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HAMILTONIAN APPROACH TO DP-BRANE NONCOMMUTATIVITY [1]

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Abstract. In this article we investigate Dp-brane noncommutativity using Hamiltonian approach. We consider separately open bosonic string and type IIB superstring which endpoints are attached to the Dp-brane. From requirement that Hamiltonian, as the time translation generator, has well defined derivatives in the coordinates and momenta, we obtain boundary conditions directly in the canonical form. Boundary conditions are treated as canonical constraints. Solving them we obtain initial coordinates in terms of the effective ones as well as effective momenta. Presence of momenta implies noncommutativity of the initial coordinates. Effective theory, defined as initial one on the solution of boundary conditions, is its Ω even projection, where Ω is world-sheet parity transformation $\Omega : \sigma \to -\sigma$. The effective background fields are expressed in terms of Ω even and squares of the Ω odd initial background fields.

1. INTRODUCTION

Quantization of the open bosonic string ending on the Dp-brane, with constant metric $G_{\mu\nu}$ and antisymmetric Neveu-Schwarz field $B_{\mu\nu}$, leads to the noncommutativity of the Dp-brane manifold (reference starting with Connes, A.). Inclusion of the dilaton field Φ linear in x^μ was studied in the second, sixth and ninth reference. In the second one, the Dirichlet boundary conditions were constructed, while in the sixth and ninth reference, the noncommutativity structure was analyzed. The ninth reference considers the conformal part of the world-sheet metric F as a dynamical variable, introducing an additional boundary condition corresponding to F. Dp-brane is introduced by choosing Neumann boundary conditions for x^i $(i = 0, 1, \dots, p)$ and Dirichlet ones for the rest ones x^a $(a = p+1, \dots, D-1)$, where D is the number of the space-time dimensions.

We consider the cases when squares of the dilaton gradient a_i, either with respect to the closed string metric G_{ij}, $a^2 = G^{ij}a_ia_j$, or with respect to the effective one G^{eff}_{ij}, $\tilde{a}^2 = (G^{-1}_{eff})^{ij}a_ia_j$, take some particular values. As a consequence of these conditions, the first class constraints and local gauge symmetries appear. Some of the initial Dp-brane coordinates change the corresponding boundary conditions from Neumann to Dirichlet and decrease the number of the Dp-brane dimensions. Also, one commutative coordinate appears (see references of Nikolic, B. and Sazdovic, B. from Phys. Rev. D).

Beside described bosonic string, we consider the non(anti)commutativity for the type IIB superstring theory. We are going to investigate pure spinor formulation of type IIB superstring theory (see group of references starting with Berkovits). In particular, we use the form introduced by de Boer, J., which corresponds to constant graviton $G_{\mu\nu}$, antisymmetric field $B_{\mu\nu}$, two gravitinos ψ^α_μ and $\bar\psi^\alpha_\mu$, and R-R field strength $F^{\alpha\beta}$. In this approach dilaton Φ and two dilatinos, λ^α and $\bar\lambda^\alpha$, are set to zero.

It occurs that effective theory, even under world-sheet parity transformation Ω, is just type I closed superstring theory defined on orientifold projection. One of the main results of the article, beside examined noncommutativity structure, contains generalized expressions of type I superstring background fields in terms of type IIB ones (Nikolić, B. and Sazdović, B., Phys. Lett. B666 (2008) 400).

In both mentioned cases we use Hamiltonian approach. We obtain the boundary conditions purely canonically, from the requirement that Hamiltonian is differentiable in its canonical variables. This is more natural approach because we intend to treat these conditions as canonical constraints. Then we perform consistency procedure for the constraints and, using Taylor expansion, obtain compact, σ dependent form of the boundary conditions.

Solving boundary conditions we find the initial coordinates in terms of the effective coordinates and effective momenta, which have strictly defined Ω parity. Presence of the momenta causes the nonzero value of Poisson brackets of the initial coordinates. Effective theory, defined as initial one on the solution of boundary conditions, is its Ω even projection.

