Publ. Astron. Obs. Belgrade No. 88 (2010), 133 - 141 *Contributed paper*

D-DEFORMED WESS-ZUMINO MODEL AND ITS RENORMALIZABILITY PROPERTIES

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Abstract. Using the twist formalism we analyze one type of deformation of the superspace. The twist we use to deform the $N = 1$ SUSY Hopf algebra is non-hermitian and it is given in terms of the covariant derivatives D_α. A SUSY invariant deformation of the Wess-Zumino action is constructed and a preliminary analysis of its renormalizability properties is done. As expected, there is no renormalization of mass and no tadpole diagrams appear.

1. INTRODUCTION

Having in mind problems which physics encounters at small scales (high energies), in recent years many attempts were made to combine supersymmetry (SUSY) with non-commutative geometry. Different models were constructed, see the list of references. Some of these models emerge naturally as low energy limits of string theories in a background with a constant Neveu-Schwarz two form and/or a constant Ramond-Ramond two form.

One way to deform symmetries of a classical (commutative) field theory is by using the twist formalism. The action of the twist operator on the Hopf algebra of classical symmetry results in a twisted (deformed) symmetry Hopf algebra. In that way the classical symmetry is deformed. At the same time, the inverse of the twist operator introduces a new product on the space of functions. This product is associative, but noncommutative and it encodes the information about deformation (noncommutativity) of the classical (commutative) spacetime.

It is interesting to see if deforming by a twist of symmetries spoils some of the renormalizability properties of SUSY invariant theories. Therefore, in this paper we analyze a simple model. The twist is given by

$$\mathcal{F} = e^{\frac{1}{2}C^{\alpha\beta}D_\alpha\otimes D_\beta}, \tag{1}$$

where $C^{\alpha\beta} = C^{\beta\alpha} \in \mathbb{C}$ is a complex constant matrix and $D_\alpha = \partial_\alpha - i\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_m$ are SUSY covariant derivatives. Following the method developed in our previous publications in the next sections we introduce the deformation and the $\star$-product which follows from it. Due to our choice of the twist the coproduct of SUSY transformations remains undeformed, leading to the undeformed Leibniz rule. Being interested in a deformation of the Wess-Zumino model, we discuss chiral fields and their products. The product of two chiral fields is not a chiral field and we have to use projectors to separate chiral and antichiral parts. All possible invariants are included in the the deformed Wess-Zumino action. Using the background field method we then analyze two point functions and their divergences. Finally, we give some comments and compare our results with the results already present in the literature.

2. D-DEFORMATION

The superspace is generated by x, θ and $\bar\theta$ coordinates that fulfill

$$[x^m, x^n] = [x^m, \theta^\alpha] = [x^m, \bar\theta_{\dot\alpha}] = 0, \quad \{\theta^\alpha, \theta^\beta\} = \{\bar\theta_{\dot\alpha}, \bar\theta_{\dot\beta}\} = \{\theta^\alpha, \bar\theta_{\dot\alpha}\} = 0, \tag{2}$$

with $m = 0, \dots 3$ and $\alpha, \beta = 1, 2$. These coordinates we call supercoordinates, to x^m we refer as bosonic and to θ^α and $\bar\theta_{\dot\alpha}$ we refer as fermionic coordinates. Also, $x^2 = x^m x_m = -(x^0)^2 + (x^1)^2 + (x^2)^2 + (x^3)^2$, that is we work in Minkowski space-time with the metric $(-, +, +, +)$.

Every function of supercoordinates can be expanded in power series in θ and $\bar\theta$. Superfields form a subalgebra of the algebra of functions on superspace. For a general superfield $F(x, \theta, \bar\theta)$ expansion in θ and $\bar\theta$ reads

$$\begin{aligned} F(x,\theta,\bar\theta) \;&= f(x) + \theta\phi(x) + \bar\theta\bar\chi(x) + \theta\theta m(x) + \bar\theta\bar\theta n(x) + \theta\sigma^m\bar\theta v_m \\ &\quad + \theta\theta\bar\theta\bar\lambda(x) + \bar\theta\bar\theta\theta\varphi(x) + \theta\theta\bar\theta\bar\theta d(x). \end{aligned} \tag{3}$$

All higher powers of θ and $\bar\theta$ vanish since these coordinates are Grassmanian.

