous studies the gauge and Yukawa couplings at the GUT scale were obtained from the assumption of a desert from the low energy SUSY scale to the GUT scale. This assumption is not always correct, since, there could be (and usually are) states lying one or more orders of magnitude below the GUT scale that could affect the relevant quantities. Secondly, the unknown values of the soft terms in the MSSM make any prediction of the fermion masses and their mixing parameters impossible. In other words, one would need to know how SUSY is broken and mediated to have a really predictive model. Also, due to the same reason, $d = 5$ proton decay cannot be accurately predicted. We have, for the very first time, systematically scanned the available parameter space to address both these issues simultaneously.

1. 1. LOW SUPERSYMMETRY SCENARIO

Let us first assume low supersymmetry scenario. To demonstrate that there exists a tension between neutrino mass scale and proton decay constraints we concentrate our attention on the type II seesaw contribution. In particular, we focus on the magnitude of the factor that multiplies the mismatch between the down-quark and charged lepton mass matrices, i.e., $M_d - M_e$, and thus controls its overall size

$$F_{II} = \frac{v}{m}\frac{\sin^2\beta}{\cos\beta}\alpha\sqrt{\frac{|\lambda|}{|\eta|}\frac{N_u^2}{N_d}}|f_{II}(x)|. \tag{5}$$

We want it to be as large as possible and phenomenologically allowed. (We will comment on the type I contribution in concrete examples to show that due to gauge coupling unification considerations it does not play a decisive role in establishing the correct neutrino mass scale.) However, as we will see, it will tend to be either too small to produce viable masses or, when it is large enough, the parameters of the model will not allow for a good fit of fermion masses and mixing parameters.

We first note that F_{II} has to be at least of the order of 0.2×10^{-9} if the model is to reach a lower bound on the heaviest light neutrino mass m_3 that is approximately $0.05 \times 10^{-9}\,\mathrm{GeV}$ (Strumia and Vissani, 2006). This is due to the well-known fact that b-τ unification happens rather naturally in supersymmetric theories at the GUT scale. We find this to still be true even after we properly incorporate the intermediate scales in the running of fermion masses in our numerical studies when the SUSY scale is low. Namely, the b and τ masses that are of the order of $1\,\mathrm{GeV}$ defer from each other by not more than $25\,\%$, i.e., $|m_b - m_\tau| \approx 0.25\,\mathrm{GeV}$, for the allowed values of $\tan\beta$. So, only if this is multiplied with $F_{II} > 0.2 \times 10^{-9}$ do we get the heaviest light neutrino mass in the right range, i.e., $m_3 > 0.05 \times 10^{-9}\,\mathrm{GeV}$.

It is actually possible to find an upper bound on F_{II}. Namely, note that the mass of proton decay mediating gauge boson $M_{(X,Y)}$ in the minimal SO(10) reads (Aulakh