At the end we give some concluding remarks.

2. BOUNDARY CONDITIONS AS CANONICAL CONSTRAINTS

2. 1. HAMILTONIAN DERIVATION OF BOUNDARY CONDITIONS

Because we intend to treat the open string boundary conditions as canonical constraints, we will derive them directly in Hamiltonian form.

The Hamiltonian H is a generator of the time translation, so it must be differentiable in coordinates x^μ and their canonically conjugated momenta π_μ. Varying the Hamiltonian depending on the x'^μ, we obtain

$$\delta H = \delta H^{(R)} - \gamma^{(0)}_\mu \delta x^\mu \Big|_0^\pi \,, \tag{1}$$

where index R denotes the regular term of the form

$$\delta H^{(R)} = \int d\sigma (A_\mu \delta x^\mu + B^\mu \delta \pi_\mu) \,. \tag{2}$$

The Hamiltonian is properly defined, when the boundary term $\gamma^{(0)}_\mu \delta x^\mu \Big|_0^\pi$ in equation (1) vanishes. That is automatically fulfilled for closed strings, because they do not have endpoints. Assuming that the variations δx^μ are arbitrary at the open string endpoints, we obtain the *Neumann* boundary conditions in the canonical form, $\gamma_\mu^{(0)}\Big|_0^\pi = 0$. On the other hand, if we suppose that the string endpoints are fixed, $\delta x^\mu\Big|_0^\pi = 0$, the boundary conditions are known as *Dirichlet* boundary conditions.

This method can be generalized directly to the case of superstring.

2. 2. CONSISTENCY OF THE CONSTRAINTS

Checking the consistency of the constraints, we obtain an infinite set of constraints

$$\gamma_\mu^{(n)} \equiv \left\{ H \, , \gamma_\mu^{(n-1)} \right\} . \quad (n = 1, 2, \dots) \tag{3}$$

Using Taylor expansion, we rewrite all the constraints at $\sigma = 0$ and $\sigma = \pi$ in a more compact, σ-dependent form

$$\Gamma_\mu(\sigma) = \sum_{n\geq 0} \frac{\sigma^n}{n!} \gamma_\mu^{(n)}(\sigma = 0) \, , \quad \tilde{\Gamma}_\mu(\sigma) = \sum_{n\geq 0} \frac{(\sigma - \pi)^n}{n!} \gamma_\mu^{(n)}(\sigma = \pi) \, . \tag{4}$$

Both in the case of open bosonic string in the presence of gravitational $G_{\mu\nu}$, antisymmetric Neveu-Schwarz $B_{\mu\nu}$ and dilaton field Φ, and in the case of pure spinor formulation of type IIB superstring, boundary conditions at $\sigma = \pi$ are solved by 2π periodicity of canonical variables.

We complete the consistency procedure finding the Poisson bracket

$$\{ H \, , \Gamma_\mu \} = \Gamma'_\mu \, . \tag{5}$$

This means there are no more constraints in the theory.

The next step is separating the constraints to the first and second class. The first class constraints generate some gauge symmetry and decrease the dimension of Dp-brane. Presence of symmetry enables us to fix gauge conditions. Solving second class constraints, first class constraints and gauge conditions we obtain the initial variables in terms of effective ones.

3. BOSONIC STRING

In this section we will consider the open bosonic string in the presence of background fields: gravitational $G_{\mu\nu}$, antisymmetric Neveu-Schwarz $B_{\mu\nu}$ and dilaton field Φ.

