

**(NON)ARCHIMEDEAN QUANTUM
COSMOLOGY AND TACHYONIC INFLATION**

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Abstract. We review the relevance of quantum rolling tachyons and corresponding inflation scenario in the frame of the standard, p -adic and adelic minisuperspace quantum cosmology. The field theory of tachyon matter proposed by Sen in a zero-dimensional version suggested by Kar leads to a model of a particle moving in a constant external field with quadratic damping. We calculate the exact quantum propagator of the model, as well as, the vacuum states and conditions necessary to construct an adelic generalization. In addition we present an overview on several important cosmological models on archimedean and nonarchimedean spaces.

1. INTRODUCTION

The main task of quantum cosmology (Wiltshire, 1996) is to describe the evolution of the universe in the very early stage. Usually one takes the universe is described by a complex-valued wave function. Since quantum cosmology is related to the Planck scale phenomena it is logical to consider various geometries (in particular the nonarchimedean (Djordjevic et al., 2002) and noncommutative (Garcia-Compean et al., 2002) ones) and parametrization of the space-time coordinates: real, p -adic, or even adelic (Vladimirov et al., 1994). In this article, we will generally maintain space-time coordinates and matter fields to be real and p -adic.

It is quite natural to consider that in the very early stage of its evolution the universe is in a quantum state, which is described by a wave function. Concerning the wave function, we will here maintain the standard point of view: the wave function takes complex values, but space-time coordinates and matter fields will be treated in a more complete way to be adelic, i.e. they have real as well as p -adic properties simultaneously. This approach is motivated by the following reasons: (i) the field of rational numbers Q , which contains all observational and experimental numerical data, is a dense subfield not only in the field of real numbers R but also in the fields of p -adic numbers Q_p (p is any prime number), (ii) there is a plausible analysis (Vladimirov et al., 1994) within and over Q_p as well as that one related to R , (iii) general mathematical methods and fundamental physical laws should be invariant under an interchange of the number fields R and Q_p (Volovich, 1987), (iv) there is a quantum gravity uncertainty (Garay, 1995) Δx while measuring distances around the

Planck length ℓ_0 ,

$$\Delta x \geq \ell_0 = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-33} \text{ cm}, \quad (1)$$

which restricts priority of archimedean geometry based on real numbers and gives rise to employment of nonarchimedean geometry related to p -adic numbers (Volovich, 1987), (v) it seems to be quite reasonable to extend compact archimedean geometries by the nonarchimedean ones in the path integral method, and (vi) adelic quantum mechanics (Dragovich, 1994) applied to quantum cosmology provides realization of all the above statements. The successful application of p -adic numbers and adeles in modern theoretical and mathematical physics started in 1987, in the context of string amplitudes (Vladimirov et al., 1987; Freund and Witten, 1987)(for a review, see Refs. (Freund and Witten, 1987; Volovich et al., 1994; Khrennikov, 1997). For a systematic research in this field it was formulated p -adic quantum mechanics (Vladimirov and Volovich, 1989; Ruelle et al., 1989) and adelic quantum mechanics (Dragovich, 1994; Dragovich, 1995). They are quantum mechanics with complex-valued wave functions of p -adic and adelic arguments, respectively. In the unified form, adelic quantum mechanics contains ordinary and all p -adic quantum mechanics. p -Adic gravity and the wave function of the universe were considered in the paper (Aref'eva et al., 1991) published in 1991. An idea of the fluctuating number fields at the Planck scale was introduced and it was suggested to restrict the Hartle-Hawking (Hartle and Hawking, 1983) proposal to summation only over algebraic manifolds. It was shown that the wave function for the de Sitter minisuperspace model can be treated in the form of an infinite product of p -adic counterparts. Another approach to quantum cosmology, which takes into account p -adic effects was proposed in 1995 (Dragovich, 1995). Like in adelic quantum mechanics, adelic eigenfunction of the universe is a product of the corresponding eigenfunctions of real and all p -adic cases. p -Adic wave functions are defined by p -adic generalization of the Hartle-Hawking path integral proposal. It was shown that in the framework of this procedure one obtains an adelic wave function for the de Sitter minisuperspace model. However, this procedure with the Hartle-Hawking p -adic prescription does not work when matter fields are included into consideration. The solution of this problem was found (Dragovich and Nesic, 1999) in treating minisuperspace cosmological models as models of adelic quantum mechanics.

Supernova Ia observations show that the expansion of the Universe is accelerating (Perlmutter et al., 1999), contrary to Friedmann-Robertson-Walker (FRW) cosmological models, with non-relativistic matter and radiation. Also, cosmic microwave background (CMB) radiation data are suggesting that the expansion of our Universe seems to be in an accelerated state which is referred to as the "dark energy" effect. The cosmological constant as the vacuum energy can be responsible for this evolution by providing negative pressure. A need for understanding these new and rather surprising facts, including (cold) "dark matter", has motivated numerous authors to reconsider different inflation scenarios. Despite some evident problems (Sami et al., 2004) such as insufficiently long period of inflation, tachyon-driven scenarios (Gibbons, 2003; Choudhury and Ghoshal, 2002) or (Tranberg et al., 2007) remain highly interesting for study.

There have been a number of attempts to understand this description of the early Universe via (classical) nonlocal cosmological models, first of all via p -adic inflation

models (Barnaby et al., 2007; Joukovskaya, 2007), which are represented by nonlocal p -adic string theory coupled to gravity. For these models, some rolling inflationary solutions were constructed and compared with CMB observations. Another example is the investigation of the p -adic inflation near a maximum of the nonlocal potential when non-local derivative operators are included in the inflaton Lagrangian. It was found that higher-order derivative operators can support a (sufficiently) prolonged phase of slow-roll inflation (Lidsey, 2007). These results are an additional strong motivation for a rather general - adelic approach to quantum cosmology.

In the unified form, adelic quantum mechanics contains ordinary and all p -adic quantum mechanics. As there is not an appropriate p -adic Schrödinger equation, there is also no p -adic generalization of the Wheeler-De Witt equation. Instead of the differential approach, Feynman's path integral method is exploited (Djordjevic and Dragovich, 1997; Djordjevic and Dragovich, 2000; Djordjevic et al., 1999; Dimitrijevic et al., 2008). p -Adic gravity and the wave function of the universe were considered (Aref'eva et al., 1991) as an idea of the fluctuating number fields at the Planck scale. Like in adelic quantum mechanics, the adelic eigenfunction of the universe is a product of the corresponding eigenfunctions of real and all p -adic cases. It was shown that in the framework of this procedure one obtains an adelic wave function for the de Sitter minisuperspace model. However, the adelic generalization with the Hartle-Hawking p -adic prescription does not work well. Consideration has been much more successful when minisuperspace cosmological models are treated as models of adelic quantum mechanics. It is a strong motivation to study a class of exactly solvable quantum mechanical models and apply them in the frame of quantum cosmology. For the review and detailed discussion see (Djordjevic et al., 2002; Djordjevic et al., 2002a). The nonarchimedean and noncommutative cosmological quantum models with extra dimensions and an accelerating phase have been considered (Djordjevic and Nestic, 2005), as well as the relevant models and techniques in a pure quantum mechanical context (Dimitrijevic et al., 2007; Djordjevic and Nestic, 2005; Dimitrijevic et al., 2004; Djordjevic and Nestic, 2003). To keep this text to a reasonable size, we will not consider noncommutative cosmology here, and a review would be given elsewhere.