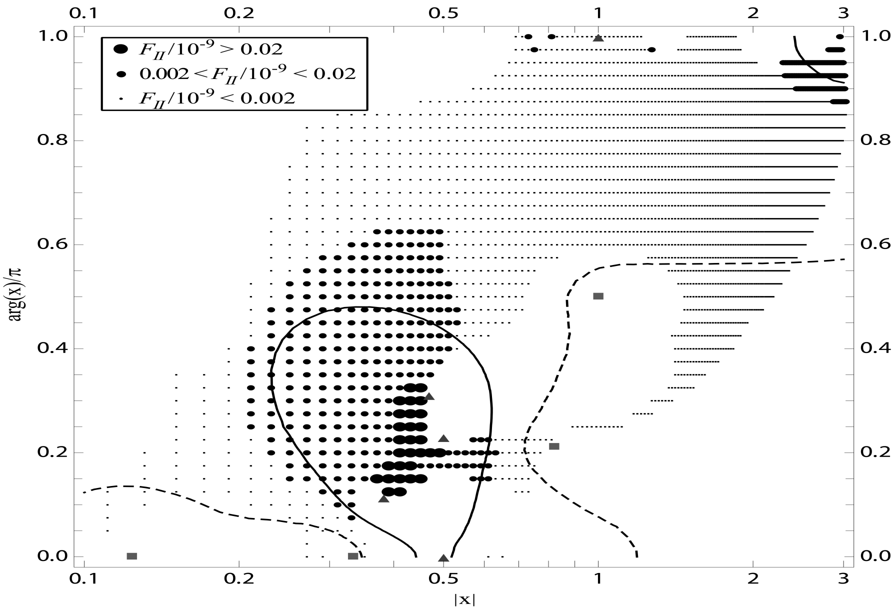

Figure 1: Lines of constant $F_{II}^{\max}/10^{-9}$ for $\eta = \lambda = \alpha = \overline{\alpha} = 1$ and $\tan\beta = 50$ and the corresponding viable gauge coupling unification points. Solid (dashed) line corresponds to $F_{II}^{\max}/10^{-9}$ of 0.02 (0.002). Dots of varying sizes mark x values for which successful one-loop gauge coupling unification takes place for a fixed SUSY scale of 1 TeV. Their size indicates the magnitude of $F_{II}/10^{-9}$. Large squares (triangles) mark the singular points of f_I (f_{II}).

and Girdhar, 2005)

$$M_{(X,Y)} = m\frac{g_{GUT}}{|\lambda|}\sqrt{4\left|\frac{2x^2+x-1}{x-1}\right|^2 + 2\left|\frac{2x(2x^2+x-1)}{(x-1)^2}\right|^2}. \tag{6}$$

If we use proton decay lifetime limits on $M_{(X,Y)}$ due to the $d = 6$ contributions we can establish the lowest phenomenologically viable mass of m and accordingly the highest possible value for F_{II} which we refer to as $F_{II}^{\max}$. Clearly, F_{II} and $F_{II}^{\max}$ do not need to necessarily coincide but one can get an idea if it is possible to have satisfactory neutrino scale or not for a given values of the parameters of the model.

We plot the lines of constant $F_{II}^{\max}$ and indicate the range of values of F_{II} for $\eta = \lambda = \alpha = \overline{\alpha} = 1$, $\tan\beta = 50$ in Fig. to show how close to the right neutrino scale one usually gets. Note that we always run $\tan\beta$ (v) from the SUSY (M_Z) scale to the GUT scale using the relevant equations to perform the numerical fit as well as to evaluate F_{II}. (For definiteness we take $M_{SUSY} = 1$ TeV.) However, whenever we specify $\tan\beta$ throughout this paper, we specify its value at the relevant SUSY scale.

Lines of constant $F_{II}^{\max}/10^{-9}$ correspond to 0.02 (solid line) and 0.002 (dashed line). These contours reflect the actual upper bounds on F_{II} that are in accord with proton decay limits. Clearly, only in a very narrow region $F_{II}^{\max}$ is within a factor of ten away from the preferred value. We can further ask if indeed unification takes place

Table 1: Range of central values of fermion masses in GeV units at the GUT scale when $\eta = \lambda = \alpha = \overline{\alpha} = 1$ and $\tan\beta = 30$ for different values of x where unification takes place. We also present the two-loop level running masses at the GUT scale in the MSSM-like setup when $M_{SUSY} = 1\,\mathrm{TeV}$ and $M_{GUT} = 2 \times 10^{16}\,\mathrm{GeV}$.

	m_u	m_c	m_t	m_d	m_s	m_b	m_e	m_μ	m_τ
min	.00053	.210	86.7	.0012	.0213	1.17	.000341	.0720	1.28
max	.00060	.237	93.2	.0014	.0240	1.29	.000367	.0775	1.37
MSSM	.00050	.198	75.7	.0011	.0202	1.07	.000357	.0754	1.34