3. 1. ACTION AND SPACE-TIME FIELD EQUATIONS

We will consider the action (see textbooks of Green, Scwarz, Witten; Polchinski)

$$S = \kappa \int_\Sigma d^2\xi \sqrt{-g} \left\{ \left[\frac{1}{2} g^{\alpha\beta} G_{\mu\nu}(x) + \frac{\varepsilon^{\alpha\beta}}{\sqrt{-g}} B_{\mu\nu}(x) \right] \partial_\alpha x^\mu \partial_\beta x^\nu + \Phi(x) R^{(2)} \right\} . \tag{6}$$

The quantum conformal invariance is determined by β functions

$$\beta^G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{4} B_{\mu\rho\sigma} B_\nu{}^{\rho\sigma} + 2 D_\mu a_\nu \, , \tag{7}$$

$$\beta^B_{\mu\nu} \equiv D_\rho B^\rho{}_{\mu\nu} - 2a_\rho B^\rho{}_{\mu\nu} \,, \tag{8}$$

$$\beta^\Phi \equiv 4\pi\kappa \frac{D-26}{3} - R + \frac{1}{12} B_{\mu\rho\sigma} B^{\mu\rho\sigma} - 4D_\mu a^\mu + 4a^2 \,, \tag{9}$$

where $R_{\mu\nu}$, R and D_μ are the space-time Ricci tensor, scalar curvature and covariant derivative, respectively, $B_{\mu\rho\sigma}$ is field strength of the field $B_{\mu\nu}$ and the vector a_μ is the gradient of the dilaton field. Vanishing of $\beta^G_{\mu\nu}$ and $\beta^B_{\mu\nu}$ implies $\partial_\mu \beta^\Phi = 0$, which means that β^Φ can be arbitrary constant.

We choose particular solution of the Eqs.(7) and (8)

$$G_{\mu\nu}(x) = G_{\mu\nu} = const \,, \quad B_{\mu\nu}(x) = B_{\mu\nu} = const \,, \quad \Phi(x) = \Phi_0 + a_\mu x^\mu \,. \quad (a_\mu = const) \tag{10}$$

Then Eq.(9) produces the condition

$$\beta^\Phi = 2\pi\kappa \frac{D-26}{6} + 4a^2 \equiv c \,, \tag{11}$$

under which the above solution is consistent with all equations of motion. On these conditions, the non-linear sigma model (6) becomes conformal field theory. There exists a Virasoro algebra with central charge c.

The remaining anomaly can be cancelled by introducing corresponding Wess-Zumino term, which in the case of the conformal anomaly takes the form of the Liouville action

$$S_L = -\frac{\beta^\Phi}{2(4\pi)^2\kappa} \int_\Sigma d^2\xi \sqrt{-g} R^{(2)} \frac{1}{\Delta} R^{(2)} \,, \quad \Delta = g^{\alpha\beta} \nabla_\alpha \partial_\beta \,, \tag{12}$$

where ∇_α is the covariant derivative with respect to the intrinsic metric $g_{\alpha\beta}$. Appropriate choice of the coefficient in front of Liouville action makes the theory fully conformally invariant and the complete action takes the form

$$S = S_{(G+B+\Phi)} + S_L \,. \tag{13}$$

We choose a particular background, decomposing the space-time coordinates $x^\mu(\xi)$ in Dp-brane coordinates denoted by $x^i(\xi)\,(i = 0, 1, ..., p)$ and the orthogonal ones $x^a(\xi)\,(a = p+1, p+2, ..., D-1)$, in such a way that $G_{\mu\nu} = 0$, $(\mu = i\,, \nu = a)$. For the other two background fields we choose: $B_{\mu\nu} \to B_{ij}$, $a_\mu \to a_i$. The part of the action describing the string oscillation in x^a directions decouples from the rest. Imposing the conformal gauge $g_{\alpha\beta} = e^{2F}\eta_{\alpha\beta}$, we obtain $R^{(2)} = -2\Delta F$ and the action (13) takes the form

$$S = \kappa \int_\Sigma d^2\xi \left[\left(\frac{1}{2} \eta^{\alpha\beta}\, {}^\star G_{ij} + \epsilon^{\alpha\beta} B_{ij} \right) \partial_\alpha x^i \partial_\beta x^j + \frac{2}{\alpha} \eta^{\alpha\beta} \partial_\alpha {}^\star F \partial_\beta {}^\star F \right] , \tag{14}$$

where we introduce

$${}^\star G_{ij} = G_{ij} - \alpha a_i a_j \,, \quad \frac{1}{\alpha} = \frac{\beta^\Phi}{(4\pi\kappa)^2} \,, \quad {}^\star F = F + \frac{\alpha}{2} a_i x^i \,. \tag{15}$$

For ${}^\star F$ we choose Neumann boundary condition.