This review is organized as follows: after the Introduction, in Chapter 2 we give basic information on “ p -adics“ and adeles. Chapter 3 is devoted to p -Adic and Adelic quantum mechanics as an underlying formalism for the corresponding approach to quantum cosmology briefly explained in the Chapter 4. Two simple cosmological models in Chapter 5 illustrate application of p -adic numbers in the Hartle-Hawking proposal, problems and possible solutions for them we are faced with in this approach. The most promising approach which generalize real and p approach to quantum cosmology is presented in Chapter 6, again through 2 simple examples. Chapter 7 and 8 are reserved for a quite intriguing and hot problems in modern cosmology and, let us add, string theory as well, namely inflation and tachyons. Following S. Kar's idea on the possibility of the examination of zero dimensional theory of the field theory of (real) tachyon matter (Kar, 2002), and motivated by successes and shortcomings of classical p -adic inflation, we consider real and p -adic aspects of a relevant model with quadratic damping. We calculated the corresponding propagator and considered vacuum states for p -adic and adelic tachyons. We end our paper with a short conclusion and a few ideas for future research. A list of references is quite subjective neither exhaustive nor complete one, but could be useful for a reader interested in having a

better insight in results in application nonarchimedean geometry in quantum theory and cosmology.

2. p -ADIC NUMBERS AND ADELES

We give here a brief survey of some basic properties of p -adic numbers and adeles, which we exploit in this work. In addition, a reach structure of nonarchimedean analysis and geometry, numerous similarities as well as differences in respect to the “standard“ ones will serve as a good basis for a general (re)consideration of mathematical foundation od modern High Energy Physics.

Completion of Q with respect to the standard absolute value $(|\cdot|_\infty)$ gives R , and an algebraic extension of R makes C . According to the Ostrowski theorem (Vladimirov et al., 1994) any non-trivial norm on the field of rational numbers Q is equivalent to the absolute value $|\cdot|_\infty$ or to the p -adic norm $|\cdot|_p$, where p is a prime number. p -Adic norm is the nonarchimedean (ultrametric) one and for a rational number, $0 \neq x \in Q$, $x = p^\nu \frac{m}{n}$, $0 \neq n, \nu, m \in Z$, has a value $|x|_p = p^{-\nu}$. Completion of Q with respect to the p -adic norm for a fixed p leads to the corresponding field of p -adic numbers Q_p . Completions of Q with respect to $|\cdot|_\infty$ and all $|\cdot|_p$ exhaust all possible completions of Q . p -Adic number $x \in Q_p$, in the canonical form, is an infinite expansion

$$x = p^\nu \sum_{i=0}^{\infty} x_i p^i, \quad x_0 \neq 0, \quad 0 \leq x_i \leq p-1. \quad (2)$$

The norm of p -adic number x in (2) is $|x|_p = p^{-\nu}$ and satisfies not only the triangle inequality, but also the stronger one

$$|x + y|_p \leq \max(|x|_p, |y|_p). \quad (3)$$

Metric on Q_p is defined by $d_p(x, y) = |x - y|_p$. This metric is the nonarchimedean one and the pair (Q_p, d_p) presents locally compact, topologically complete, separable and totally disconnected p -adic metric space. In the metric space Q_p , p -adic ball $B_\nu(a)$, with the centre at the point a and the radius p^ν is the set

$$B_\nu(a) = \{x \in Q_p : |x - a|_p \leq p^\nu, \nu \in Z\}. \quad (4)$$

p -Adic sphere $S_\nu(a)$ with the centre a and the radius p^ν is

$$S_\nu(a) = \{x \in Q_p : |x - a|_p = p^\nu, \nu \in Z\}. \quad (5)$$

Elementary p -adic functions (Schikhof, 2006) are given by the series of the same form as in the real case, e.g.

$$\exp x = \sum_{k=0}^{\infty} \frac{x^k}{k!}, \quad \sinh x = \sum_{k=0}^{\infty} \frac{x^{k+1}}{(2k+1)!}, \quad \cosh x = \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!}, \quad (6)$$

$$\tanh x = \sum_{k=2}^{\infty} \frac{2^k(2^k - 1)B_k x^{k-1}}{k!}, \quad \coth x = \frac{1}{x} + \sum_{k=2}^{\infty} \frac{2^k B_k x^{k-1}}{k!}, \quad (7)$$

where B_k are Bernoulli's numbers. These functions have the same domain of convergence $G_p = \{x \in Q_p : |x|_p < |2|_p\}$.

Real and p -adic numbers are unified in the form of the adèles (Gel'fand et al., 1966). An adèle is an infinite sequence

$$a = (a_\infty, a_2, \dots, a_p, \dots), \quad (8)$$

where $a_\infty \in Q_\infty$, and $a_p \in Q_p$, with restriction to $a_p \in Z_p$ ($Z_p = \{x \in Q_p : |x|_p \leq 1\}$) for all but a finite set S of primes p . If we introduce $\mathcal{A}(S) = Q_\infty \times \prod_{p \in S} Q_p \times \prod_{p \notin S} Z_p$

then the space of all adèles is $\mathcal{A} = \bigcup_S \mathcal{A}(S)$, which is a topological ring. Namely, \mathcal{A} is a ring with respect to the componentwise addition and multiplication. A principal adèle is a sequence $(r, r, \dots, r, \dots) \in \mathcal{A}$, where $r \in Q$. Thus, the ring of principal adèles, which is a subring of \mathcal{A} , is isomorphic to Q . An important function on \mathcal{A} is the additive character $\chi(x)$, $x \in \mathcal{A}$, which is a continuous and complex-valued function with basic properties:

$$|\chi(x)|_\infty = 1, \quad \chi(x+y) = \chi(x)\chi(y). \quad (9)$$

This additive character may be presented as

$$\chi(x) = \prod_v \chi_v(x_v) = \exp(-2\pi i x_\infty) \prod_p \exp(2\pi i \{x_p\}_p), \quad (10)$$

