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GRAVITATIONAL LENSING STATISTIC AND COSMOLOGY

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Abstract. The goal of this paper is mainly to present a short overview of some of our recent investigations of gravitational lensing and their applications to observational cosmology. We are mostly focused on the optical depth of gravitational lensing (i.e. the chance of seeing these effects) and on its applications for estimation of the dark matter fraction in the Universe and for constraining cosmological parameters. We also performed a statistical investigation of optical depth of gravitationally lensed quasars in order to study different cosmological models in the case when the large scale spatial curvature of the Universe Ω_{k} is zero (flat cosmological models), and hence: $\Omega_0 + \Omega_\Lambda = 1$. We found that the most popular cosmological model ($\Omega_0 = 0.3, \Omega_\Lambda = 0.7$) achieves the best fit and therefore it is the most probable one.

1. INTRODUCTION

An important prediction of general relativity is the deflection of light in the gravitational field of a massive object. This is the basic principle of gravitationl lensing and it was proved for the first time during total solar eclipse in 1919 by Eddington and collaborators. The distant quasar QSO 0957+0561 was the first extragalactic gravitational lens which was discovered by Walsh, Carswell and Weymann exactly 30 years ago, in 1979 (Walsh et al. 1979). Since then the theory, observations, and applications of gravitational lensing became one of the most rapidly growing fields in astrophysics and nowadays, gravitational lensing has developed into a versatile tool for observational cosmology.

Basically, there are two types of gravitational lensing: 1. strong - where an observed image of a distant source is split into several separate images (macrolensing), or at least where the apparent brightness of distant sources is magnified by certain factor (microlensing) and 2. weak - where the observed images of distant objects are slightly distorted as a result of light deflection by mass fluctuations along the line of sight. Both, strong and weak gravitational lensing are very important for estimation of visible and dark matter fractions in the Universe, and therefore for determination of the cosmological parameters. Besides, there are also so called pixel and nano-lensing which are very extensively used nowadays for detections of extrasolar planets (exoplanets) not only in our Galaxy, but even in other near galaxies such as Andromeda and Magellanic clouds (see e.g. Ingrosso et al. 2009).

The phenomenology of gravitational lensing effects and introduction to this field has been given in several review papers and books. References to some of them can be found in e.g. Popović et al. (2002). Therefore, here we will not discuss all aspects of such effects in the universe, but instead, we will briefly present some basic concepts and applications of gravitational lensing for constraining the cosmological parameters and dark matter detection.