Under the infinitesimal SUSY transformation a general superfield transforms as follows

$$\delta_\xi F = \big(\xi Q + \bar\xi\bar Q\big)F, \tag{4}$$

where ξ and $\bar\xi$ are constant anticommuting parameters and Q and $\bar Q$ are SUSY generators

$$Q_\alpha = \partial_\alpha - i\sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_m, \quad \bar Q^{\dot\alpha} = \bar\partial^{\dot\alpha} - i\theta^\alpha\sigma^m_{\alpha\dot\beta}\varepsilon^{\dot\beta\dot\alpha}\partial_m. \tag{5}$$

Note that the connection between SUSY covariant derivatives and SUSY generators is given by

$$Q_\alpha = D_\alpha - 2i\sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_m, \quad \bar Q_{\dot\alpha} = \bar D_{\dot\alpha} + 2i\theta^\alpha\sigma^m_{\alpha\dot\beta}\varepsilon^{\dot\beta\dot\alpha}\partial_m. \tag{6}$$

Transformations (4) close in the algebra with the bosonic partial derivatives $\partial_m = \frac{\partial}{\partial x^m}$

$$[\delta_\xi, \delta_\eta] = -2i(\eta\sigma^m\bar\xi - \xi\sigma^m\bar\eta)\partial_m. \tag{7}$$

The product of two superfields is a superfield again

$$\delta_\xi(F\cdot G) = \big(\xi Q + \bar\xi\bar Q\big)(F\cdot G) = (\delta_\xi F)\cdot G + F\cdot(\delta_\xi G). \tag{8}$$

We said in the previous section that a well defined way of deforming symmetries of a theory is via the twist formalism, see references at the end of the text. The deformation is intoduced via a bidiferential operator $\mathcal{F}$ which is acting on the Hopf algebra of symmetries. In this procedure the algebra relations remain unchanged while the comultiplication (which coresponds to the Leibniz rule of the symmetry generators) in general changes.

Let us now apply this formalism in order to introduce a deformation of the Hopf algebra of infinitesimal SUSY transformations. We choose the twist $\mathcal{F}$ in the following way

$$\mathcal{F} = e^{\frac{1}{2}C^{\alpha\beta}D_\alpha\otimes D_\beta}, \tag{9}$$

with the complex constant matrix $C^{\alpha\beta} = C^{\beta\alpha} \in \mathbb{C}$. Note that this twist is not hermitian, $\mathcal{F}^* \neq \mathcal{F}$. The usual complex conjugation is denoted by "$*$". The Hopf algebra of SUSY transformation does not change since

$$\{Q_\alpha, D_\beta\} = \{\bar{Q}_{\dot\alpha}, D_\beta\} = 0. \tag{10}$$

and is given by

- algebra

$$\begin{aligned}\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0, \quad \{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2i\sigma^m_{\ \alpha\dot\beta}\partial_m,\\ [\partial_m, \partial_n] = [\partial_m, Q_\alpha] = [\partial_m, \bar{Q}_{\dot\alpha}] = 0.\end{aligned} \tag{11}$$

- coproduct

$$\begin{aligned}\Delta Q_\alpha = Q_\alpha \otimes 1 + 1 \otimes Q_\alpha, \quad \Delta\bar{Q}_{\dot\alpha} = \bar{Q}_{\dot\alpha} \otimes 1 + 1 \otimes \bar{Q}_{\dot\alpha},\\ \Delta\partial_m = \partial_m \otimes 1 + 1 \otimes \partial_m.\end{aligned} \tag{12}$$

- counit and antipode

$$\begin{aligned}&\varepsilon(Q_\alpha) = \varepsilon(\bar{Q}_{\dot\alpha}) = \varepsilon(\partial_m) = 0,\\ &S(Q_\alpha) = -Q_\alpha, \quad S(\bar{Q}_{\dot\alpha}) = -\bar{Q}_{\dot\alpha}, \quad S(\partial_m) = -\partial_m.\end{aligned} \tag{13}$$

This means that the full supersymmetry is preserved[1].