where $v = \infty, 2, \dots, p, \dots$, and $\{x\}_p$ is the fractional part of the p -adic number x . Map $\varphi : \mathcal{A} \rightarrow C$, which has the form

$$\varphi(x) = \varphi_\infty(x_\infty) \prod_{p \in S} \varphi_p(x_p) \prod_{p \notin S} \Omega(|x_p|_p), \quad (11)$$

where $\varphi_\infty(x_\infty) \in D(Q_\infty)$ is an infinitely differentiable function on Q_∞ and falls to zero faster than any power of $|x_\infty|_\infty$ as $|x_\infty|_\infty \rightarrow \infty$, $\varphi_p(x_p) \in D(Q_p)$ is a locally constant function with compact support, and

$$\Omega(|x|_p) = \begin{cases} 1, & |x|_p \leq 1, \\ 0, & |x|_p > 1, \end{cases} \quad (12)$$

is called an elementary function on \mathcal{A} . Finite linear combinations of elementary functions (11) make the set of the Schwartz-Bruhat functions $D(\mathcal{A})$. The existence of Ω -function is unavoidable for a construction of any adelic model, in particular for quantum mechanical or quantum cosmological model. The Fourier transform is

$$\tilde{\varphi}(\xi) = \int_{\mathcal{A}} \varphi(x) \chi(\xi x) dx \quad (13)$$

and it maps one-to-one $D(\mathcal{A})$ onto $D'(\mathcal{A})$. It is worth noting that Ω -function is a counterpart of the Gaussian in the real case, since it is invariant with respect to the Fourier transform. It is also an important issue for consideration of the ground state(s) of quantum mechanical systems at high energies, where the use of p -adic numbers and nonarchimedean geometry in ‘‘modelling’’ should be fully justified.

The integrals of the Gauss type over the p -adic sphere S_ν , p -adic ball B_ν and over any Q_ν are (for $|4\alpha|_p \geq p^{2-2\nu}$):

$$\int_{S_\nu} \chi_p(\alpha x^2 + \beta x) dx = \begin{cases} \lambda_p(\alpha) |2\alpha|_p^{-1/2} \chi_p\left(-\frac{\beta^2}{4\alpha}\right), & \left|\frac{\beta}{2\alpha}\right|_p = p^\nu, \\ 0, & \left|\frac{\beta}{2\alpha}\right|_p \neq p^\nu, \end{cases} \quad (14)$$

$$\int_{B_\nu} \chi_p(\alpha x^2 + \beta x) dx = \begin{cases} p^\nu \Omega(p^\nu |\beta|_p), & |\alpha|_p p^{2\nu} \leq 1, \\ \frac{\lambda_p(\alpha)}{|2\alpha|_p^{1/2}} \chi_p\left(-\frac{\beta^2}{4\alpha}\right) \Omega\left(p^{-\nu} \left|\frac{\beta}{2\alpha}\right|_p\right), & |\alpha|_p p^{2\nu} > 1, \end{cases} \quad (15)$$

$$\int_{Q_\nu} \chi_p(\alpha x^2 + \beta x) dx = \lambda_\nu(\alpha) |2\alpha|_\nu^{-1/2} \chi_\nu\left(-\frac{\beta^2}{4\alpha}\right), \quad \alpha \neq 0. \quad (16)$$

The arithmetic functions $\lambda_\nu(a) : Q_\nu \mapsto C$, where $\nu = \infty, 2, 3, 5, \dots$, have the following properties:

$$|\lambda_\nu(a)|_\infty = 1, \quad \lambda_\nu(0) = 1, \quad \lambda_\nu(ab^2) = \lambda_\nu(a), \quad (17)$$

$$\lambda_\nu(a)\lambda_\nu(b) = \lambda_\nu(a+b)\lambda_\nu(a^{-1} + b^{-1}), \quad (18)$$

where $a \neq 0, b \neq 0$. The physical implication of λ_p function could be very important, but still far away to be well understood. One of the most intriguing consequences would be multidimensional nature of time (at Planck scale (Vladimirov et al., 1994)).

3. p -ADIC AND ADELIC QUANTUM MECHANICS

In foundations of standard quantum mechanics (over R) one usually starts with a representation of the canonical commutation relation

$$[\hat{q}, \hat{k}] = i\hbar, \quad (19)$$

where q is a spatial coordinate and k is the corresponding momentum. It is well-known that the procedure of quantization is not unique. In formulation of p -adic quantum mechanics (Vladimirov and Volovich, 1989; Ruelle et al., 1989) the multiplication $\hat{q}\psi \rightarrow x\psi$ has no meaning for $x \in Q_p$ and $\psi(x) \in C$. Also, there is no possibility to define p -adic "momentum" or "Hamiltonian" operator. In the real case they are infinitesimal generators of space and time translations, but, since Q_p is disconnected field, these infinitesimal transformations become meaningless. However, finite transformations remain meaningful and the corresponding Weyl and evolution operators are p -adically well defined. Canonical commutation relation in p -adic case can be represented by the Weyl operators ($\hbar = 1$)

$$\hat{Q}_p(\alpha)\psi_p(x) = \chi_p(\alpha x)\psi_p(x) \quad (20)$$

$$\hat{K}_p(\beta)\psi(x) = \psi_p(x + \beta). \quad (21)$$

Now, instead of the relation (19) in the real case, we have

$$\hat{Q}_p(\alpha)\hat{K}_p(\beta) = \chi_p(\alpha\beta)\hat{K}_p(\beta)\hat{Q}_p(\alpha) \quad (22)$$

in the p -adic one.

Dynamics of a p -adic quantum model is described by a unitary operator of evolution $U(t)$ without using the Hamiltonian. Instead of that, the evolution operator has been formulated in terms of its kernel $\mathcal{K}_t(x, y)$

$$U_p(t)\psi(x) = \int_{Q_p} \mathcal{K}_t(x, y)\psi(y)dy. \quad (23)$$

In this way (Vladimirov and Volovich, 1989) p -adic quantum mechanics is given by a triple

$$(L_2(Q_p), W_p(z_p), U_p(t_p)). \quad (24)$$

Keeping in mind that standard quantum mechanics can be also given as the corresponding triple, ordinary and p -adic quantum mechanics can be unified in the form of adelic quantum mechanics (Dragovich, 1994; Dragovich, 1995)

$$(L_2(\mathcal{A}), W(z), U(t)). \quad (25)$$

$L_2(\mathcal{A})$ is the Hilbert space on \mathcal{A} , $W(z)$ is a unitary representation of the Heisenberg-Weyl group on $L_2(\mathcal{A})$ and $U(t)$ is a unitary representation of the evolution operator on $L_2(\mathcal{A})$. The evolution operator $U(t)$ is defined by

$$U(t)\psi(x) = \int_{\mathcal{A}} \mathcal{K}_t(x, y)\psi(y)dy = \prod_v \int_{Q_v} \mathcal{K}_t^{(v)}(x_v, y_v)\psi^{(v)}(y_v)dy_v. \quad (26)$$