in that particular region. The dots in Fig. mark places where successful one-loop gauge coupling unification takes place assuming that the GUT scale M_{GUT} is given by the scale of the lightest proton decay mediating (X, Y) gauge bosons $M_{(X,Y)}$. We plot only the points that are in agreement with the experimental limits on proton decay lifetimes and yield a perturbative unification below the Planck scale. There are three different dot sizes; the largest dots represent unification scenario where $F_{II}/10^{-9} > 0.02$, medium size dots correspond to $0.002 < F_{II}/10^{-9} < 0.02$ and the smallest dots are for $F_{II}/10^{-9} < 0.002$. Here we note that not even one point corresponds to the case where $F_{II}/10^{-9}$ reaches or exceeds 0.2. In other words, we can be rather certain that no successful fermion mass fit with dominant type II scenario for neutrino masses can be found for these particular values of model parameters due to a too small neutrino mass scale. As far as the pure type I contributions are concerned we show in Fig. that all the singular points of f_I, represented with squares, do not overlap with the viable unification points. In other words, whenever type I contribution is potentially significant for light neutrino masses, unification does not happen.

We have already indicated that in all previous studies the gauge and Yukawa couplings at the GUT scale were obtained from the assumption of a desert from the low energy SUSY scale to the GUT scale. Since our numerical analysis takes care of all possible thresholds to the gauge couplings and Yukawa couplings (at one-loop level) we can check how correct that assumption was. In Table 1 we present the ranges of the central values of masses of quarks and charged leptons after we extrapolate them to the GUT scale for the unification points presented in Fig. and confront them with the central values when we assume MSSM-like scenario with $\tan\beta = 30$ and $M_{GUT} = 2 \times 10^{16}\,\mathrm{GeV}$. As one can see the ranges of values we obtain by taking into account threshold corrections do not even bracket the central values of charged fermion masses at the GUT scale of the MSSM-like setup.

Secondly, m is just another parameter in the theory and as such must be allowed to vary as long as the proton decay lifetime constraints are satisfied in order to cover all possible unification scenarios. It actually varies from $4 \times 10^{14}\,\mathrm{GeV}$ to $4 \times 10^{17}\,\mathrm{GeV}$ for unification points shown in Fig. . Thus, fixing m to a particular value as was done in all previous studies would reproduce only a portion of the available parameter space of the theory.

The exact procedure that we use to check for unification of gauge couplings for data in Fig. is as follows. First, we fix $\eta = \lambda = \alpha = \overline{\alpha} = 1$ and then vary x and g_{GUT}, where g_{GUT} is the gauge coupling at the GUT scale. Once all these parameters are given we numerically determine the masses of the superheavy fields, including the gauge ones, in arbitrary units of m. Then we solve the relevant renormalization group equations taking into account the usual one-loop coefficients of β functions of all the thresholds to find m. We ask that the gauge couplings unify within 4% of their values at the GUT scale to reflect the fact that we consider one-loop running only. We take $\alpha_3 = 0.0895$, $\alpha_2 = 0.0326$ and $\alpha_1 = 0.0174$ to be the input values at the SUSY scale of 1 TeV. These reflect the two-loop running effects from M_Z to 1 TeV. We finally check that α_{GUT}, if there is unification for given x, is perturbative, proton decay constraints are satisfied and GUT scale itself is below the Planck scale. To check the proton decay viability we look at $p \to \pi^0 e^+$ channel and the $d = 6$ operator contribution. To accurately evaluate the prediction for $p \to \pi^0 e^+$ we numerically find the relevant short distance coefficients of the $d = 6$ operators taking care of all intermediate scales using the results of Refs. (Buras et al., 1978; Wilczek and Zee, 1979; Ellis at al., 1979; Ibanez and Munoz, 1984; Munoz, 1986) and use g_{GUT}, $M_{(X,Y)}$ and $M_{(X',Y')}$ as deduced from unification.

If we vary η, λ, α, $\overline{\alpha}$ within reasonable range we find values of F_{II} that are large enough and for which there is unification. However, it turns out that the numerical fit of fermion masses and their mixing parameters is not satisfactory. (To be precise, we have found that both the correct neutrino masses and unification of gauge couplings can coexist even in the low energy SUSY framework. However, we then also have unsuccessful fit of charged fermion masses and their mixing parameters.) In this way we can state that low supersymmetric scenario within the framework of the minimal SO(10) model is not viable.