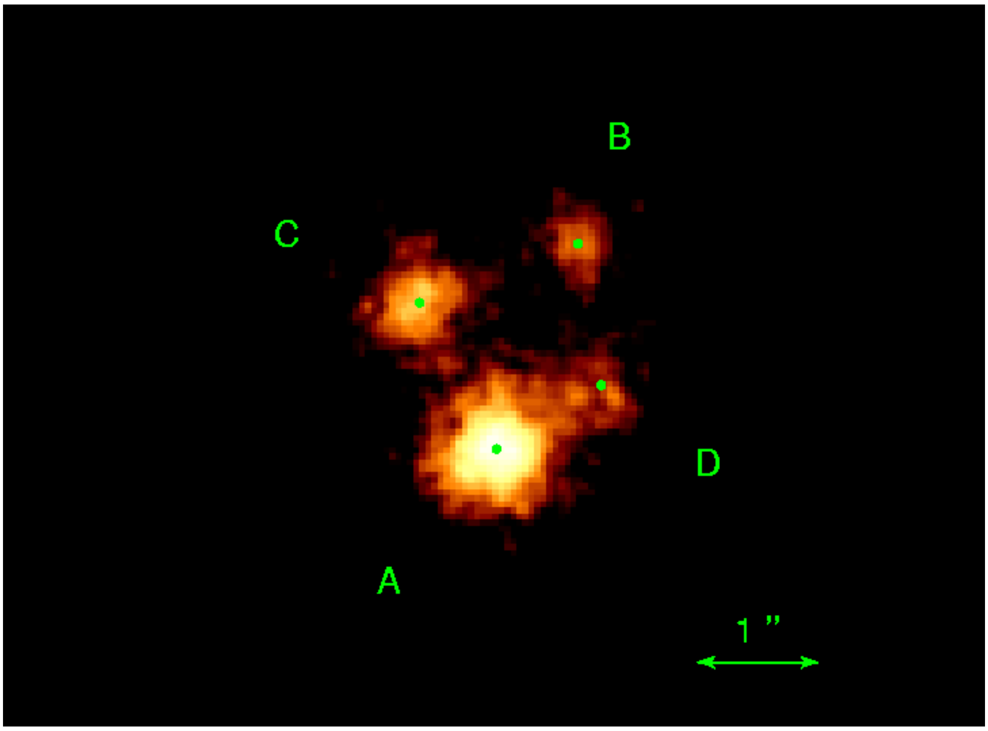

Figure 1: Gravitationally lensed system QSO 2237+0305 (Einstein Cross) observed by the *Advanced CCD Imaging Spectrometer (ACIS)* onboard the *Chandra X-ray Observatory* (Dai et al. 2003). The green circles in each image are the corresponding *Hubble Space Telescope (HST)* image positions provided by *CASTLES* (http://cfawww.harvard.edu/glensdata/Individual/Q2237.html).

2. GRAVITATIONAL LENSING

Gravitational lensing is an universal natural phenomenon where the gravitational force of lensing object induce either the amplification of some background source (microlensing), or the appearance of its multiple images (macrolensing), due to light bending in a gravitational field of the deflector. Separation angle between the images of background source depends on the mass of gravitational lens and therefore, multiple images can be observed only in the case of a massive lensing object, such as galaxy. One of the most famous multiple image lens systems is quasar QSO 2237+0305, also known as Einstein Cross, which is located at redshift $z = 1.695$ (see Fig. 1). Its four images are due to lensing effect of galaxy ZW2237+030, located between us and the quasar at redshift $z = 0.0394$. In the case of a small mass lens (e.g. a star), the separation angle is also small and therefore, different images of background source cannot be resolved. Instead, its intensity is amplified, causing the changes in the observed light curve. Common name for both, macrolensing (or simply lensing) and microlensing is strong lensing, contrary to weak lensing, which causes distortions in observed images of distant objects which can be then used for studying the mass distribution along the line of sight.

Some recent observational and theoretical studies suggest that gravitational microlensing can induce variability in the X-ray emission of Active Galactic Nuclei (AGN), especially in the case of gravitationally lensed quasars. They can be also used for studying the effects of strong gravitational fields in vicinity of supermassive black holes in the centers of AGN. Here we will not discuss these problems, but more details can be found in e.g. Jovanović (2005, 2006); Jovanović & Popović (2008, 2009); Popović et al. (2003ab, 2006ab); Zakharov et al. (2004), etc. In last several years we developed three different models of gravitational microlenses: point-like, straight-fold caustic and quadrupole microlenses. Below we give a short overview of these theoretical models.

2. 1. POINT-LIKE MICROLENS

The first of these models is called point-like microlens, and it is applied when an isolated compact object (e.g. a star) plays a role of gravitational microlens. Such microlens is characterized by its Einstein Ring Radius in the lens plane (Jovanović et al. 2008):

$$ERR = \sqrt{\frac{4Gm}{c^2}\frac{D_l D_{ls}}{D_s}} \tag{1}$$

or by the corresponding projection to the source plane:

$$\eta_0 = \frac{D_s}{D_l} ERR = \sqrt{\frac{4Gm}{c^2}\frac{D_s D_{ls}}{D_l}}, \tag{2}$$