The eigenvalue problem for $U(t)$ reads

$$U(t)\psi_{\alpha\beta}(x) = \chi(E_\alpha t)\psi_{\alpha\beta}(x), \quad (27)$$

where $\psi_{\alpha\beta}$ are adelic eigenfunctions, $E_\alpha = (E_\infty, E_2, \dots, E_p, \dots)$ is the corresponding energy, indices α and β denote energy levels and their degeneration. Note that any adelic eigenfunction has the form

$$\Psi(x) = \Psi_\infty(x_\infty) \prod_{p \in S} \Psi_p(x_p) \prod_{p \notin S} \Omega(|x_p|_p), \quad x \in \mathcal{A}, \quad (28)$$

where $\Psi_\infty \in L_2(R)$, $\Psi_p \in L_2(Q_p)$. A suitable way to calculate p -adic propagator $\mathcal{K}_p(x'', t''; x', t')$ is to use Feynman's path integral method, i.e.

$$\mathcal{K}(x'', t''; x', t') = \int_{x', t'}^{x'', t''} \chi_p \left(-\frac{1}{h} \int_{t'}^{t''} L(\dot{q}, q, t) dt \right) \mathcal{D}q. \quad (29)$$

It has been evaluated (Djordjevic and Dragovich, 1997; Djordjevic et al., 2003) for quadratic Lagrangians in the same way for real and p -adic case and the following exact general expression is obtained:

$$\mathcal{K}_v(x'', t''; x', t') = \lambda_v \left(-\frac{1}{2h} \frac{\partial^2 \bar{S}}{\partial x'' \partial x'} \right) \left| \frac{\partial^2 \bar{S}}{\partial x'' \partial x'} \right|_v^{\frac{1}{2}} \chi_v \left(-\frac{1}{h} \bar{S}(x'', t''; x', t') \right), \quad (30)$$

where λ_v functions satisfy relations (17) and (18). When one has a system with more than one dimension with uncoupled spatial coordinates, then the total propagator is the product of the corresponding one-dimensional propagators. As an illustration of p -adic and adelic quantum-mechanical models, the following one-dimensional systems with the quadratic Lagrangians were considered: a free particle and harmonic oscillator (Vladimirov et al., 1994; Dragovich, 1994; Dragovich, 1995), a particle in a constant field, a free relativistic particle (Djordjevic et al., 1999) and a harmonic oscillator with time-dependent frequency (Djordjevic and Dragovich, 2000). Adelic quantum mechanics takes into account ordinary as well as p -adic quantum effects and may be regarded as a starting point for construction of a more complete superstring and M-theory. In the low-energy limit adelic quantum mechanics becomes the ordinary one (Djordjevic et al., 1999).

4. QUANTUM COSMOLOGY

According to the so-called standard cosmological model, in the very beginning the universe was very small, dense, hot and started to expand very fast. This initial period of evolution should be unavoidably described by quantum cosmology. In the path integral approach to quantum cosmology over the field of real numbers R , the starting point is the idea that the amplitude to go from one state with intrinsic metric h'_{ij} , and matter configuration ϕ' on an initial hypersurface Σ' , to another state with metric h''_{ij} , and matter configuration ϕ'' on a final hypersurface Σ'' , is given by a functional integral of the form

$$\langle h''_{ij}, \phi'', \Sigma'' | h'_{ij}, \phi', \Sigma' \rangle = \int \mathcal{D}g_{\mu\nu} \mathcal{D}\Phi e^{-S[g_{\mu\nu}, \Phi]}, \quad (31)$$

over all four-geometries $g_{\mu\nu}$, and matter configurations Φ , which interpolate between the initial and final configurations. In this expression $S[g_{\mu\nu}, \Phi]$ is an Einstein-Hilbert action for the gravitational and matter fields (which can be massless, minimally or conformally coupled with gravity). This expression stays valid in the p -adic case too, because of its form invariance under change of real to the p -adic number fields.

Among many cosmological models, there is one very important type of models, the so-called de Sitter models. The de Sitter are the models with the cosmological constant Λ and without matter fields. Models of this type are exactly soluble models and because of that, they play a role similar to a linear harmonic oscillator in ordinary quantum mechanics. The corresponding Einstein-Hilbert action is (Halliwell and Myers, 1989)

$$S = \frac{1}{16\pi G} \int_M d^D x \sqrt{-g} (R - 2\Lambda) + \frac{1}{8\pi G} \int_{\partial M} d^{D-1} x \sqrt{h} K, \quad (32)$$

where R is the scalar curvature of D -manifold M , K is the trace of the extrinsic curvature K_{ij} of the boundary ∂M of the D -manifold M . The general form of the metric for these models is

$$ds^2 = \sigma^2 [-N^2 dt^2 + a^2(t) d\Omega_{D-1}^2], \quad (33)$$

where $d\Omega_{D-1}^2$ denotes the metric on the unit $(D-1)$ -sphere

$$\sigma^{D-2} = \frac{8\pi G}{V^{D-1}(D-1)(D-2)}$$

and V^{D-1} is the volume of the unit $(D-1)$ -sphere. In the $D=3$ case, this model is related to the multiple sphere configuration and wormhole solutions. v -Adic ($v = \infty$ for the real, and $v = p$ in the p -adic cases) classical action for this model is

$$\bar{S}_v(a'', N; a', 0) = \frac{1}{2\sqrt{\lambda}} \left[N\sqrt{\lambda} + \lambda \left(\frac{2a''a'}{\sinh(N\sqrt{\lambda})} - \frac{a'^2 + a''^2}{\tanh(N\sqrt{\lambda})} \right) \right]. \quad (34)$$

Let us note that a denotes a scale factor and λ denotes here the appropriately rescaled cosmological constant Λ , i.e. $\lambda = \sigma^2 \Lambda$. This model was investigated in all aspects (p -adic, real and adelic) in Ref. (Djordjevic et al., 2002). Especially, for this model, the

adelic wave function (which unifies the wave function over the field of real numbers and wave functions over the field of p -adic numbers), is in the form

$$\Psi(a) = \Psi_\infty(a) \prod_p \Psi_p(a_p), \quad (35)$$

where $\Psi_\infty(a)$ is a standard wave function and $\Psi_p(a_p)$ are p -adic wave functions. It is very important that only for finite numbers of p , p -adic wave functions can be different from Ω function which is defined by the (12).