The inverse of the twist (9)

$$\mathcal{F}^{-1} = e^{-\frac{1}{2}C^{\alpha\beta}D_\alpha\otimes D_\beta}, \tag{14}$$

defines the $\star$-product. For two arbitrary superfields F and G the $\star$-product reads

$$\begin{aligned}F \star G &= \mu_\star\{F \otimes G\}\\ &= \mu\{\mathcal{F}^{-1}\, F \otimes G\}\\ &= \mu\{e^{-\frac{1}{2}C^{\alpha\beta}D_\alpha\otimes D_\beta} F \otimes G\}\\ &= F\cdot G - \frac{1}{2}(-1)^{|F|}C^{\alpha\beta}(D_\alpha F)\cdot(D_\beta G)\\ &\quad -\frac{1}{8}C^{\alpha\beta}C^{\gamma\delta}(D_\alpha D_\gamma F)\cdot(D_\beta D_\delta G),\end{aligned} \tag{15}$$

[1]Strictly speaking, the twist given by (9) does not belong to the universal enveloping algebra of the Lie algebra of infinitesimal SUSY transformations. Therefore, to be mathematically correct we should enlarge the algebra (11) by introducing the relations for the operators D_α as well.

where $|F| = 1$ if F is odd (fermionic) and $|F| = 0$ if F is even (bosonic). In the second line the definition of the $\mu_\star$ multiplication is given. No higher powers of $C^{\alpha\beta}$ appear since the derivatives D_α are Grassmanian. The $\star$-product (15) is associative, noncommutative and in the zeroth order in the deformation parameter $C_{\alpha\beta}$ it reduces to the usual pointwise multiplication. One should also note that it is not hermitian,

$$(F \star G)^* \neq G^* \star F^*. \tag{16}$$

The deformed infinitesimal SUSY transformation is defined in the following way

$$\delta^\star_\xi F \quad = \quad \big(\xi Q + \bar\xi\bar Q\big)F. \tag{17}$$

Since the coproduct (12) is undeformed, the usual (undeformed) Leibniz rule follows. Then the $\star$-product of two superfields is again a superfield. Its transformation law is given by

$$\begin{aligned} \delta^\star_\xi(F\star G) &= \big(\xi Q + \bar\xi\bar Q\big)(F\star G) \\ &= (\delta^\star_\xi F)\star G + F\star(\delta^\star_\xi G). \end{aligned} \tag{18}$$

A chiral superfield Φ fulfills $\bar D_{\dot\alpha}\Phi = 0$, where $\bar D_{\dot\alpha} = -\bar\partial_{\dot\alpha} - i\theta^\alpha\sigma^m_{\alpha\dot\alpha}\partial_m$ and $\bar D_{\dot\alpha}$ is related to D_α by the usual complex conjugation. In terms of the component fields the chiral superfield Φ is given by

$$\begin{aligned} \Phi(x,\theta,\bar\theta) &= A(x) + \sqrt{2}\theta^\alpha\psi_\alpha(x) + \theta\theta H(x) + i\theta\sigma^l\bar\theta(\partial_l A(x)) \\ &\quad - \frac{i}{\sqrt{2}}\theta\theta(\partial_m\psi^\alpha(x))\sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha} + \frac{1}{4}\theta\theta\bar\theta\bar\theta(\Box A(x)). \end{aligned} \tag{19}$$

