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MODULI STABILISATION IN HETEROTIC STRING COMPACTIFICATIONS

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Abstract. In this paper I present a toy model which can be obtained from heterotic string compactifications on manifolds with SU(3) structure and analyse its vacuum structure.

The Standard Model of Particle Physics is believed to describe physical processes with high accuracy at/up to energy scales of order of 100 GeV. It is nevertheless generally accepted that it is not a fundamental theory and various theoretical issues point towards certain extensions at higher energies. Some of the most popular theories beyond the standard model include supersymmetric extensions (Minimal Supersymmetric Standard Model - MSSM) or Grand Unified Theories (GUT). Powered mostly by recent developements in string theory, supersymmetry is one of the features many people expect to see just around the corner at the LHC. Moreover it has been known that without low energy supersymmetry unification of the three coupling constants of the standard model does not happen, while this is a well established feature of MSSM.

String theory is yet another setup which has developed independently of the possible extensions of the Standard model and it is supposed to be valid at energy scales as high as the Planck scale – $10^{19} Gev$ (Green et al., 1986). Since all the known consistent string theories contain supersymmetry it is natural to try to relate string theory to the standard model via supersymmetry.

Matter fields in minimal supersymmetric theories in 4d come in so called chiral supermultiplets, Φ, which have as on-shell degrees of freedom a complex scalar ϕ field and a spin 1/2 field ψ. Gauge fields A come in vector multiplets which also contain spin 1/2 fields called gaugini. The Lagrangean for a theory containing matter fields charged under some gauge group (everything coupled to $N = 1$ supergravity) is given entirely by three functions (Wess and Bagger, 1992)

- the Kähler potential $K(\Phi, \bar{\Phi})$
- the superpotential $W(\Phi)$
- the gauge kinetic function $f_{ab}(\Phi)$

The Kähler potential is a real function of the (anti) chiral superfields Φ and $\bar{\Phi}$ while the superpotential and the gauge kinwtic function are holomorphic functions of the chiral superfields Φ. The Lagrangean for the bosonic components then reads

$$\begin{aligned}
\mathcal{L} &\sim -g_{i\bar{j}}\partial_\mu\phi^i\partial^\mu\bar{\phi}^{\bar{j}} - \tfrac{1}{4}Imf_{ab}F^a_{\mu\nu}F^{b\,\mu\nu} + \tfrac{i}{4}Ref_{ab}F^a_{\mu\nu}\tilde{F}^{b\,\mu\nu} - V \ , \\
g_{i\bar{j}} &= \partial_{\Phi^i}\partial_{\bar{\Phi}^{\bar{j}}}K(\Phi,\bar{\Phi}) \ , \\
V &= e^K\left(D_iW\overline{D_jW}g^{i\bar{j}} - 3|W|^2\right) + \tfrac{1}{2}Imf^{-1}_{ab}D^aD^b \\
D_iW &= \partial_{\Phi^i}W + (\partial_{\Phi^i}K)W \ .
\end{aligned}$$

One important aspect which we shall use later on is that the supersymmetric solutions in these theories (which are automatically extremal points of the potential) are given by the equations $D_iW = 0$.

In string theory there are instances where these functions can be computed under certain assumptions. Consistent string theories are supersymmetric and live in 10 space-time dimensions. In order to obtain a 4d theory one usually takes the route of compactifying six extra dimensions on some compact manifold. Then the details of the 4d theory depend on this internal manifold. In particular there will be parameters in the compactification which will appear as fields in four-dimensions. Such fields are called moduli as their potential is flat. Their vevs are free parameters in 4d and many of the relevant 4d quantities (masses, couplings) depend on these vevs. All 4d models derived from string theory contain these moduli and therefore it is necessary that one finds a mechanism of giving these fields a potential which eventually has a minimum which will determine all their vevs. It has been realised recently that modifying the compactification Annsatz by turning on fluxes for the field strengths of the various fields in the 10d theory one can actually stabilise many of the moduli fields (Grana, 2006). Also one can introduce what was called geometric fluxes by deforming the geometry in a particular way to include torsion. We shall concentrate on such models, without entering too many details.