At this place we indicate a considerable similarity between the action (34) for the de Sitter model in 2+1 dimensions and the action (66) for the tachyon field in the zero dimensional model, i.e. “quadratically damped particle under gravity“. Tachyon fields, inflation and their classical and quantum aspects are discussed in Chapters 8 and 9.

5. p -ADIC MODELS IN THE HARTLE-HAWKING PROPOSAL

The Hartle-Hawking proposal for the wave function of the universe is generalized to p -adic case in Refs. (Dragovich and Nestic, 1996; Dragovich, 1995). In this approach, p -adic wave function is a solution of the integral

$$\Psi_p(q^\alpha) = \int_{|N|_p \leq 1} dN \mathcal{K}_p(q^\alpha, N; 0, 0), \quad (36)$$

where p -adic integration has to be performed over the p -adic ball B_0 .

5. 1. MODELS OF THE DE SITTER TYPE

It is well-known that there are many, many cosmological models. It happens, surprisingly or not, that one of the very first models, the de Sitter model, despite its simplicity, and at the first glance artificiality, is still very important and actual one. There are also many variations of the “original“ de Sitter model, all of them named as models of “de Sitter type“. As it is mentioned and well-known, models of the de Sitter type are models with cosmological constant Λ and without matter fields. We consider two minisuperspace models of this type, with $D = 4$ and $D = 3$ space-time dimensions. For $D = 3$ using (30) for the propagator of this model we have

$$\mathcal{K}_v(a'', N; a', 0) = \lambda_v \left(-\frac{2\sqrt{\lambda}}{\sinh(N\sqrt{\lambda})} \right) \left| \frac{\sqrt{\lambda}}{\sinh(N\sqrt{\lambda})} \right|_v^{1/2} \chi_v(-\bar{S}_v(a'', N; a', 0)). \quad (37)$$

The p -adic Hartle-Hawking wave function is

$$\Psi_p(a, \lambda) = \int_{|N|_p \leq 1} dN \frac{\lambda_p(-2N)}{|N|_p^{1/2}} \chi_p \left(\frac{\sqrt{\lambda} \coth(N\sqrt{\lambda})}{2} a^2 \right), \quad (38)$$

which after p -adic integration becomes

$$\Psi_p(a, \lambda) = \begin{cases} \Omega(|a|_p), & |\lambda|_p \leq p^{-2} \\ \frac{1}{2} \Omega(|a|_2), & |\lambda|_2 \leq 2^{-4}. \end{cases} \quad (39)$$

The de Sitter model in $D = 4$ space-time dimensions is described by the metric (Halliwell and Luoko, 1989)

$$ds^2 = \sigma^2 \left(-\frac{N^2}{q(t)} dt^2 + q(t) d\Omega_3^2 \right). \quad (40)$$

For the v -adic classical action

$$\bar{S}_v(q'', T; q', 0) = \frac{\lambda^2 T^3}{24} - [\lambda(q' + q'') - 2] \frac{T}{4} - \frac{(q'' - q')^2}{8T} \quad (41)$$

the corresponding propagator is

$$\mathcal{K}_v(q'', T|q', 0) = \frac{\lambda_v(-8T)}{|4T|_v^{1/2}} \chi_v(-\bar{S}_v(q'', T|q', 0)). \quad (42)$$

5. 2. MODEL WITH A HOMOGENEOUS SCALAR FIELD

To deal with the models of the de Sitter type is very instructive. However, it is also important to consider models with some matter content. If we use metric in the form (Garray et al., 1991)

$$ds^2 = \sigma^2 \left(-N^2(t) \frac{dt^2}{a^2(t)} + a^2(t) d\Omega_3^2 \right), \quad (43)$$

the gravitational part of the action in the form (32) (with $D = 4$), and a suitable action for a scalar field

$$S_{matter} = -\frac{1}{2} \int_M d^4x \sqrt{-g} [g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + V(\Phi)], \quad (44)$$

then, after some substitutions, we get the corresponding classical action and propagator as follows:

$$\begin{aligned} \bar{S}_p(x'', y'', N|x', y', 0) &= \frac{\alpha^2 - \beta^2}{24} N^3 + \frac{1}{4} (2 - \alpha(x' + x'') - \beta(y' + y'')) N \\ &\quad + \frac{-(x'' - x')^2 + (y'' - y')^2}{8N}, \end{aligned} \quad (45)$$

$$\mathcal{K}_p(x'', y'', N|x', y', 0) = \frac{1}{|4N|_p} \chi_p(-\bar{S}_p(x'', y'', N; x', y', 0)). \quad (46)$$

As we have shown (Dragovich and Nesić, 1999) for this model, a p -adic Hartle-Hawking wave function in the form of Ω - function does not exist. This leads to the conclusion that either the above model is not adelic, or that p -adic generalization of the Hartle-Hawking proposal is not an adequate one. However, if in the action (45) we take $\beta = 0$, $y = 0$, then we get classical action for the de Sitter model (41), and such model, as we showed it, is the adelic one. The similar conclusion holds also for some other models in which minisuperspace is not one-dimensional. This is a reason to regard p -adic and adelic minisuperspace quantum cosmology just as the corresponding application of p -adic and adelic quantum mechanics without the Hartle-Hawking proposal.

We demonstrated that p -adic (and adelic) to the de Sitter model(s) works quite well, at least at a “formal“ level, and can be used in higher dimensional spaces. These results can be useful for examination of the accelerating scenarios of an expanding Universe.

6. MINISUPERSPACE MODELS IN p -ADIC AND ADELIC QUANTUM MECHANICS

In this approach we investigate conditions under which quantum-mechanical p -adic ground state exists in the form of Ω -function and some other eigenfunctions. This approach leads to the desired result and it enables adelization of all exactly soluble minisuperspace cosmological models, usually with some restrictions on the parameters of the models. One can speculate, but also continue a study, that nonarchimedean geometry or “nonarchimedean phase“ in evolution of the Universe restricts a set of initial conditions and a set of Lagrangians related to a realistic dynamics of our Universe (Djordjevic et al. 2000). The necessary condition for the existence of an adelic model is an existence of p -adic quantum-mechanical ground state $\Omega(|q_\alpha|_p)$, i.e.

$$\int_{|q_\alpha'|_p \leq 1} \mathcal{K}_p(q_\alpha'', N; q_\alpha', 0) dq_\alpha' = \Omega(|q_\alpha''|_p), \quad (47)$$

and, analogously, if a system is in the state $\Omega(p^\nu |q_\alpha|_p)$. If p -adic ground state is of the form of the δ -function, we will investigate conditions under which the corresponding kernel of the model satisfies equation

$$\int_{Q_p} \mathcal{K}_p(q_\alpha'', T; q', 0) \delta(p^\nu - |q'_\alpha|_p) dq'_\alpha = \chi_p(ET) \delta(p^\nu - |q''_\alpha|_p), \quad (48)$$

with zero energy $E = 0$. In the following, we apply (47) and (48) to the some minisuperspace models.