3. 2. NONCOMMUTATIVITY

We will consider three cases: (1) $a^2 \neq \frac{1}{\alpha}$, $\tilde{a}^2 \neq \frac{1}{\alpha}$; (2) $a^2 = \frac{1}{\alpha}$, $\tilde{a}^2 \neq \frac{1}{\alpha}$; (3) $a^2 \neq \frac{1}{\alpha}$, $\tilde{a}^2 = \frac{1}{\alpha}$. We applied described Hamiltonian approach on the model given by the action (14) and obtained the following results:

- Because ${}^\star F$ decouples from the rest terms in the action in all three cases, it is commutative variable (in limit $\alpha \to \infty$ the role of commutative coordinate takes $a_i x^i$), while other ones are noncommutative.

- Case (1) - all constraints originating from boundary conditions are of the second class (Dirac constraints do not appear). The number of Dp-brane dimensions is unchanged.

- Case (2) - one Dirac constraint of the first class appears as a consequence of the fact that metric ${}^\star G_{ij}$ is singular. The number of Dp-brane dimensions decreases by 2.

- Case (3) - two constraints originating from boundary conditions are of the first class. The number of Dp-brane dimensions decreases by 2.

4. TYPE IIB SUPERSTRING THEORY

4. 1. PURE SPINOR FORMULATION OF THE TYPE IIB SUPERSTRING THEORY - ACTION AND BOUNDARY CONDITIONS

We will investigate the type IIB superstring theory using pure spinor formulation. Originally, theory is formulated using BRST charge and contains ghost fields. For our purpose, as well as in reference article of de Boer, it is enough to consider ghost independent part of the action. The action is of the form

$$
\begin{aligned}
S \quad &= \kappa \int_\Sigma d^2\xi \left[\frac{1}{2}\eta^{ab} G_{\mu\nu} + \varepsilon^{ab} B_{\mu\nu}\right] \partial_a x^\mu \partial_b x^\nu \\
+ \quad &\int_\Sigma d^2\xi \left[-\pi_\alpha(\partial_\tau - \partial_\sigma)(\theta^\alpha + \psi^\alpha_\mu x^\mu) + (\partial_\tau + \partial_\sigma)(\bar\theta^\alpha + \bar\psi^\alpha_\mu x^\mu)\bar\pi_\alpha\right] \\
+ \quad &\frac{1}{2\kappa} \int_\Sigma d^2\xi \pi_\alpha F^{\alpha\beta} \bar\pi_\beta \,, \qquad (16)
\end{aligned}
$$

where $D = 10$ dimensional space-time is parameterized by coordinates x^μ ($\mu = 0, 1, 2, \ldots, D-1$). The fermionic part of superspace is spanned by same chirality fermionic coordinates θ^α and $\bar\theta^\alpha$, while the variables π_α and $\bar\pi_\alpha$ are their canonically conjugated momenta. The gravitational field $G_{\mu\nu}$, antisymmetric Neveu-Schwarz field $B_{\mu\nu}$, gravitinos ψ^α_μ, $\bar\psi^\alpha_\mu$ and bispinor $F^{\alpha\beta}$ are constant.