where G is the gravitational constant, c is the speed of light, m is the microlens mass and D_l, D_s and D_{ls} are the cosmological angular distances between observer-lens, observer-source and lens-source, respectively. In this case, the microlensing amplification is given by the following relation (Popović et al. 2001; Jovanović et al. 2003):

$$A(X,Y) = \frac{u^2(X,Y)+2}{u(X,Y)\sqrt{u^2(X,Y)+4}}, \tag{3}$$

where X and Y are the impact parameters (i.e. the coordinates) which describe the apparent position of each point on an observer's celestial sphere,

$$u(X,Y) = \frac{\sqrt{(X-X_0)^2+(Y-Y_0)^2}}{\eta_0} \tag{4}$$

which corresponds to the angular separation between the microlens and a source and X_0, Y_0 are the coordinates of the microlens with respect to the source.

2. 2. STRAIGHT-FOLD CAUSTIC MICROLENS

In most cases we cannot simply consider that microlensing is caused by an isolated compact object, but we should take into account that such micro-deflector is located in an extended object (typically, the lens galaxy). Therefore, when the size of the microlens projected Einstein Ring Radius is larger than the size of an extended source and when a number of microlenses form a caustic net, we describe the microlensing effects by a straight-fold caustic approximation (Jovanović et al. 2008; Popović et al. 2003), where the amplification at a point of an extended source close to the caustic is given by (Chang & Refsdal, 1984):

$$A(X,Y) = A_0 + K\sqrt{\frac{r_{caustic}}{\kappa(\xi-\xi_c)}} \cdot H(\kappa(\xi-\xi_c)). \tag{5}$$

In above expression A_0 is the amplification outside the caustic and $K = A_0\beta$ is the caustic amplification factor, where β is constant of order of unity. ξ is the distance perpendicular to the caustic and ξ_c is the minimum distance from the source center to the caustic. The "caustic size" $r_{caustic}$ is the distance ξ for which the caustic amplification is equal to 1, and therefore this parameter defines a typical linear scale for the caustic. $H(\kappa(\xi-\xi_c))$ is the Heaviside function which equals 1 for $\kappa(\xi-\xi_c) > 0$ and otherwise it is 0. κ is ± 1 depending on the direction of caustic motion.

2. 3. QUADRUPOLE MICROLENS

The most complex model for gravitational microlensing is so called quadrupole microlens. This model is applied to obtain a spatial distribution of magnifications in the source plane, produced by a random star field placed in the lens plane. Such spatial distribution of magnifications is called microlensing map, microlensing pattern or caustic network. If we consider a set of N compact objects (e.g. stars) which are characterized by their positions x_i and their masses m_i, then the normalized lens equation is given by (Schneider et. al 2006; Jovanović & Popović 2009):

$$\vec{y} = \sum_{i=1}^{N} m_i \frac{\vec{x}-\vec{x}_i}{|\vec{x}-\vec{x}_i|^2} + \left[\begin{array}{cc} 1-\kappa_c+\gamma & 0 \\ 0 & 1-\kappa_c-\gamma \end{array} \right] \vec{x}, \tag{6}$$

where $\vec{x}$ and $\vec{y}$ are the normalized image and source positions, respectively. The sum describes the light deflection by the stars and the last term is a quadrupole contribution from the galaxy containing the stars, where κ_c is a smooth surface mass density and γ is an external shear. The total surface mass density or convergence can be written as $\kappa = \kappa_* + \kappa_c$, where κ_* represents the contribution from the compact microlenses. The corresponding microlensing map is then defined by two parameters: the convergence - κ, and the shear due to the external mass - γ.

For some specific lensing event one can model the corresponding magnification map using numerical simulations based on inverse ray-shooting techniques (see e.g. Jovanović et al. 2008; Popović et al. 2006ab and references therein), in which the rays are shot from the observer to the source, through the randomly generated star field in the lens plane. An example of a magnification map for image A of QSO 2237+0305 is presented in Fig. 2. The size of this magnification map is 16 $\eta_0 \times 16$ η_0 and it is generated using the following values for convergence and shear: $\kappa = 0.36$ and $\gamma = 0.4$ (Jovanović et al. 2008).