6.1. THE DE SITTER MODEL IN $D = 3$ DIMENSIONS

Let us demonstrate quantum mechanical approach to p -adic (real and adelic as well) cosmological models through two simple but relevant and instructive examples. In case we choose the de Sitter model in 3 dimensions, application of the above exposed formalism of p -adic quantum mechanics enable us to calculate (Dragovich and Nestic, 1999) the ground state of the Universe

$$\Psi_p(a, N) = \begin{cases} \Omega(|a|_p), & |N|_p \leq 1, \quad p \neq 2, \\ \Omega(|a|_2), & |N|_2 \leq \frac{1}{4}, \quad p = 2, \end{cases} \quad (49)$$

with conditions $|\lambda|_p \leq 1$ and $|\lambda|_2 \leq 2$. We also found

$$\Psi_p(a, N) = \begin{cases} \Omega(p^\nu |a|_p), & |N|_p \leq p^{-2\nu}, \quad |\lambda|_p \leq p^{4\nu} \\ \Omega(2^\nu |a|_2), & |N|_2 \leq 2^{-2-2\nu}, \quad |\lambda|_2 \leq 2^{1+4\nu}, \end{cases} \quad (50)$$

where $\nu = 1, 2, \dots$. The existence of the ground state in the form of the δ -function may be investigated by the equation (48), i.e.

$$\int_{Q_p} \mathcal{K}(a'', N; a', 0) \delta(p^\nu - |a'|) da' = \delta(p^\nu - |a''|), \quad (51)$$

with the kernel (37), what leads to the equation

$$\lambda_p \left(-\frac{\sqrt{\lambda}}{2 \sinh(N\sqrt{\lambda})} \right) \Big|_{\frac{\sqrt{\lambda}}{\sinh(N\sqrt{\lambda})}}^{1/2} \chi_p \left(-\frac{N}{2} + \frac{\sqrt{\lambda}}{2 \tanh(N\sqrt{\lambda})} a''^2 \right)$$

$$\times \int_{|a'|_p=p^\nu} \chi_p \left(\frac{\sqrt{\lambda}}{2 \tanh(N\sqrt{\lambda})} a'^2 - \frac{\sqrt{\lambda}}{\sinh(N\sqrt{\lambda})} a'' a' \right) da' = \delta(p^\nu - |a''|_p). \quad (52)$$

The above integration is performed over p -adic sphere with the radius p^ν and for $|\frac{N}{2}|_p \leq p^{2\nu-2}$, $\nu = 1, 0, -1, \dots$. As a result, on the left hand side we have

$$\chi_p \left(-\frac{N}{2} + \frac{\sqrt{\lambda}}{2} \tanh(N\sqrt{\lambda}) a''^2 \right).$$

To have an equality, the norm of argument of the additive character must be equal or less than unity. This requirement leads to the condition

$$\left| \frac{\sqrt{\lambda} \tanh(N\sqrt{\lambda}) a''^2}{2} \right|_p = p^{4\nu-2} |\lambda|_p \leq 1, \Leftrightarrow |\lambda|_p \leq p^{2-4\nu}.$$

This (for the p -adic norms of N and λ) is also related to the domain of convergence of the analytic function $\tanh x$,

$$|N\sqrt{\lambda}|_p \leq |N|_p |\lambda|_p^{1/2} \leq p^{2\nu-2} \cdot p^{1-2\nu} = p^{-1}, \forall \nu.$$

If $p = 2$, then condition $|N|_2 \leq 2^{2\nu-3}$ holds, for $\nu = 1, 0, -1, -2, \dots$, and we are in the domain of convergence. Finally, we conclude that also p -adic ground state

$$\Psi_p(a, N) = \begin{cases} \delta(p^\nu - |a|_p), & |N|_p \leq p^{2\nu-2}, & |\lambda|_p \leq p^{2-4\nu}, \\ \delta(2^\nu - |a|_2), & |N|_2 \leq 2^{2\nu-3}, & |\lambda|_2 \leq 2^{-4\nu}, \end{cases} \quad (53)$$

exists for $\nu = 1, 0, -1, -2, \dots$

We can note that in this approach there is Ω function as a ground state - wave function of the Universe. In fact, there is more than one state we can consider as the vacuum state. It also means that structure of p -adic (adelic) vacuum is quite rich and can be a source of several scenarios in the inflation theory, including tachyons.

6. 2. MODEL WITH A HOMOGENEOUS SCALAR FIELD

When one considers the two-dimensional minisuperspace model with two decoupled degrees of freedom (introduced in Subsection 5.2) we find that the corresponding ground state is of the form $\Omega(|x|_p)\Omega(|y|_p)$, i.e.

$$\Psi_p(x, y, N) = \begin{cases} \Omega(|x|_p)\Omega(|y|_p), & |N|_p \leq 1, \\ \Omega(|x|_2)\Omega(|y|_2), & |N|_2 \leq \frac{1}{2}, \end{cases} \quad (54)$$

with $\alpha = 4 \cdot 3 \cdot l_1$, $\beta = 4 \cdot 3 \cdot l_2$, $l_1, l_2 \in \mathbb{Z}$, and also

$$\Psi_p(x, y, N) = \begin{cases} \Omega(p^\nu |x|_p)\Omega(p^\mu |y|_p), & |\alpha|_p \leq |3|_p^{1/2} p^{3\nu}, & |\beta|_p \leq |3|_p^{1/2} p^{3\mu}, \\ \Omega(2^\nu |x|_2)\Omega(2^\mu |y|_2), & |\alpha|_2 \leq 2^{3\nu-1}, & |\beta|_2 \leq 2^{3\mu-1}, \end{cases} \quad (55)$$

where $\nu, \mu = 1, 2, 3, \dots$. As in the previous cases, we can also investigate the existence of the vacuum state of the form $\delta(p^\nu - |x|_p)\delta(p^\mu - |y|_p)$. After some calculations we find p -adic wave function for the ground state to be in the form

$$\Psi_p(x, y, N) = \begin{cases} \delta(p^\nu - |x|_p)\delta(p^\mu - |y|_p), & |N|_p \leq p^{2\nu, \mu-2}, & |\alpha, \beta|_p \leq p^{2-3\nu, \mu}, \\ \delta(2^\nu - |x|_2)\delta(2^\mu - |y|_2), & |N|_2 \leq 2^{2\nu, \mu-1}, & |\alpha, \beta|_2 \leq 2^{-3\nu, \mu}, \end{cases} \quad (56)$$

where $\nu, \mu = 0, -1, -2, \dots$

It means that the “quantum mechanical“ approach to quantum cosmology does not have an obvious contradiction and instability. As we know, in the p -adic generalization of Hartle-Hawking approach, for the multidimensional minisuperspace, a ground state (in particular Ω function) is missing.