To numerically fit fermion masses we specify the pull, χ_i, for each output value p_i in the following way:

$$\chi_i = \frac{p_i - \tilde{p}_i}{f_i \tilde{p}_i}$$

where $\tilde{p}_i$ are masses and angles which have been run to the GUT scale. f_i are the percentages that specify the errors and are listed in Tables 2 and 3 . The minimization function is then a sum of squared pulls

$$\chi^2 = \sum_{i=1}^{14} \left(\frac{p_i - \tilde{p}_i}{f_i \tilde{p}_i} \right)^2 .$$

We take into account s_{13}^{PMNS} contribution to χ^2 only if s_{13}^{PMNS} exceeds the current experimental bound. All $\tilde{p}_i$ at the M_Z and M_{GUT} are given in Table 4 in columns two and four, respectively.

Table 2: Fitted parameters of the charged fermions and their relative errors.

i	1	2	3	4	5	6	7	8	9	10
p_i	m_u	m_c	m_t	m_d	m_s	m_b	s_{12}^{CKM}	s_{23}^{CKM}	s_{13}^{CKM}	δ_{13}^{CKM}
$f_i(\%)$	30	10	10	35	35	10	7	4	1	20

Table 3: Fitted parameters of the neutral fermions and their relative errors.

i	11	12	13	14
p_i	m_2/m_3	s_{12}^{PMNS}	s_{23}^{PMNS}	s_{13}^{PMNS}
$f_i(\%)$	11	15	10	20

1. 2. SPLIT SUPERSYMMETRY SCENARIO

The minimal SO(10) theory fails to describe the current data on fermion masses and their mixing angles with generic parameters. The primarily reason is that the neutrino scale happens to be right where the fit of fermion masses is wrong and vice versa. We thus resort to a different strategy. We maximize $F_{II}^{\max}$ in such a fashion in terms of x, η, λ, α and $\overline{\alpha}$ in order to make sure not only that $F_{II}^{\max}$ is large enough for reasonable value of $\tan\beta$ to yield the correct neutrino mass scale but that x, η, λ, α and $\overline{\alpha}$ are such that the satisfactory numerical fit of fermion masses is possible. (We vary η, λ, α and $\overline{\alpha}$ in the following range: 0.03–7.) This in turn fixes the value of m and thus sets the overall scale of the theory that is relevant for proton decay in terms of the scale that is relevant for the description of light neutrinos. We then test for gauge coupling unification. This we do by splitting M_{SUSY} into two scales (Giudice and Romanino, 2004) to have more freedom. The first scale stands for the common scale $M_{1/2}$ of gauginos and higgsinos while the second one describes the common scale $M_{\tilde{f}}$ of matter field superpartners. With this we simply solve the three relevant RGEs for $M_{1/2}$, $M_{\tilde{f}}$ and g_{GUT} to insure gauge coupling unification at the one-loop level. Finally, if and when unification works, we run the charged fermion masses, CKM parameters, $\tan\beta$ and v to the GUT scale and perform the numerical fit to check the viability of the theory.

We find a solution that yields both successful unification and satisfactory χ^2 which we explicitly spell out in Table 4. There we also specify our input values for fermions, and CKM and PMNS parameters at M_Z. These M_Z values reflect the results of Ref. (Yao et al., 2006. The numerical fit determines also leptonic angle $\sin\theta_{13}(\equiv s_{13}^{PMNS})$, the only angle in the PMNS matrix that is yet to be determined. Our solution needs a strongly split SUSY breaking scenario with gauginos and higgsinos around 100 TeV, sfermions close to 10^{14} GeV and a low GUT scale of around 6×10^{15} GeV. Note that in this regime the influence of the unknown values of the soft terms on predicted values of the fermion masses and mixing parameters is negligible. Moreover, the problematic $d=5$ operators are negligible altogether.