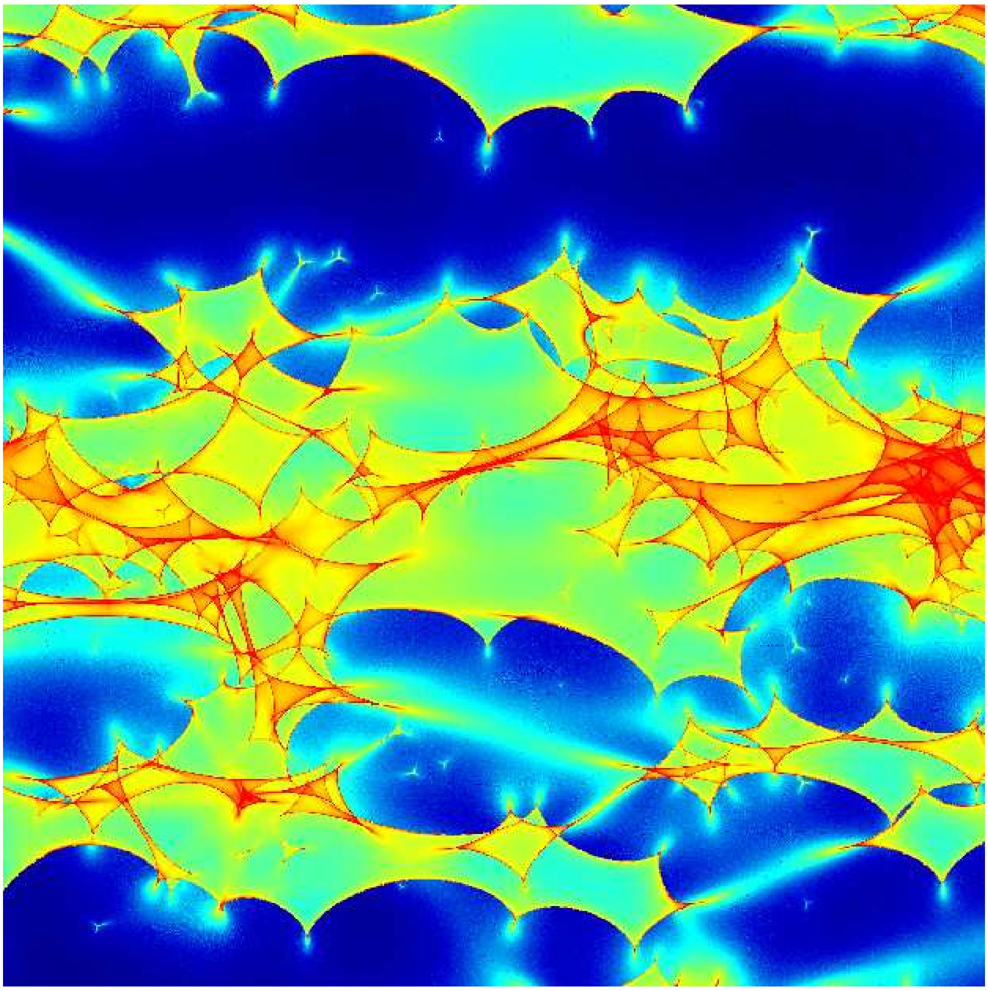

Figure 2: Magnification map of image A of QSO 2237+0305 which is also known as Einstein Cross (Jovanović et al. 2008).

3. RESULTS

In this section we present some of our results concerning the optical depth of gravitational lensing. We studied two cases: i) the optical depth of deflectors at cosmological distances between observer and a source, which enabled us to derive certain conclusions about the dark mater fraction in the Universe and ii) the optical depth of multiply imaged quasars, using which we were able to constrain the cosmological parameters.