7. p -ADIC INFLATION

Cosmological inflation has become an integral part of the standard model of the universe. It provides important clues for structure formation in the universe and is capable of removing the shortcomings of standard cosmology.

Many string theorists and cosmologists have turned their attention to building and testing stringy models of inflation in recent years. The goals have been to find natural realizations of inflation within string theory, and novel features which would help to distinguish the string-based models from their more conventional field theory counterparts. In most examples to date, string theory has been used to derive an effective 4D field theory operating at energies below the string scale and all the inflationary predictions are made within the context of this low energy effective field theory. This is a perfectly valid approach to string cosmology, but, at least, a few problems still exist. For instance it is often very difficult to identify features of string theory inflation that cannot be reproduced in more conventional models. Thus, there is motivation to consider models in which inflation takes place at higher energy scales where stringy corrections to the low energy effective action are playing an important role. This is usually daunting since the field theory description should be supplemented by an infinite number of higher dimensional operators at energies above the string scale, whose detailed form is not known. Because of that, to study nonlocality (intimately connected with p -adic and nonarchimedean themes (Dragovich, 2009), as ubiquitous in string field theory, and to consider a broad class of nonlocal inflationary models is a quite interesting area of research.

Gibbons (Gibbons, 2003) has emphasized the cosmological implication of tachyonic condensate rolling towards its ground state. The tachyonic matter might provide an explanation for inflation at the early epochs and could contribute to a new form of dark matter at later times. A recent paper on p -adic inflation (Barnaby et al., 2007) gives rise to the hopes that nonlocal inflation can succeed where the real string theory fails. p -Adic string theory, initiated by Volovich and his pioneering paper (Volovich, 1987) and developed by Arefeva, Dragovich, Goshal, Frampton, Freund, Sen, Witten and many other, despite some open and serious problems is an interesting and wide field of research. For a review and a considerable list of references see (Dragovich et al., 2009). In our approach we will just use some results relevant for real and p -adic tachyons, suitable for further study in inflation and suggest to a motivated reader to find more details in the noted references.

Starting from the action of the p -adic string, with m_s the string mass scale and g_s the open string coupling constant,

$$S = \frac{m_s^4}{g_p^2} \int d^4x \left(-\frac{1}{2} \phi p^{-\frac{-\partial_x^2 + \nabla^2}{2m_s^2}} \phi + \frac{1}{p+1} \phi^{p+1} \right), \quad \frac{1}{g_p^2} = \frac{1}{g_s^2} \frac{p^2}{p-1}, \quad (57)$$

for the open string tachyon scalar field $\phi(x)$, it has been shown that a p -adic tachyon

drives a sufficiently long period of inflation while rolling away from the maximum of its potential. Even though this result is constrained by $p \gg 1$ and obtained by an approximation, it is a good motivation to consider p -adic inflation for different tachyonic potentials. In particular, it would be interesting to study p -adic inflation in quantum regime and in adelic framework to overcome the constraint $p \gg 1$, with an unclear physical meaning. For more details, and further development see (Joukovskaya 2007).

8. CLASSICAL AND QUANTUM TACHYONS

A. Sen proposed a field theory of tachyon matter a few years ago (Sen, 2002, 2005). The action is given as:

$$S = - \int d^{D+1}x V(T) \sqrt{1 + \eta^{ij} \partial_i T \partial_j T} \quad (58)$$

where $\eta_{00} = -1$ and $\eta_{\alpha\beta} = \delta_{\alpha\beta}$, $\alpha, \beta = 1, 2, 3, \dots, D$, $T(x)$ is the scalar tachyon field and $V(T)$ is the tachyon potential which unusually appears in the action as a multiplicative factor and has (from string field theory arguments) exponential dependence with respect to the tachyon field $V(T) \sim e^{-\alpha T/2}$. In this paper we will focus our attention on this type of the potential. It is very useful to understand and to investigate lower dimensional analogs of this tachyon field theory. The corresponding zero dimensional analog of a tachyon field can be obtained by the correspondence: $x^i \rightarrow t, T \rightarrow x, V(T) \rightarrow V(x)$. The action reads

$$S = - \int dt V(x) \sqrt{1 - \dot{x}^2}. \quad (59)$$

In what follows, all variables and parameters can be treated as real or p -adic without a formal change in the obtained forms. It is not difficult to see that action (59), with some appropriate replacement leads to the equation of motion for a particle with mass m , under a constant external force, in the presence of quadratic damping:

$$m\ddot{y} + \beta\dot{y}^2 = mg. \quad (60)$$

This equation of motion can be obtained from two Lagrangians (Jain et al., 2007; Dimitrijevic et al., 2008):

$$L(y, \dot{y}) = \left(\frac{1}{2} m \dot{y}^2 + \frac{m^2 g}{2\beta} \right) e^{2\frac{\beta}{m} y}, \quad (61)$$

$$L(y, \dot{y}) = -e^{-\frac{\beta}{m} y} \sqrt{1 - \frac{\beta}{mg} \dot{y}^2}. \quad (62)$$

Despite the fact that different Lagrangians can give rise to nonequivalent quantization, we will choose the form (61) that can be handled easily. The first one is better because of the presence of the square root in the second one. The general solution of the equation of motion is

$$y(t) = C_2 + \frac{m}{\beta} \ln[\cosh(\sqrt{\frac{g\beta}{m}} t + C_1)]. \quad (63)$$

For the initial and final conditions $y' = y(0)$ and $y'' = y(T)$, for the v -adic classical action we obtain

$$\bar{S}_v(y'', T; y', 0) = \frac{\sqrt{mg\beta}}{2 \sinh(\sqrt{\frac{g\beta}{m}}T)} \left[(e^{\frac{2\beta}{m}y'} + e^{\frac{2\beta}{m}y''}) \cosh(\sqrt{\frac{g\beta}{m}}T) - 2e^{\frac{\beta}{m}(y'+y'')} \right]. \quad (64)$$

In the p -adic case, we get a constraint which arises from the investigation of the domain of a convergence analytical function which appears during the derivation of the formulae (63). This constraint is $|\dot{y}|_p \leq \frac{1}{p} |\sqrt{\frac{gm}{\beta}}|_p$.

