

**MANUSCRIPT COLLECTION OF SOLVED PROBLEMS OF
GENERAL ASTRONOMY BY VOJISLAV MIŠKOVIĆ**

N. PEJOVIĆ

*Faculty of Mathematics, Studentski trg 16, 11000 Belgrade, Serbia
E-mail: nada@matf.bg.ac.yu*

Abstract. In this paper we present the first university problems book concerning astronomy written in Serbian. The topic is "**Zbirka rešenih zadataka iz opšte astronomije**" (A Collection of Solved Problems of General Astronomy) by Prof. Vojislav Mišković. The first part of this collection was published in 1956 in Belgrade. The second one is still in the form of a manuscript. Though completely prepared for publishing, it has never been published. From the methodical point of view the collection was interestingly and nicely arranged. All astronomical notions and formulae are treated in details and well explained so that no complementary textbook is needed. A special attention is devoted to the numerical solution of the problems, which is not only missing in Serbian schools, but also, as said by Mišković in the Preface, is not properly valued. This paper treats both parts of this Collection which have been digitized and are available in the Virtual Library of the National Digitization Centre (Virtual Library, <http://elib.matf.bg.ac.yu:8080/virlib/>). By means of the manuscript digitization the second part of the Collection has become available to public use.

1. INTRODUCTION

The first part of "**Zbirka rešenih zadataka iz opšte astronomije**" (A Collection of Solved Problems of General Astronomy) by Prof. Vojislav Mišković (hereafter referred to as the Collection) is the first university collection of astronomical problems printed in Serbian. It appeared in the middle of the last century. The need to publish this book arose, as written in the Preface by Mišković, on one hand due to an increase in the number of students who studied astronomy, and, on the other hand, due to the insufficient number of solved problems, not only in Serbian literature, but also abroad. Whereas the number of astronomical textbooks was sufficient in all European languages, the lack of collections containing solved problems was evident. The Collection was approved of printing through an act of the Textbook Commission of Belgrade University No 896 of August 10, 1956 as a textbook for students of the Faculty of Sciences. Though the approval concerned the Collection as a whole, for technical reasons, as written by Mišković, it had to be divided into two parts. The first one was published by *Naučna knjiga* in Belgrade in 1956 in 2000 copies, whereas the second part should have been published in the following year. Unfortunately, the second part of the Collection is still in the manuscript form. The manuscript was

given to the author of this paper in 1995 by Professor of Astronomy Jovan Simovljević (1929-2007) when he got retired. Since 1995 many problems contained in the manuscript have been solved by the students as exercises or exams of the subject General Astronomy at the Astronomy Department of the Faculty of Mathematics in Belgrade. Since Mišković was a European scholar and knew several foreign languages very well, he wrote the Collection using the most contemporaneous textbooks and scientific literature of that time. In its *Introduction* one finds about ten references – foreign textbooks, mostly in French, English and German. The problems taken from other authors have a designation, which indicates their origin. The digitization comprising the whole Collection has just been completed and in this way Mišković's original manuscript became available to the public, after half a century.

2. CONTENTS OF THE BOOKS

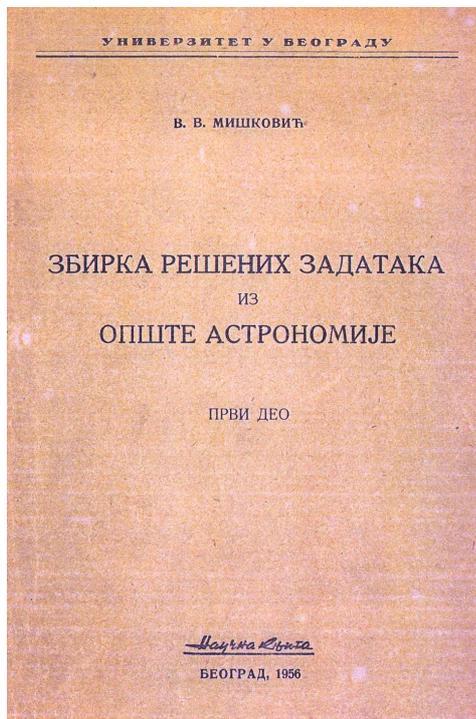


Figure 1: Cover page of the first part of the Collection.

The first chapter *Problems* contains 136 problems, whereas the second *Solutions*, much larger, contains methods, explanations and solutions concerning the problems of the manuscript. The *Manuscript* also contains an instruction how to prepare a single volume of the first part (already printed) and the second one (manuscript). According to *the enclosed plan*, Mišković planned a different arrangement of the whole collection.