3. 1. GRAVITATIONAL LENSING AND DARK MATTER DETECTION

Most of the matter in the Universe consists of a peculiar kind of matter that we neither see nor detect by any means (Massey et al. 2007). Such kind of matter is called dark matter. It is believed that the dark matter of the Universe is constituted, in large part, by non-baryonic particles with very small primordial velocity dispersion which are so weakly interacting that they move purely under the influence of gravity.

Gravitational lensing of distant objects by foreground structures has proven to be a powerful tool for studing the mass distribution in the Universe (Massey et al. 2007). As photons travel through the Universe, they are deflected by mass along the line of sight (gravitational lensing), and at the moment, this is the only mean which allows us to study the properties of the dark matter directly, providing us with a unique way to study the projected mass distribution along the line of sight.

The probability of observing a gravitational lensing effect is usually expressed in terms of the optical depth τ. The optical depth τ (the chance of seeing a gravitational lens) is the probability that at any instant of time a distant source is covered by the Einstein ring of a deflector. Here we will consider deflectors at cosmological distances between observer and a source. To evaluate their optical depth, we assume that a source is located at some redshift z. The corresponding expression for optical depth is (Zakharov et al. 2004):

$$\begin{aligned}\tau &= \frac{3}{2}\frac{\Omega_L}{\lambda(z)}\int_0^z dw \frac{(1+w)^3[\lambda(z)-\lambda(w)]\lambda(w)}{\sqrt{\Omega_0(1+w)^3+\Omega_\Lambda}},\\ \lambda(z) &= \int_0^z \frac{dw}{(1+w)^2\sqrt{\Omega_0(1+w)^3+\Omega_\Lambda}}.\end{aligned} \tag{7}$$

Here, $\lambda(z)$ is the affine distance (in units of cH_0^{-1}), H_0 - Hubble constant at the present epoch, c - the speed of light, Ω_0, Ω_Λ and Ω_L - dimensionless density parameters corresponding to the mass density of the Universe at the present epoch, cosmological constant Λ and the matter fraction in compact lenses, respectively.

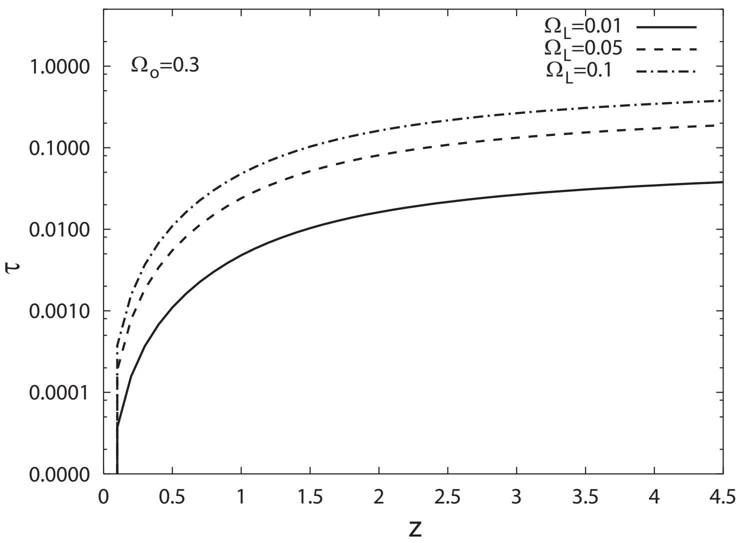

Figure 3: The calculated optical depth as a function of red-shift for 3 different values of Ω_L and for $\Omega_0 = 0.3$ (Zakharov et al. 2004).