There exist five consistent string theories in 10d: type IIA/B, type I, heterotic $SO(32)$ and $E_8 \times E_8$. Generally speaking, we shall be interested to obtain a 4d theory which resembles a supersymmetric GUT as much as possible. Therefore our purpose is to study a theory which has a gauge group G which is big enough to contain the standrad model gauge group $SU(3) \times SU(2) \times U(1)$ and which has chiral matter. However we shall not impose other restrictions like having only three families of matter or obtaining the correct couplings, but rather we shall be interested in studying moduli stabilisation in such models.

In order to obtain a theory with a non-Abelian gauge group in 4d, in type II theories one needs additional constructions like introducing branes or singularities in the internal manifold. Moreover $SO(32)$ turns out not to have the right representations in order to give chiral matter in 4d and therefore we shall concentrate in the following on the $E_8 \times E_8$ heterotic string.

Let us study more closely the stringy origin of the models we want to consider (Gurrieri et al., 2004; 2007). The bosonic fields in ten dimensions are the metric g_{MN}, an antisymmetric tensor field B_{MN}, the dilaton ϕ and the gauge fields in the adjoint of $E_8 \times E_8$. The ten dimensional theory is chiral and the gauge anomalies are cancelled via the so called Green-Schwarz mechanism which requires that the B-field transforms non-trivially under the gauge transformations. The action can be constructed in terms of the gauge invariant field strength H which has to obey the following Bianchi idnetity

$$dH = trF \wedge F - trR \wedge R \ , \tag{1}$$

where F denote the field strengths of the gauge fields and R denotes the Riemann curvature tensor viewed as a two-form.

Asking for supersymmetry in 4d imposes some constraints on the compactification manifold which has to be Calabi-Yau (has SU(3) holonomy). Since such manifolds are curved, $trR \wedge R \neq 0$ and one needs to choose a gauge background to cancel this contribution in the Bianchi identity. Therefore the gauge group will be broken to some subgroup which depends on the background gauge fields. One simple choice which one can always choose is the so called *standard embedding* which amounts to set $F = R$. Since, on Calabi–Yau manifolds, R is an $SU(3)$ valued two-form, the identification above will fix a $SU(3)$ subgroup inside E_8 and therefore the surviving gauge group in 4d will be the maximal commutant of $SU(3)$ inside E_8. Thus by the above simple considerations we have found a class of compactifications which lead to a $E_6 \times E_8$ gauge theory in 4d.

Let us briefly see what are the other fields we will encounter in the 4d theory. There will be moduli fields associated with possible deformations of the compactification geometry and there will be also matter fields which originate from the 10d gauge fields. The moduli fields are the usual Calabi–Yau moduli, namely, the complex structure deformations – complex fields Z^a, $a = 1..h^{2,1}$ – the complexified Kähler moduli – complex fields T^i, $i = 1..h^{1,1}$ – and the axio-dilaton, S. In the above, $h^{2,1}$ and $h^{1,1}$ stand for the corresponding Hodge numbers, ie the dimensions of the cohomology groups $H^{2,1}$ and $H^{1,1}$ respectively. There will also be moduli associated with the gauge bundle, but their treatment is more complicated and we shall ignore them in the following.

Now let us consider the matter fields in four dimensions. Such fields come from the 10d gaugini which are spin 1/2 fields which transform in the adjoint representation of E_8. This decomposes under the maximal subgroup $E_6 \times SU(3)$ as (Slansky, 1981)

$$\mathbf{248} = (\mathbf{78}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8}) \oplus (\mathbf{27}, \mathbf{3}) \oplus (\overline{\mathbf{27}}, \bar{\mathbf{3}}) \ . \tag{2}$$

We see therefore that the 4d matter fields transform in either $\mathbf{27}$ or $\overline{\mathbf{27}}$ of E_6 and we need to expand the 10d gaugini in spinors on the internal manifold which transform in the (anti)fundamental of the $SU(3)$ holonomy group. Moreover, we are interested in massless matter fields which due to the decomposition of the 10d Dirac operator

$$\not{\nabla}_{10} = \not{\nabla}_4 + \not{\nabla}_6 \ , \tag{3}$$

have to be zero eigenspinors of the 6d Dirac operator. Mathematically, for the standard embedding case, this means

$$\not{\nabla}_6 \psi_{\mathbf{3}} \quad = \quad 0 \quad \leftrightarrow \quad H^{0,1}(T^{1,0}X) \equiv H^{2,1}(X) \ ; \tag{4}$$

$$\not{\nabla}_6 \psi_{\bar{\mathbf{3}}} \quad = \quad 0 \quad \leftrightarrow \quad H^{0,1}(T^{0,1}X) \equiv H^{1,1}(X) \ , \tag{5}$$

so again, like the moduli fields, the matter fields are in one to one corrrespondence with the $(1,1)$ and $(2,1)$ (de Rahm) cohomology groups.