According to the definition of canonical Hamiltonian, $\mathcal{H} = \dot{x}^\mu\pi_\mu + \dot\theta^\alpha\pi_\alpha + \dot{\bar\theta}^\alpha\bar\pi_\alpha - \mathcal{L}$, we have

$$
H = \int d\sigma \mathcal{H} \,, \quad \mathcal{H} = T_- - T_+ \,, \quad T_\pm = t_\pm - \tau_\pm \,, \qquad (17)
$$

where

$$
\begin{aligned}
t_\pm &= \mp\frac{1}{4\kappa}G^{\mu\nu}I_{\pm\mu}I_{\pm\nu}\,, \quad I_{\pm\mu}=\pi_\mu+2\kappa\Pi_{\pm\mu\nu}x'^\nu+\pi_\alpha\psi^\alpha_\mu-\bar\psi^\alpha_\mu\bar\pi_\alpha\,,\\
\tau_+ &= (\theta'^\alpha+\psi^\alpha_\mu x'^\mu)\pi_\alpha-\frac{1}{4\kappa}\pi_\alpha F^{\alpha\beta}\bar\pi_\beta\,,\\
\tau_- &= (\bar\theta'^\alpha+\bar\psi^\alpha_\mu x'^\mu)\bar\pi_\alpha+\frac{1}{4\kappa}\pi_\alpha F^{\alpha\beta}\bar\pi_\beta\,, \quad \Pi_{\pm\mu\nu}=B_{\mu\nu}\pm\frac{1}{2}G_{\mu\nu}\,,
\end{aligned}
\tag{18}
$$

and π_μ denotes momentum canonically conjugated to the coordinate x^μ.

Using canonical approach described in the Section 2, we will derive boundary conditions directly in terms of canonical variables. Varying Hamiltonian H we obtain

$$\delta H=\delta H^{(R)}-[\gamma^{(0)}_\mu\delta x^\mu+\pi_\alpha\delta\theta^\alpha+\delta\bar\theta^\alpha\bar\pi_\alpha]\big|_0^\pi\,, \tag{19}$$

where $\delta H^{(R)}$ is regular term without τ and σ derivatives of supercoordinates and supermomenta, while

$$\gamma^{(0)}_\mu=\Pi_{+\mu}{}^\nu I_{-\nu}+\Pi_{-\mu}{}^\nu I_{+\nu}+\pi_\alpha\psi^\alpha_\mu+\bar\psi^\alpha_\mu\bar\pi_\alpha\,. \tag{20}$$

As a time translation generator Hamiltonian must have well defined derivatives in its variables. Consequently, boundary term has to vanish and we obtain

$$\Big[\gamma^{(0)}_\mu\delta x^\mu+\pi_\alpha\delta\theta^\alpha+\delta\bar\theta^\alpha\bar\pi_\alpha\Big]\Big|_0^\pi=0\,. \tag{21}$$

For bosonic coordinates x^μ we choose Neumann boundary conditions implying

$$\gamma^{(0)}_\mu\big|_0^\pi=0\,. \tag{22}$$

In order to preserve $N=1$ SUSY of the initial $N=2$ SUSY, for fermionic coordinates we chose

$$(\theta^\alpha-\bar\theta^\alpha)\Big|_0^\pi=0\,,\qquad (\pi_\alpha-\bar\pi_\alpha)\big|_0^\pi=0\,, \tag{23}$$

where the first condition produces the second one.

4. 2. NONCOMMUTATIVITY AND EFFECTIVE THEORY

The solution of the constraints gives

$$x^\mu(\sigma)=q^\mu-2\Theta^{\mu\nu}\int_0^\sigma d\sigma_1 p_\nu+\frac{\Theta^{\mu\alpha}}{2}\int_0^\sigma d\sigma_1(p_\alpha+\bar p_\alpha)\,, \tag{24}$$

$$
\begin{aligned}
\theta^\alpha(\sigma) &= \eta^\alpha-\Theta^{\mu\alpha}\int_0^\sigma d\sigma_1 p_\mu-\frac{\Theta^{\alpha\beta}}{4}\int_0^\sigma d\sigma_1(p_\beta+\bar p_\beta)\,,\\
\bar\theta^\alpha(\sigma) &= \bar\eta^\alpha-\Theta^{\mu\alpha}\int_0^\sigma d\sigma_1 p_\mu-\frac{\Theta^{\alpha\beta}}{4}\int_0^\sigma d\sigma_1(p_\beta+\bar p_\beta)\,,
\end{aligned}
$$