Based on the contents presented above it is seen that the whole Collection should have covered six fields: spherical trigonometry, Earth as a celestial body, apparent diurnal motion of celestial sphere, astronomical refraction, elements of theory of motion of planets and comets and apparent annual motion of the Sun. The importance

The book printed is the first part of the Collection. It has 150 pages and consists of *Preface*, *Introduction* and two chapters. *Introduction* consists of three parts: *Review of Formulae of Spherical Trigonometry*, *Series* and *Review of Formulae for Transforming Astronomical Coordinate Systems*. The first chapter *Problems* concerns three fields: *Spherical Trigonometry*, *Earth as a Celestial Body* and *Apparent Diurnal Motion of Celestial Sphere*. The second, much larger, chapter *Solutions* contains the instructions and solutions for 126 problems of the Collection.

The manuscript contains the second part of the Collection consisting of *Introduction* and two chapters. The introduction to this part concerns three fields: *Astronomical Refraction*, *Elements of Theory of Motion of Planets and Comets* and *Apparent Annual Motion of the Sun*.

of 262 problems from these fields, which are explained and completely solved, is very great even today because there are no other collections containing solved astronomical problems written in Serbian. There are textbooks, both for secondary schools and universities, but often without any solved problem or example. For this reason the Collection appears as a real jewel in Serbian astronomical literature.

3. BIOGRAPHY OF PROFESSOR VOJISLAV MIŠKOVIĆ



Figure 2: Vojislav Mišković.

Vojislav Mišković (1892-1976) was born at Fužine in Croatia. He started to study astronomy at the Universities of Budapest and Goettingen shortly before the First World War. When the war began, he came illegally to Serbia, became a member of the volunteer squad and took an active part in the war till the defeat of the Serbian Army. After the demobilization he went to France to finish his studies. He graduated in 1919 and became assistant at the Astronomical Observatory in Marseille. From 1922 he was engaged as an astronomer at the Observatory of Nice. He got his PhD degree in 1924 at the University of Montpellier. In France he was very active as an astronomer. He organized new services at observatories, was editor of a journal in astronomy, organized and performed the astronomical measurement for the purpose of triangulation connecting

Corse with the French Eastern Alps, presented the first variant of his original astronomical instrument. For his contributions to stellar statistics he was awarded Valz prize by the French Academy of Sciences. Following an invitation he was elected in 1925 as Associate Professor of Practical Astronomy at the Faculty of Philosophy in Belgrade. He came to Belgrade in 1926 as an already recognized astronomer. At the same time he was appointed Director of the newly founded Astronomical Observatory in Belgrade. It may be said that an intensive development of astronomy began with Mišković's arrival from France to Belgrade. He became a full Professor in 1936. In 1929 Mišković was elected a corresponding member of the Serbian Royal Academy and in 1939 a full member. In his honour a minor planet was named Mišković.

4. FIRST PART OF THE COLLECTION - PRINTED VOLUME

The first part of the Collection is well known to astronomers because they used it during the exercises in the framework of subject General Astronomy when they were students. Since there are no other collections of astronomical problems in Serbian even today, this book is still in use. Naturally, today, students utilize calculators and computers for solving the problems. In the collection the use of logarithmic tables (Mišković wrote a well known book containing logarithmic tables) is recommended. For this reason, the Collection appears as an excellent base for controlling the solutions in the case that a new collection appeared where modern computing software, such as MATHEMATICA and MATLAB, would be used.

It should be emphasized that in *Preface* Mišković writes words which could be written also today: *include again astronomy in the plans for secondary schools*. Namely, he says that astronomy was excluded from schools shortly before and that its topics

are unjustifiably joined the framework of a descriptive-narrative discipline, geography. Unfortunately, today also, in the space era, in the time of the Hubble Space Telescope, astronomy is not present in secondary schools because it was excluded 19 years ago. Its topics are now scattered not only in the framework of geography, but also in that of physics. Of course, efforts are made to reintroduce astronomy as a separate subject, in the framework of the education-system reform which is taking place now. The chances are encouraging because the proposal to include astronomy with one lesson a week in real gymnasiums has been accepted.

In *Preface* it is seen that Mišković for the assistance in the formation of the Collection thanks his colleague Prof. R. Kašanin, for the manuscript redaction his assistants J. Simovljević and R. Djordjević for verifying the large numerical work (then there were neither calculators nor computers), as well as M. Čavčić from the Mathematical Institute of the Serbian Academy of Science and Art for making all the drawings in the Collection.

Introduction in the Collection has a review character. Necessary formulae of spherical trigonometry are given, both for oblique spherical triangle and for right one, also special formulae and differential ones for polar spherical triangle. Series (of trigonometric and other special functions) used in astronomy are also given, as well as a review of standard formulae for transforming astronomical coordinate systems.