We use realistic cosmological parameters to evaluate the Equation (7). According the cosmological SN (Supernovae) Ia data and Cosmic Microwave Background (CMB) anisotropy one could take that $\Omega_\Lambda \approx 0.7, \Omega_0 \approx 0.3$ (see Zakharov et al. 2004 and references therein). The optical depth for 3 realistic values of Ω_L as a function of redshift is presented in Fig. 3 and the corresponding numerical values are given in Table 1. Here we assume that almost all baryonic matter could form compact lenses ($\Omega_L = 0.05$), or 20% of baryonic matter forms such lenses ($\Omega_L = 0.01$). As one can see from Fig. 3 and Table 1, for both of these cases and for distant objects ($z \sim 2.0$) the optical depth could reach $\tau \sim 0.01 - 0.1$. If about 30% of non-baryonic dark matter forms objects with stellar masses, $\Omega_L = 0.1$ could be adopted as a realistic value and then $\tau \sim 0.2$ at $z \sim 2$.

Table 1: Numerical values of calculated optical depth from Fig. 3 (Zakharov et al. 2004).

$z \backslash \Omega_L$	0.01	0.05	0.10
0.5	0.001100	0.005499	0.010998
1.0	0.004793	0.023967	0.047934
1.5	0.010310	0.051550	0.103100
2.0	0.016196	0.080980	0.161959
2.5	0.021667	0.108334	0.216669
3.0	0.026518	0.132590	0.265180
3.5	0.030770	0.153852	0.307703
4.0	0.034504	0.172521	0.345042
4.5	0.037804	0.189018	0.378037
5.0	0.040742	0.203712	0.407424

3. 2. GRAVITATIONAL LENSING STATISTICS

The gravitational lensing statistics was extensively used as a method for determining the cosmological parameters during the last several decades (see e.g. Schneider et. al 2006 for more details about history and basics of this method). Using Véron & Véron catalogue of quasars and active nuclei (Véron-Cetty & Véron 2006) we perform a statistical investigation of optical depth of gravitationally lensed quasars in order to study different cosmological models. We discuss the cosmological parameters in the case when the large scale spatial curvature of the universe Ω_k is zero (flat cosmological models), and hence: $\Omega_0 + \Omega_\Lambda = 1$.

The integrated probability, referred as gravitational macrolensing optical depth, of a quasar at a redshift z being multiply imaged by a deflector along the line of sight is (Turner 1990):

$$\tau_{GL}(z) = \frac{F}{30}\left[\int\limits_1^{z+1} \frac{dw}{\sqrt{\Omega_0 w^3 - \Omega_0 + 1}}\right]^3, \tag{8}$$

where F represents the gravitational lensing effectiveness (for more details see e.g. Zakharov et al. 2004 and references therein). The integral in equation (8) may be analytically solved in the case $\Omega_0 = 0$, giving the following expression for optical depth: $\tau_{GL}(z) = \dfrac{F}{30} z^3$, as well as in the case $\Omega_0 = 1$: $\tau_{GL}(z) = \dfrac{4F}{15}\left(1 - \dfrac{1}{\sqrt{1+z}}\right)^3$.

The effective optical depth of lenses (modeled by singular isothermal spheres and producing multiple images of distant quasars) is related to the number $N_{GL}(z)$ of multiply imaged quasars (gravitational lens sources) within a sample of $N_{QSO}(z)$ quasars with redshifts z by the following expression (Surdej et al. 1993):

$$\tau_{GL}(z) = N_{GL}(z) / N_{QSO}(z) . \tag{9}$$

In order to study the different cosmological models using the gravitational lensing statistics, we analyzed a complete sample of 85 221 quasars from Véron & Véron catalogue (Véron-Cetty & Véron 2006), among which 69 are gravitationally lensed. We calculated the empirical distributions of both, all and lensed quasars and then, for three different values of Ω_0, we fitted the optical depth $\tau_{GL}(z)$ given by equation (8) to the fraction of lensed quasars in respect to the total number of quasars: $N_{GL}(z)/N_{QSO}(z)$. Assuming the spatially flat cosmological models ($\Omega_\mathrm{k} = 0$), we obtained the following values for the effectiveness of gravitational lensing by quasars: i) $F = 0.0165$ for $\Omega_0 = 0.3$, ii) $F = 0.0012$ for $\Omega_0 = 0$ and iii) $F = 0.0556$ for $\Omega_0 = 1$. These values are something less than those for gravitational lensing by local galaxies: $0.023 \leq F \leq 0.047$, as reported by several authors (Surdej et al. 1993).