The $\star$-product of two chiral fields reads

$$\begin{aligned} \Phi\star\Phi &= \Phi\cdot\Phi - \frac{1}{8}C^{\alpha\beta}C^{\gamma\delta}D_\alpha D_\gamma\Phi D_\beta D_\delta\Phi \\ &= \Phi\cdot\Phi - \frac{1}{32}C^2(D^2\Phi)(D^2\Phi) \\ &= A^2 - \frac{C^2}{2}H^2 + 2\sqrt{2}A\theta^\alpha\psi_\alpha \\ &\quad -i\sqrt{2}C^2 H\bar\theta_{\dot\alpha}\bar\sigma^{m\dot\alpha\alpha}(\partial_m\psi_\alpha) + \theta\theta\Big(2AH - \psi\psi\Big) \\ &\quad +C^2\bar\theta\bar\theta\Big(-H\Box A + \frac{1}{2}(\partial_m\psi)\sigma^m\bar\sigma^l(\partial_l\psi)\Big) \\ &\quad +i\theta\sigma^m\bar\theta\Big(\partial_m(A^2) + C^2 H\partial_m H\Big) \\ &\quad +i\sqrt{2}\theta\theta\bar\theta_{\dot\alpha}\bar\sigma^{m\dot\alpha\alpha}\big(\partial_m(\psi_\alpha A)\big) \\ &\quad +\frac{\sqrt{2}}{2}\bar\theta\bar\theta C^2(-H\theta\Box\psi + \theta\sigma^m\bar\sigma^n\partial_n\psi\partial_m H) \\ &\quad +\frac{1}{4}\theta\theta\bar\theta\bar\theta(\Box A^2 - \frac{1}{2}C^2\Box H^2), \end{aligned} \tag{20}$$

where $C^2 = C^{\alpha\beta}C^{\gamma\delta}\varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta}$. Due to the $\bar\theta$, $\bar\theta\bar\theta$ and the $\theta\bar\theta\bar\theta$ terms (20) is not a chiral field. Therefore we decompose the $\star$-products of chiral fields into their irreducible components by using the projectors to antichiral, chiral and transfers components

$$P_1 = \frac{1}{16}\frac{D^2\bar{D}^2}{\Box}, \quad P_2 = \frac{1}{16}\frac{\bar{D}^2 D^2}{\Box}, \quad P_T = -\frac{1}{8}\frac{D\bar{D}^2 D}{\Box}. \tag{21}$$

Using these irreducible components, we construct all possible terms that under the deformed SUSY transformations (17) transform covariantly. Here we just list all covariant terms. One can check explicitly that all of them indeed transform covariantly under (17).

$$I_1 = P_2(\Phi\star\Phi)\Big|_{\theta\theta} = 2AH - \psi\psi, \tag{22}$$

$$I_2 = P_1(\Phi\star\Phi)\Big|_{\bar\theta\bar\theta} = -C^2\Big(H\Box A - \frac{1}{2}(\partial_m\psi)\sigma^m\bar\sigma^n(\partial_n\psi)\Big), \tag{23}$$

$$I_3 = P_2(P_2(\Phi\star\Phi)\star\Phi)\Big|_{\theta\theta} = 3(A^2H - A\psi\psi), \tag{24}$$

$$\begin{aligned} I_4 &= P_1(P_2(\Phi\star\Phi)\star\Phi)\Big|_{\bar\theta\bar\theta} \\ &= C^2\Big(- AH\Box A - \frac{1}{2}H\Box A^2 \\ &\quad + \frac{1}{2}\psi\psi\Box A + \partial_m(A\psi)\sigma^m\bar\sigma^n(\partial_n\psi)\Big), \end{aligned} \tag{25}$$

$$\begin{aligned} I_7 &= P_2(P_1(\Phi\star\Phi)\star\Phi)\Big|_{\theta\theta\bar\theta\bar\theta} \\ &= -\frac{C^2}{16}\Big(A\Box H^2 + 5H^2\Box A - 4H(\partial_m\psi)\sigma^m\bar\sigma^l(\partial_l\psi) + 2\psi\sigma^m\bar\sigma^l\partial_m(H(\partial_l\psi))\Big). \end{aligned} \tag{26}$$