Therefore, the effective 4d theory contains $h^{1,1}+h^{2,1}+1$ neutral chiral fields, $h^{2,1}$ charged fields in the $\mathbf{27}$ of E_6 and $h^{1,1}$ charged fields in the $\overline{\mathbf{27}}$. In general, we expect that matter and antimatter pairs up at the GUT scale and we are left with a net number of $|h^{1,1}-h^{2,1}|=|\chi|/2$ generations.

As explained before, one can compute from the compactification the functions which fully specify the 4d theory. The superpotential is cubic in the charged fields ($\mathbf{27}^3$) and does not depend on the moduli fields. However, a mass term $\mathbf{27}\cdot\overline{\mathbf{27}}$ is not present even if in field theory one may expect that any gauge invariant combination like this mass term is present. Next, the gauge kinetic function is given by the axio-dilaton alone $f_{ab}=S\delta_{ab}$. Finally, the Kähler potential for the moduli is just the typical one for string compactifications while the one for the matter fields has almost canonical form $\mathbf{27}\cdot\mathbf{27}^*$ with a coefficient which depends on the moduli fields.

It is possible to generalise such compactifications in order to study the question of moduli stabilisation. For this purpose one can turn on H-fluxes and consider manifolds with $SU(3)$ structure rather than Calabi–Yau manifolds (Gurrieri et al., 2004; 2007; Ali and Cleaver, 2008). One can still find a way of solving the Bianchi identity by a variant of the standard embedding and many of the features (including the spectrum and the Kähler potentials) derived in the case of Calabi–Yau compactifications remain unchanged. The additional features which appear are superpotential terms for the moduli fields and one may also find mass terms for the matter fields of the form $\mathbf{27}\cdot\overline{\mathbf{27}}$.

In the following we shall analyse a simple example of models which can be derived as above. Specifically, we shall consider a model with $h^{1,1}=1$ and $h^{2,1}=0$. Calabi–Yau manifolds which have such Hodge numbers are not known to exist, but two comments can be made here in order to argue for the relevance of these toy models. First of all, the generalisation to arbitrary $h^{1,1}$ is completelely straightforward and the results obtained here are unchanged. Second of all, we have argued that by considering certain manifolds with $SU(3)$ structure one can pair up matter fields in $\mathbf{27}$ and in $\overline{\mathbf{27}}$ and give them GUT scale masses. Similar mass terms in the suprepotential can be found also between Kähler and complex structure moduli and so one can imagine that for a theory with $h^{1,1}>h^{2,1}>0$, $h^{2,1}$ pairs of matter fields ($\mathbf{27}$ and $\overline{\mathbf{27}}$) and moduli (Kähler and complex structure) get large masses and decouple from the theory and we are effectively left with a 4d theory which formally has $h^{1,1}\neq 0$ and $h^{2,1}=0$.

Let us now continue with the model which has $h^{1,1}=1$ and $h^{2,1}=0$ (Micu, 2009). For simplicity we shall also ignore the axio-dilaton which does not play any role in our analysis. We denote the (complex) Kähler modulus by T and the charged field in $\overline{\mathbf{27}}$ by C^A, where A is the index running in the $\overline{\mathbf{27}}$ of E_6. Then, at the first order in α' the 4d model is specified by the following Kähler potential (Witten, 1985)

$$K=-3\log(T+\bar{T})+\alpha'\frac{6}{(T+\bar{T})}C^A\bar{C}_A+\mathcal{O}(\alpha'^2)\,, \tag{6}$$

and the superpotential

$$W=ieT-\frac{\alpha'}{3}j_{ABC}C^AC^BC^C+\mathcal{O}(\alpha'^2)\,. \tag{7}$$

In the above, e denotes the geometric flux parameter and j_{ABC} is the (totally symmetric) cubic invariant of E_6.