Table 4: Input and output parameters of the numerical fit at the GUT scale with χ^2/d.o.f. = 9.6/13, $m_3 = 0.049\,\mathrm{eV}$, $m_2 = 0.0087\,\mathrm{eV}$ and $m_1 = 0.0012\,\mathrm{eV}$. Unification takes place for $|x| = 0.109$, $arg(x)/\pi = 0.52$, $\alpha = 1.26788$, $\overline{\alpha} = 7$, $\eta = 6.54112$, $\lambda = 0.03$, $M_{1/2} = 1.5 \times 10^5$ GeV, $M_{\tilde{f}} = 9.0 \times 10^{13}$ GeV, $M_{GUT} = 5.8 \times 10^{15}$ GeV and $g_{GUT} = 1.3$. The input data at M_Z are also given in the second column.

	DATA @ M_Z	FIT @ M_{GUT}	RGE @ M_{GUT}	pull @ M_{GUT}
m_e(MeV)	0.4866613	0.42279	0.42279	—
m_μ(GeV)	0.10273	0.08925	0.08925	—
m_τ(GeV)	1.746	1.534	1.534	—
m_u(MeV)	1.6	0.56	0.54	+0.07
m_c(GeV)	0.628	0.218	0.214	+0.18
m_t(GeV)	171.5	72.0	67.4	+0.68
m_d(MeV)	3.5	0.24	1.2	−2.31
m_s(MeV)	62	27.2	21.8	+0.70
m_b(GeV)	2.89	0.917	0.910	+0.08
s_{12}^{CKM}	0.2272	0.2423	0.2272	+0.95
s_{23}^{CKM}	0.0422	0.0478	0.0474	+0.20
s_{13}^{CKM}	0.00399	0.00447	0.00448	−0.02
δ^{CKM}	0.995	1.149	0.995	+0.60
s_{12}^{PMNS}	0.50	0.42	0.50	−1.09
s_{23}^{PMNS}	0.60	0.55	0.60	−0.76
s_{13}^{PMNS}	< 0.17	0.103	< 0.17	—
δ^{PMNS}	—	6.2	—	—
m_2/m_3	0.180	0.178	0.180	−0.10

It is now easy to generate predictions for proton decay due to exchange of the gauge bosons. These we show in Table 5.

2. CONCLUSION

We have established that the minimal supersymmetric SO(10) is still viable in view of all experimental constraints. Our solution needs a strongly split SUSY breaking scenario. It predicts fast proton decays through SO(10) type of $d = 6$ operators and the leptonic mixing angle $\sin\theta_{13} \approx 0.1$. Of course, we cannot claim that the above solution is unique. In fact, we have found that both the correct neutrino masses and unification of gauge couplings can coexist even in the low energy SUSY framework. The unsuccessful fit of charged fermion masses and mixing parameters with low energy SUSY that we then find might be corrected by loops due to soft SUSY breaking terms. Both our results—in the strongly split SUSY and low energy SUSY regime—shed completely new light on the status of the minimal SO(10).

Table 5: Predictions for partial lifetimes for the most significant p decay modes for the fit shown in Table 4 and current bounds expressed in 10^{33} years units. For the matrix element we take $\alpha = 0.003\,\mathrm{GeV}^3$. The predicted associated branching ratios are also shown.

	Partial mean life (10^{33} years)		
p decay modes	Model predictions	Lifetime bounds	Fraction (Γ_i/Γ)
$p \to \pi^0 e^+$	12.1	> 8.2	44.3 %
$p \to \pi^0 \mu^+$	518.7	> 6.6	1.0 %
$p \to K^0 e^+$	3277.2	> 1.0	0.2 %
$p \to K^0 \mu^+$	113.4	> 1.3	4.7 %
$p \to \eta e^+$	1765.6	$> .313$	0.3 %
$p \to \eta \mu^+$	75499.3	$> .126$	0.007 %
$p \to K^+ \bar{\nu}$	1190.2	> 2.3	0.5%
$p \to \pi^+ \bar{\nu}$	10.9	$> .025$	49.1 %

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