Then our deformation of the commutative Wess-Zumino model is given by

$$\begin{aligned} L &= \Phi^+\star\Phi\Big|_{\theta\theta\bar\theta\bar\theta} + \Big[\frac{m}{2}\Big(P_2(\Phi\star\Phi)\Big|_{\theta\theta} + aP_1(\Phi\star\Phi)\Big|_{\bar\theta\bar\theta}\Big) \\ &\quad + \frac{\lambda}{3}\Big(P_2(P_2(\Phi\star\Phi)\star\Phi)\Big|_{\theta\theta} + bP_1(P_2(\Phi\star\Phi)\star\Phi)\Big|_{\bar\theta\bar\theta} \\ &\quad + 2c(P_1+P_2)(P_1(\Phi\star\Phi)\star\Phi)\Big|_{\theta\theta\bar\theta\bar\theta}\Big) + \text{ c.c. }\Big]. \end{aligned} \tag{27}$$

Terms $P_1(P_1(\Phi\star\Phi)\star\Phi)\Big|_{\theta\theta\bar\theta\bar\theta}$ and $P_2(P_1(\Phi\star\Phi)\star\Phi)\Big|_{\theta\theta\bar\theta\bar\theta}$ are equal up to a total derivative term and are therefore included with the same coefficient.

3. RENORMALIZABILITY PROPERTIES OF THE MODEL

The action following from (27) written in component fields reads

$$\begin{aligned} S &= \int \mathrm{d}^4x \Big\{ A^*\Box A + i\partial_m\bar\psi\bar\sigma^m\psi + H^*H \\ &+m(AH - \frac{1}{2}\psi\psi) + m(A^*H^* - \frac{1}{2}\bar\psi\bar\psi) \\ &+\lambda(A^2H - A\psi\psi) + \lambda\big((A^*)^2H^* - A^*\bar\psi\bar\psi\big) \\ &+\Big[C^2\Big(ma_1(\frac{1}{2}\psi\Box\psi - H\Box A) + \lambda a_2(-AH\Box A - \frac{1}{2}H(\Box A^2) \\ &+\frac{1}{2}\psi\psi(\Box A) + A\psi\Box\psi) + \lambda a_3(-\frac{3}{2}H^2\Box A + \frac{3}{2}H(\partial_m\psi)\sigma^m\bar\sigma^l(\partial_l\psi))\Big) \\ &+ \text{c.c.} \Big]\Big\}. \end{aligned} \tag{28}$$

The coefficients a, b and c are related to a_1, a_2 and a_3: $a/2 = a_1$, $b/3 = a_2$ and $c = a_3$. Note that (28) is the full action, i.e. no higher order terms in the deformation parameter $C^{\alpha\beta}$ appear.

Having the action of our deformed model at hand, we now investigate some of the renormalizability properties of our model. We use the background field method and the dimensional reduction and calculate the divergent part of the effective action up to second order in fields. To start with, we rewrite the deformed action (28) introducing the real fields S, P, E and G as

$$A = \frac{S+iP}{\sqrt{2}}, \quad H = \frac{E+iG}{\sqrt{2}} \tag{29}$$

and the Majorana spinor[2]

$$\psi_M = \begin{pmatrix}\psi_\alpha \\ \bar\psi^{\dot\alpha}\end{pmatrix}. \tag{30}$$

In order to simplify our calculation we will assume[3] that $C^2 - \bar C^2 = 0$.

With all this and introducing $g = \frac{\lambda}{\sqrt{2}}$ the action (28) becomes

$$S = S_0 + S_2$$

[2]The index M on Majorana spinors will be omitted in the forthcoming formulas.

[3]The deformation parameter $C_{\alpha\beta}$ can be written in the following way

$$C_{\alpha\beta} = K_{ab}(\sigma^{ab}\varepsilon)_{\alpha\beta}, \quad \bar C_{\dot\alpha\dot\beta} = K^*_{ab}(\varepsilon\bar\sigma^{ab})_{\dot\alpha\dot\beta}. \tag{31}$$

Since K_{ab} is a self dual tensor we write it as

$$K_{ab} = \kappa_{ab} + \frac{i}{2}\epsilon_{abcd}\kappa^{cd}, \tag{32}$$

where κ_{ab} is a real antisymmetric tensor. In this way we obtain

$$C^2 + \bar C^2 = 4\kappa_{ab}\kappa^{ab} \tag{33}$$
$$C^2 - \bar C^2 = 2i\epsilon_{abcd}\kappa^{ab}\kappa^{cd}. \tag{34}$$