$$\pi_\mu=p_\mu\,,\qquad \pi_\alpha=\frac{p_\alpha}{2}\,,\qquad \bar\pi_\alpha=\frac{\bar p_\alpha}{2}\,, \tag{25}$$

where

$$\eta^\alpha \equiv \frac{1}{2}(\theta^\alpha + \Omega\bar{\theta}^\alpha)\,,\ \bar{\eta}^\alpha \equiv \frac{1}{2}(\Omega\theta^\alpha + \bar{\theta}^\alpha)\,, p_\alpha \equiv \pi_\alpha + \Omega\bar{\pi}_\alpha\,,\ \ \bar{p}_\alpha \equiv \Omega\pi_\alpha + \bar{\pi}_\alpha \tag{26}$$

and

$$\begin{aligned}
\Theta^{\mu\nu} &= -\frac{1}{\kappa}(G^{-1}_{eff}BG^{-1})^{\mu\nu}\,, \quad \Theta^{\mu\alpha} = 2\Theta^{\mu\nu}(\Psi_{eff})^\alpha_\nu - \frac{1}{2\kappa}G^{\mu\nu}\psi^\alpha_{-\nu}\,,\\
\Theta^{\alpha\beta} &= \frac{1}{2\kappa}F_s^{\alpha\beta} + 4(\Psi_{eff})^\alpha_\mu\Theta^{\mu\nu}(\Psi_{eff})^\beta_\nu\\
&- \frac{1}{\kappa}\psi^\alpha_{-\mu}(G^{-1}BG^{-1})^{\mu\nu}\psi^\beta_{-\nu} + \frac{G^{\mu\nu}}{\kappa}\left[\psi^\alpha_{-\mu}(\Psi_{eff})^\beta_\nu + \psi^\beta_{-\mu}(\Psi_{eff})^\alpha_\nu\right]\,.
\end{aligned}\tag{27}$$

Here we have

$$G^{eff}_{\mu\nu} = G_{\mu\nu} - 4(BG^{-1}B)_{\mu\nu}\,, \quad (\Psi_{eff})^\alpha_\mu = \frac{1}{2}\psi^\alpha_{+\mu} + (BG^{-1})_\mu{}^\nu\psi^\alpha_{-\nu}\,, (\psi^\alpha_{\pm\mu} = \psi^\alpha_\mu \pm \bar{\psi}^\alpha_\mu)\,, \tag{28}$$

$$F_s^{\alpha\beta} = \frac{1}{2}(F^{\alpha\beta} + F^{\beta\alpha})\,. \tag{29}$$

On the solutions of the boundary conditions original string variables depend both on effective coordinates and effective momenta. This is a source of noncommutativity relations

$$\begin{aligned}
\{x^\mu(\sigma)\,, x^\nu(\bar{\sigma})\} &= 2\Theta^{\mu\nu}\theta(\sigma+\bar{\sigma})\,,\\
\{x^\mu(\sigma)\,,\theta^\alpha(\bar{\sigma})\} = -\Theta^{\mu\alpha}\theta(\sigma+\bar{\sigma}) \quad &, \qquad \{\theta^\alpha(\sigma)\,,\bar{\theta}^\beta(\bar{\sigma})\} = \frac{1}{2}\Theta^{\alpha\beta}\theta(\sigma+\bar{\sigma})\,,
\end{aligned}\tag{30}$$

where $\theta(\sigma+\bar{\sigma})$ is the step function.

Substituting the solutions of the constraints (25) into the expression for canonical Hamiltonian (17) we obtain effective one. It easily produces the effective Lagrangian, which on the equations of motion for momentum p_μ, turns into

$$\begin{aligned}
\mathcal{L}^{eff} &= \frac{\kappa}{2}G^{eff}_{\mu\nu}\eta^{ab}\partial_a q^\mu\partial_b q^\nu +\\
&- \pi_\alpha(\partial_\tau - \partial_\sigma)\left[\eta^\alpha + (\Psi_{eff})^\alpha_\mu q^\mu\right] + (\partial_\tau + \partial_\sigma)\left[\bar{\eta}^\alpha + (\Psi_{eff})^\alpha_\mu q^\mu\right]\bar{\pi}_\alpha\\
&+ \frac{1}{2\kappa}\pi_\alpha F^{\alpha\beta}_{eff}\bar{\pi}_\beta\,,
\end{aligned}\tag{31}$$