By the transformation $X = \frac{m}{\beta} e^{\frac{\beta}{m}y}$, we can convert Lagrangian (61) in a more suitable, quadratic form

$$L(X, \dot{X}) = \frac{m\dot{X}^2}{2} + \frac{g\beta X^2}{2}. \quad (65)$$

For the conditions $X' = X(0)$, and $X'' = X(T)$, action for the classical v -adic solution $\bar{X}(t)$ is

$$\bar{S}_v(X'', T; X', 0) = \frac{\sqrt{mg\beta}}{2 \sinh(\sqrt{\frac{g\beta}{m}}T)} \left[(X'^2 + X''^2) \cosh(\sqrt{\frac{g\beta}{m}}T) - 2X'X'' \right]. \quad (66)$$

We note that this action is different from the action (34) only in one constant term. Because action (66) is quadratic one (with respect to the initial and final point), the corresponding kernel is (Djordjevic and Dragovich 1997)

$$\mathcal{K}_v(X'', T; X', 0) = \lambda_v \left(\frac{1}{2h} \frac{\sqrt{g\beta m}}{\sinh(\sqrt{\frac{g\beta}{m}}T)} \right) \left| \frac{1}{h} \frac{\sqrt{g\beta m}}{\sinh(\sqrt{\frac{g\beta}{m}}T)} \right|_v^{1/2} \chi_v \left(-\frac{1}{h} \bar{S}_v \right), \quad (67)$$

where χ_v is the v -adic additive character (Vladimirov 1994).

In what follows, we apply (47) and (48) to our model. As a result for the p -adic wave functions (in the case $p \neq 2$), we get

$$\Psi_p(X) = \Omega(|X|_p), \quad |T|_p \leq \left| \frac{m}{2h} \right|_p, \quad \left| \frac{g\beta m}{4h^2} \right|_p < 1 \quad (68)$$

$$\Psi_p(X) = \Omega(p^\nu |X|_p), \quad |T|_p \leq \left| \frac{m}{2h} \right|_p p^{-2\nu}, \quad \left| \frac{g\beta m}{4h^2} \right|_p \leq p^{3\nu} \quad (69)$$

$$\Psi_p(X) = \delta(p^\nu - |X|_p), \quad \left| \frac{T}{2} \right|_p \leq \left| \frac{m}{h} \right|_p p^{2\nu-2}, \quad \left| \frac{g\beta m}{h^2} \right|_p \leq p^{2-3\nu}. \quad (70)$$

The above conditions are in accordance with the conditions for the convergence of the p -adic analytical functions which appear in the solution of the equation of motion (63) and the classical action (64). We see there is a wide freedom in choosing the parameters of the model, such as mass of the tachyon field m , damping factor β , parameter g related to the ‘‘strength of the constant gravity‘‘, and cosmological constant Λ which appears in the de Sitter (2 + 1) dimensional model. A relevant physical conclusion served from these relations still needs a more realistic model with tachyon matter and with a precise form of metrics.

9. CONCLUSION

In this paper, we find applications of p -adic numbers in quantum cosmology very interesting. It gives new possibilities to investigate the structure of space-time at the Planck scale. In the Hartle-Hawking approach the wave function of a spatially closed universe is defined by Feynman's path integral method. The action is a function of the gravitational and matter fields, and integration is performed over all compact real four-metrics connecting two three-space states. According to Feynman's integration over all real compact metrics, this approach generalizes to all corresponding compact p -adic metrics. However, it does not lead to the adequate adelic picture and generalization for a wide class of the minisuperspace models. From the other side, the consideration of minisuperspace models in the framework of adelic quantum mechanics gives the appropriate adelic generalization. Moreover, we can conclude that all these models lead to the picture of space-time as a discrete one. Namely, for all the above models there exists adelic wave function

$$\Psi(q^1, \dots, q^n) = \prod_{\alpha=1}^n \Psi_{\infty}(q_{\infty}^{\alpha}) \prod_p \prod_{\alpha=1}^n \Omega(|q_p^{\alpha}|_p), \quad (71)$$

where $\Psi_{\infty}(q_{\infty}^{\alpha})$ are the corresponding wave functions of the universe in standard cosmology. Adopting the usual probability interpretation of the wave function (71) in rational points of q^{α} , we have

$$|\Psi(q^1, \dots, q^n)|_{\infty}^2 = \prod_{\alpha=1}^n |\Psi_{\infty}(q^{\alpha})|_{\infty}^2 \prod_p \prod_{\alpha=1}^n \Omega(|q_p^{\alpha}|_p), \quad (72)$$

because $(\Omega(|q_p^{\alpha}|_p))^2 = \Omega(|q_p^{\alpha}|_p)$. As a consequence of Ω -function properties we have

$$|\Psi(q^1, \dots, q^n)|_{\infty}^2 = \begin{cases} |\Psi_{\infty}(q^{\alpha})|_{\infty}^2, & q^{\alpha} \in Z, \\ 0, & q^{\alpha} \in Q \setminus Z. \end{cases} \quad (73)$$

This result leads to the discretization of minisuperspace coordinates q^{α} , because probability is nonzero only in the integer points of q^{α} . Keeping in mind that Ω function is invariant with respect to the Fourier transform, this conclusion is also valid for the momentum space. Note that this kind of discreteness depends on adelic quantum state of the universe. When system is in an excited state, then the sharp discrete structure disappears, and minisuperspace, as well as configuration space in quantum mechanics, demonstrate usual properties of real space.

In spite of the very attractive features of the tachyonic inflation, first of all, the rolling tachyon condensate, this approach faces difficulties such as reheating (Sami, 2004). It seems that both mechanisms, based on real tachyons - conventional reheating mechanism and quantum mechanical particle production during inflation - do not work. Recent results in nonlocal (p -adic) tachyon inflation (Barnaby et al., 2007; Joukovskaya, 2007), in which a p -adic tachyon drives a sufficiently long period of inflation while rolling away from the maximum of its potential deserve much more attention. The *classical* p -adic models succeed with inflation where the real string theory fails. In this paper we have calculated a *quantum* propagator for the p -adic and adelic tachyons, found conditions for the existence of the vacuum state of p -adic and adelic tachyons, noted interesting relations with the minisuperspace closed

homogenous isotropic model in $(2 + 1)$ dimensions using Einstein gravity with a cosmological constant and an antisymmetric tensor field matter source (Djordjevic et al., 2002, Halliwell and Myers, 1989). We have shown that the new results can give rise to a better understanding of the p -adic and real quantum tachyons, their relation via Freund-Witten formula and a possible role of tachyon field as a dark matter. Our results can also be used as a basis for further investigation of (p -adic) quantum mechanical damped systems and corresponding wave functions of the universe in the minisuperspace models based on the tachyonic matter with different potentials. Further investigation should contribute to the better understanding of quantum rolling tachyon scenario in a real (Ambjorn and Janik 2004) and p -adic case.

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