It is always possible to choose $C^2 - \bar C^2 = 0$ by setting $\kappa_{0i} = 0$.

where

$$\begin{aligned} S_0 &= \int \mathrm{d}^4x\,\Big\{\frac{1}{2}S\Box S + \frac{1}{2}P\Box P - \frac{1}{2}(i\bar\psi\gamma^m\partial_m\psi + m\bar\psi\psi) + \frac{1}{2}(E^2+G^2) \\ &\quad + m(SE - PG) - gS\bar\psi\psi + gP\bar\psi\gamma^5\psi \\ &\quad + g(ES^2 - EP^2 - 2SPG)\Big\}, \qquad (35) \\ S_2 &= C^2\int \mathrm{d}^4x\,\Big\{ma_1(\frac{1}{2}\bar\psi\Box\psi - E\Box S + G\Box P) \\ &\quad + ga_2(PG\Box S - SE\Box S + PE\Box P + SG\Box P \\ &\quad - \frac{1}{2}(S^2\Box E - P^2\Box E - 2SP\Box G) + \frac{1}{2}\bar\psi\psi\Box S \\ &\quad - \frac{1}{2}\bar\psi\gamma^5\psi\Box P + \bar\psi\Box\psi S - \bar\psi\gamma^5\Box\psi P) \\ &\quad + \frac{3}{2}ga_3(-E^2\Box S + G^2\Box S + 2EG\Box P - E\partial_m\bar\psi\partial^m\psi \\ &+ G\partial_m\bar\psi\gamma^5\partial^m\psi - 2\bar\psi\Sigma^{mn}\partial_n\psi\partial_m E + 2\bar\psi\Sigma^{mn}\gamma^5\partial_n\psi\partial_m G)\Big\}. \qquad (36) \end{aligned}$$

Next we split the fields into their classical and quantum parts, for example $E \to E + \mathcal{E}$. The action quadratic in quantum fields is

$$S^{(2)} = \frac{1}{2}\begin{pmatrix}\bar\Psi & \mathcal{S} & \mathcal{P} & \mathcal{E} & \mathcal{G}\end{pmatrix} M \begin{pmatrix}\Psi \\ \mathcal{S} \\ \mathcal{P} \\ \mathcal{E} \\ \mathcal{G}\end{pmatrix}, \qquad (37)$$

where $\Psi, \bar\Psi,\ \mathcal{S},\ \mathcal{P},\ \mathcal{E},\ \mathcal{G}$ are quantum fields. The one loop effective action is then

$$\Gamma = \frac{i}{2}\mathrm{STr}\ln\Big[1 + (\Box - m^2)^{-1}MC\Big], \qquad (38)$$

with

$$C = \begin{pmatrix} -i\partial\!\!\!/ + m & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -m & 0 \\ 0 & 0 & 1 & 0 & m \\ 0 & -m & 0 & \Box & 0 \\ 0 & 0 & m & 0 & \Box \end{pmatrix}. \qquad (39)$$

The matrix MC we decompose into three parts

$$MC = N + T + V. \qquad (40)$$

The matrix N is zeroth order in fields and in $C^{\alpha\beta}$, the matrix T is zeroth order in fields but 2nd order in $C^{\alpha\beta}$ and the matrix V is linear in fields and 2nd order in $C^{\alpha\beta}$.

Expanding (38) gives

$$\begin{aligned}\Gamma &= \frac{i}{2}\mathrm{STr}\ln\Big[1+(\Box-m^2)^{-1}(N+T+V)\Big]\\ &= \frac{i}{2}\Big[\mathrm{STr}((\Box-m^2)^{-1}(N+T+V))\\ &\quad -\frac{1}{2}\mathrm{STr}((\Box-m^2)^{-1}N(\Box-m^2)^{-1}N)\\ &\quad -\mathrm{STr}((\Box-m^2)^{-1}N(\Box-m^2)^{-1}(T+V))\\ &\quad +\mathrm{STr}(((\Box-m^2)^{-1}N)^2(\Box-m^2)^{-1}T)\Big]. \end{aligned} \tag{41}$$

The calculation of divergent parts of supertraces is tedious but straightforward and here we only give the results. The details of these calculations and also the explicit form of the matrices N, T and V are given in our previous paper, see the list of references.

