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INVESTIGATION OF THE DYNAMICAL STRUCTURE AND DIFFUSION IN A SYSTEM OF HAMILTONIAN TYPE: 4-DIMENSIONAL SYMPLECTIC MAP

N. TODOROVIĆ

Astronomical Observatory, Volgina 7, 11060 Belgrade 38, Serbia
E–mail: ntodorovic@aob.bg.ac.yu

Abstract. The Nekhoroshev theorem (Nekhoroshev 1977) is one of the most important theorems in modern Hamiltonian dynamics. This theorem applies to quasi integrable Hamiltonian systems of type $H(I, \varphi) = h(I)+\varepsilon f(I, \varphi)$, where $h(I)$ is the integrable approximation, $f(I, \varphi)$ the perturbing function, ε is a small perturbing parameter, $I \in R^n$ are the actions and $\varphi \in T^n$ the angles of the system. With some additional geometrical and analytical properties, the theorem provides the stability of actions in exponentially long times. In recent years it has been shown that with some modifications the Nekhoroshev theorem can be applied to the problems in Solar system dynamic (Morbidelli and Guzzo 1997, Guzzo et al 2002, Efthymiopoulos and Sàndor 2005, Pavlović and Guzzo 2008).

In this work, we are interested to observe numerically a Nekhoroshev like behavior on a model given with a 4-dimensional symplectic map. The model is not in the quasi-integrable form, i.e. independently from the perturbation it contains some additional hyperbolic structures (they appear in the model as primary resonances). Since the hyperbolic structures exist even for zero perturbation, the system will belong to the class of the so called *a priori unstable systems*.

The main numerical tool used here was the Fast Lyapunov Indicator- *FLI*, introduced in (Froschlé et al. 1997, 2000). As an indicator of chaotic motion, FLI gives very precise and fast information about the chaoticity of an orbit. Also, among regular orbits, FLI is able to differentiate resonant from nonresonant motions. This property of FLI allows us to visualize the studied system and to obtain the Arnold web of the model (Froschlé et al. 2000). In such a way it was possible to observe the transition from a stable Nekhoroshev like structure (regular orbits dominate) to a globally chaotic system where resonances overlap, also known as Chirikov regime. Numerically, this transition happens when between 50% and 60% of orbits become chaotic ($FLI > 1.5 \; log \; t, t$ is the number of iteration).

In the second part of of the thesis the phenomen of diffusion was studied. The techniques that where used are described in Froeschlé et al. (2005) and Lega at al. (2003). For lower values of the perturbing parameter the slow diffusion along the chaotic border of a chosen low order resonance is observed and measured. This diffusion is of Arnold type (named by a model in Arnold, 1964), i.e. very slow. The diffusion coefficients show a broken power low against ε, which indicates an exponential change. This result could be treated as some kind of numerical confirmation of the Nekhoroshev behavior in the given model, even if the model does not strictly satisfy the Nekhoroshev theorem hypothesis (the quasi-integrability). For somewhat higher values of the perturbation, the diffusion becomes faster, leaving the initial resonance, and has a global character (the global Arnold diffusion). Numerically, such a phenomenon was observed in Lega et al. (2003). In this model, instead of global Arnold diffusion, the coexistence of two different kinds of diffusion is observed: a very slow one that we find in Nekhoroshev structures and a fast diffusion on a special subset of the Arnold web

made of low-order primary resonances. The slow diffusion can drive orbits to the fast web of the primary resonances where it takes quickly a global character.

The results of this work with more details can be find in Todorović et al. (2008).

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