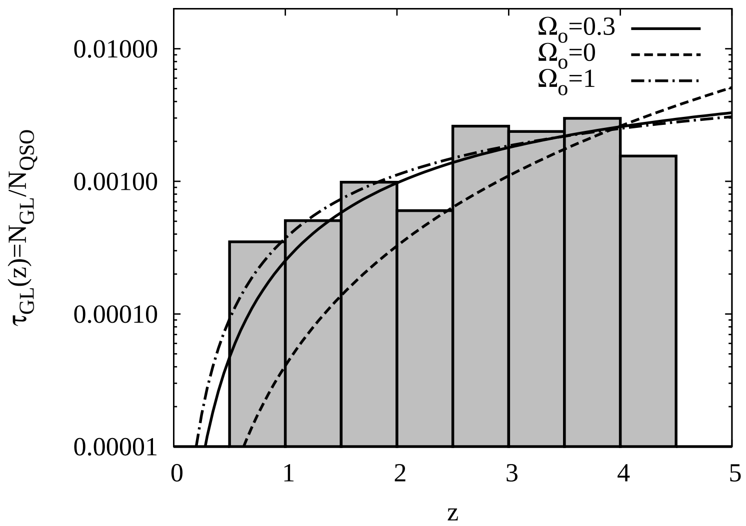

Figure 4: The ratio of lensed (N_{GL}) to total number of quasars (N_{QSO}) taken from Véron & Véron Catalogue (Véron-Cetty & Véron 2006) of AGN (histograms) and the gravitational lensing optical depth (solid and dashed lines), calculated for three different flat ($\Omega_\mathrm{k} = 0$) cosmological models as a function of quasar redshift.

The corresponding results are presented in Fig. 4, from which it can be seen that the optical depth for lensing by quasars is increased by several orders of magnitude at high redshifts in all three analyzed cases. At redshifts $z \approx 1-2$, the difference between extreme cases of matter dominated universe ($\Omega_0 = 1, \Omega_\Lambda = 0$, dash-dot curve) and cosmology constant Λ dominated universe ($\Omega_0 = 0, \Omega_\Lambda = 1$, dashed curve) is about an order of magnitude. While the first model gives the good fit, the second one achieves the worst and thus, it is the most unlikely. The most popular cosmological model ($\Omega_0 = 0.3, \Omega_\Lambda = 0.7$, solid line) achieves the best fit and therefore it is the most probable one.

4. CONCLUSIONS

We studied the optical depth of gravitational lensing for the cosmologically distributed deflectors and multiply imaged quasars from Véron & Véron catalogue and we found that:

1. The optical depth for gravitational lensing caused by cosmologically distributed deflectors could be significant and could reach $10^{-2} - 0.1$ (Zakharov et al. 2004). Taking into account that the upper limit of the optical depth ($\tau \sim 0.1$) corresponds to the case where dark matter forms cosmologically distributed deflectors, one can conclude that gravitational lensing can be used as a tool for the estimation of the dark matter fraction in the Universe.

2. Statistical investigation of optical depth in the case of different flat cosmological models showed that the matter dominated universe ($\Omega_0 = 1, \Omega_\Lambda = 0$) gave the good fit, while the cosmology constant Λ dominated universe ($\Omega_0 = 0, \Omega_\Lambda = 1$) achieved the worst one and thus, it was the most unlikely. The most popular cosmological model ($\Omega_0 = 0.3, \Omega_\Lambda = 0.7$) achieved the best fit and therefore it was the most probable one.

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