Let us observe that in $\mathrm{STr}(K^{-1}NK^{-1}V)$ terms $(\Box S)^2$ and $(\Box P)^2$ appear. Since these terms do not have classical counterparts we take $a_2 = 0$. Then the divergent part of the one loop effective action is finally given by

$$\begin{aligned}\Gamma_1 &= \frac{g^2}{\pi^2\epsilon}\int \mathrm{d}^4x\,\Big[\frac{1}{4}(S\Box S+P\Box P+\bar\psi i\partial\!\!\!/\psi+E^2+G^2)\\ &\quad +\frac{3}{4}a_3C^2m^2(2P\Box G+\bar\psi\Box\psi-2S\Box E)\\ &\quad -C^2a_1m^2(S\Box S+P\Box P-\bar\psi i\partial\!\!\!/\psi+E^2+G^2)\Big]. \end{aligned} \tag{42}$$

4. DISCUSSION

Let us now discuss the one loop renormalizability properties of our model. To cancel the divergences we have to add counterterms to the classical action

$$S_B = S_0 + S_2 - \Gamma_1. \tag{43}$$

In this way we obtain the bare action S_B. It is important to note that the term I_7 in the classical action (28) produces the divergences proportional to the term I_2 so both of them are needed in order to absorb the divergences in the effective action. From the form of the bare Lagrangian we see that all fields in the theory are renormalized in the same way:

$$S_0 = \sqrt{Z}S,\ P_0 = \sqrt{Z}P,\ \psi_0 = \sqrt{Z}\psi,\ E_0 = \sqrt{Z}E,\ G_0 = \sqrt{Z}G, \tag{44}$$

with

$$Z = 1 - \frac{g^2}{2\pi^2\epsilon}(1-4a_1m^2C^2). \tag{45}$$

The tadpole contributions add up to zero as in the commutative case. Also, $\delta m = 0$, i.e. there are no $\delta m\bar\psi\psi$ and $\delta m(SE+PG)$ counterterms. It is obvious that the deformation parameter has to be renormalized in the following way

$$C_0^2 = \Big(a_1 - \frac{3a_3g^2}{2\pi^2\epsilon}\Big)C^2\ . \tag{46}$$

Our present analysis is not complete and we plan to consider the vertex corrections in the forthcoming publication. They should tell us something about the renormalization of the coupling constant g and about the renormalizability of the full model.

Finally we give a comment concerning the non-renormalization theorem. From (42) we see that the divergent part of the one loop effective action consists of the usual term $(\Phi^+\Phi)\Big|_{\theta\theta\bar\theta\bar\theta}$ and a new (compared with the undeformed case) term $P_1(\Phi\star\Phi)\Big|_{\bar\theta\bar\theta}$. Both of them are expressible as an integral over the whole superspace. Especially, in the case of the new term we have

$$\begin{aligned} P_1(\Phi\star\Phi)\Big|_{\bar\theta\bar\theta} &= \int \mathrm{d}^4x\ \mathrm{d}^2\ \bar\theta \mathrm{d}^2\ \theta\ \theta\theta P_1(\Phi\star\Phi) \\ &= -\frac{1}{32}C^2\int \mathrm{d}^4x\ \mathrm{d}^2\ \bar\theta \mathrm{d}^2\ \theta\ \theta\theta(D^2\Phi)(D^2\Phi) \\ &= \frac{1}{8}C^2\int \mathrm{d}^4x\ \mathrm{d}^2\ \bar\theta \mathrm{d}^2\ \theta\ \Phi(D^2\Phi). \end{aligned} \tag{47}$$

We see that at the level of two point Green functions there is no need to deform the nonrenormalization theorem.

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