Let us now study the supersymmetric solutions of this theory which are given by

$$D_T W = ie - \frac{3}{(T+\bar{T})} W - \alpha' \frac{6C^A \bar{C}_A}{(T+\bar{T})^2} W + \mathcal{O}(\alpha'^2) \; ; \tag{8}$$

$$D_{C^A} W = \alpha' \left(-j_{ABC} C^B C^C + \frac{6\bar{C}_A}{(T+\bar{T})} W \right) + \mathcal{O}(\alpha'^2) \; . \tag{9}$$

We are not interested in the most general solution of these equations, but in some solution which preserves a gauge group which is large enough to include the Standard Model gauge group.

One obvious choice is a solution which preserves the full E_6 group. This necessarily has vanishing vev's for the charged fields, $< C^A >= 0$, and therefore equation (9) is identically satisfied. However, a quick inspection of equation (8) shows that it can only have a solution provided $e = 0$ which means that T remains a modulus. Therefore this solution is not satisfactory as it does not satisfy the requirement of moduli stabilisation (de Carlos et al., 2006).

Let us now concentrate on solutions which break the E_6 gauge group and we shall analyse the following choices

a. $E_6 \supset SO(10) \times U(1)$,

b. $E_6 \supset SU(3) \times SU(3) \times SU(3)$.

The corresponding decomposition of the $\overline{\mathbf{27}}$ under these subgroups is (Slansky, 1981)

a. $\overline{\mathbf{27}} = \overline{\mathbf{16}}^1 \oplus \mathbf{10}^{-2} \oplus \mathbf{1}^4$,

b. $\overline{\mathbf{27}} = (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}) \oplus (\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{3}}) \oplus (\mathbf{1}, \mathbf{3}, \mathbf{3})$.

In the case a, the interesting solution is when the singlet field in the corresponding decomposition acquires a non-vanishing vev, $< \mathbf{1} > \neq 0$, while the vev's of the other fields vanish $< \overline{\mathbf{16}} >= 0$ and $< \mathbf{10} >= 0$. This breaks the E_6 gauge group to $SO(10)$. However one can see that in this case there is no $\mathbf{1}^3$ coupling in the superpotential (in other words, $j_{1,1,1} = 0$ in (7)) and so equation (9) for the single field reads

$$D_1 W = 0 + \frac{3\bar{C}_1}{T+\bar{T}} W = 0 \; , \tag{10}$$

which only has a solution for vanishing $< W >$. Inserting this into (8) one again finds that a solution is possible only if $e = 0$ which is not satisfactory.

Let us consider in case b, a non-vanishing vev for $(\mathbf{1}, \mathbf{3}, \mathbf{3})$ which breaks the gauge group to $SU(3) \times SU(2) \times SU(2)$. In this case a coupling $(\mathbf{1}, \mathbf{3}, \mathbf{3})^3$ does exist and therefore the corresponding equation no longer implies $< W >= 0$. Let us denote the field which acquires a vev in this case by B^a. Then its corresponding equation reads

$$D_B W = B \cdot B + \bar{B} \cdot W = 0 \; . \tag{11}$$

The important point to notice here is that the approximation in which the effective theory was deerived assumes that the charged fields represent small fluctuations around the zero background and therefore we have to take $B \ll 1$. From the equation above we conclude that $W \sim B$, while from (8) we find that $W \sim eT$. Moreover, the supergravity approximation we have been assuming requires that $T+\bar{T} \gg 1$. Finally, since the fluxes are quantised in units of $1/sqrt\alpha'$ we conclude that this solution is out of the validity range of our approximations.

In conclusion we have analysed one of the simplest setups which contains a matter sector as well as a superpotential which has a chance to stabilise the closed string moduli. However we have shown that in this case one can not stabilise moduli at a point where all the approximations used in deriving the model hold. This is a serious drawback in the attempt to stabilise moduli in realistic string models, as, despite its simplicity, it may be relevant in certain classes of string compactifications. In particular the conclusions above immediately generalise to the case of multiple Kähler moduli and matter fields in $\overline{\mathbf{27}}$ and this latter situation can be understood as an effective theory where additional pairs of matter fields $(\mathbf{27}, \overline{\mathbf{27}})$ and, complex structure and Kähler moduli, have been given large masses and have decoupled from the spectrum.

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