where

$$F^{\alpha\beta}_{eff} = \frac{1}{2}(F^{\alpha\beta} - F^{\beta\alpha}) - (\psi_{-\mu}G^{\mu\nu}\psi_{-\nu})^{\alpha\beta}\,. \tag{32}$$

Consequently, in effective theory only Ω symmetric parts survive: graviton $G^{eff}_{\mu\nu}$ in the NS-NS sector, gravitino $(\Psi_{eff})^\alpha_\mu$ in NS-R sector and antisymmetric part of R-R field strength, $F^{\alpha\beta}_{eff}$. In our approach the effective theory has been obtained from initial IIB one on the solution of boundary conditions. As its Ω symmetric part, it corresponds to the type I superstring theory, but with improved background fields, which beside standard linear terms in Ω even fields contain bilinear terms in Ω odd fields. As a consequence of boundary conditions at $\sigma = \pi$, this is a closed string theory.

5. CONCLUDING REMARKS

In this article we demonstrated Hamiltonian approach on two examples: open bosonic string in the presence of gravitational $G_{\mu\nu}$, Neveu-Schwarz field $B_{\mu\nu}$ and dilaton field Φ, and type IIB superstring theory. We derived boundary conditions using canonical methods. Demanding that Hamiltonian as time translation generator is differentiable in its canonical variables, we obtain the boundary conditions purely canonically.

Boundary conditions are treated as canonical constraints. Solving boundary conditions we obtained the initial coordinates in terms of effective ones and effective momenta. The general form of the solution is

$$x^\mu = q^\mu - 2\Theta^{\mu\nu} \int d\sigma p_\nu \,, \tag{33}$$

where q^μ and p_μ are Ω even parts of x^μ and π_μ and $\Theta^{\mu\nu}$ is some coefficient. It is obvious that presence of momenta p_μ in the solution for x^μ implies

$$\{x^\mu(\sigma)\,, x^\nu(\bar\sigma)\} = 2\Theta^{\mu\nu}\theta(\sigma+\bar\sigma) \neq 0\,. \tag{34}$$

In all considered cases effective theory, defined as initial one on the solution of boundary conditions, is its Ω even part.

In the case of bosonic string we discussed three cases: (1) $a^2 \neq \frac{1}{\alpha}$, $\tilde a^2 \neq \frac{1}{\alpha}$; (2) $a^2 = \frac{1}{\alpha}$, $\tilde a^2 \neq \frac{1}{\alpha}$; (3) $a^2 \neq \frac{1}{\alpha}$, $\tilde a^2 = \frac{1}{\alpha}$. We showed that there is one commutative coordinate in all cases, ${}^\star F$. All other coordinates are noncommutative. In the cases (2) and (3) a first class constraints appeared. In the case (2) constraint is of Dirac type, while in the case (3) two of the constraints originating from boundary conditions which initially were of the second class, became the first class ones. As a consequence of the presence of the first class constraints, two coordinates which initially satisfy Neumann, turned into Dirichlet boundary conditions and decreased the number of Dp-brane dimensions.

For the type IIB superstring we obtained that Poisson bracket $\{x^\mu\,, x^\nu\}$ have the same form as in the dilaton free bosonic case. The explanation is that solution for x^μ depends on the fermionic momenta but not on the corresponding canonically conjugated coordinates. The noncommutativity relations we obtained are generalization of the results obtained by de Boer. For $\psi^\alpha_\mu = \bar\psi^\alpha_\mu$ we confirmed results of the mentioned article.

The background fields of the effective theory consist of standard linear terms, which are even under Ω projection, and the improvements, which are squares of Ω odd fields of the initial theory.

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