There is a common property in the solutions of the problems contained in the Collection. Namely, the problems are explained in details, their general solutions are given and the numerical part is presented. Most frequently the same problems are solved numerically by using alternative methods or by applying alternative formulae so that in this way their verification is carried out. The problems are illustrated by the following examples:

A Problem from Spherical Trigonometry. This field contains 33 problems. For instance, Problem 15 is: *The sides of a spherical triangle $b = 37^{\circ}47'18''$ and $c = 74^{\circ}51'50''$ and the angle between them $A = 44^{\circ}10'40''$ are given. One should determine: 1) third side a and one of the angles along it, say C ; 2) all the three unknown elements.*

A Problem from Earth as a Celestial Body. Here 42 problems are treated. Problem 39 on page 66 is: *The observer is aboard a balloon at an altitude $h = 12$ km above a topographical feature with geographic coordinates $L = 0^{\circ}$, $\phi = +60^{\circ}$. Assuming that the Earth is a sphere one should find the coordinates of the points along the meridian and the parallel reached by the observer's seeing.*

A problem from Apparent Diurnal Motion of Celestial Sphere. Here 51 problems are treated. For instance, Problem 114 is: *At what moment of sidereal time for an observer at latitude $\phi = +36^{\circ}47'50''$ will the stars $\sigma_1(\alpha_1 = 1^h35^m57^s.2, \delta_1 = +67^{\circ}36'31'')$ and $\sigma_2(\alpha_2 = 3^h39^m20^s.8, \delta_2 = +42^{\circ}18'28'')$ reach the same elevation simultaneously? What is the interval of sidereal time beginning with this moment after which the difference of their zenith distances attains $1''$? (Sorbonne 1922).*

5. SECOND PART OF THE COLLECTION - MANUSCRIPT

This part of the Collection has never been published. The Manuscript in its original form was published recently for the first time in an electronic form. This part contains solved problems from the following fields: *astronomical refraction, elements of theory of motion for planets and comets and apparent annual motion of the Sun.* Like the

preceding part the problems are explained in details, they contain necessary formulae and numerical examples. The importance of these fully treated problems to the teaching of astronomy in our midst at the university level is great because we cannot find them elsewhere in the literature.

To mention that in the electronic form of *Manuscript* in the solution to every problem at the place, where the figure should have been, there is a hyperlink opening this figure.

A problem from Astronomical Refraction. In addition to the definitions here one finds formulae for calculating astronomical refraction, the tables, then the expressions treating the refraction influence on the change of coordinates of a celestial body, as well as that treating the horizon depression. In this field there are 23 problems treated fully. Here Problem 134 is presented: *A circumpolar star was observed in both culminations at apparent zenith distances $z_s = 17^\circ 18' 48''.2$ and $z_i = 73^\circ 01' 11''.6$ at a topographical feature of latitude $\phi = +44^\circ 48' 13''$, 2. Determine:*

213) Пошто и места гледа на меридијану Београда, која нам је географска ширнина позната, ϕ - ако астрономски гледа Земља сферне - помоћу датих удаљених места од Београда, тако исто одређују географске ширине тих места и Београда, па, арена оне, и како географске ширине. Ако аренамо да су оваква места од Београда њихови косинуси кривога на Земљи сфери (апоцентрична $R = 6372 \text{ km}$) и означимо их са s , за разне географске ширине имаћемо:

$$\Delta p = \frac{R}{s} \cdot \frac{1}{R} = 57' 29'' 578 \times \frac{1}{6372} = 57' 29'' 578 \times 0.0001569 = 8'' 49' 458 = 29' 40''$$

Географске ширине тих места су, арена оне, северније - $g_1 = +45^\circ 17' 53''$; јужније - $g_2 = +44^\circ 16' 33''$.
 За Сунце меридијанске висине имамо:
 $h = (90^\circ - \phi) + \Delta p$

а за некое фреквенције имамо:
 у гледа екваторској - $\odot = 0^\circ$;
 на гледа летњег соланостица - $\odot = +23^\circ 27'$;
 на гледа зимског соланостица - $\odot = -23^\circ 27'$.
 Сунчеве меридијанске висине у обе гледе, у Београду су:
 $h_1 = 90^\circ - 44^\circ 48' + 29' 40'' = 65^\circ 12''$;
 $h_2 = 90^\circ - 44^\circ 48' + 23^\circ 27' = 68^\circ 39''$;
 $h_3 = 90^\circ - 44^\circ 48' - 23^\circ 27' = 21^\circ 45''$.

Формуле Δ меридијанских сенки у сферном случају, функције $L = 10^\circ$, саће $\Delta_e = L \cdot \text{ctg } h_e$, $\Delta_1 = L \cdot \text{ctg } h_1$, $\Delta_2 = L \cdot \text{ctg } h_2$,
 ако јесте, арена горњој реквизици, ако геодимензију на гледа соланостица означимо ϵ ,
 $\Delta_e = L \cdot \text{ctg } \phi$, $\Delta_1 = L \cdot \text{ctg } (\phi + \epsilon)$, $\Delta_2 = L \cdot \text{ctg } (\phi - \epsilon)$.
 Разлика разлике у функцијама сенки у нацило вектора одређује се са,
 $\Delta \Delta_e = L (\text{ctg } \phi - \text{ctg } \epsilon)$, $\Delta \Delta_1 = L (\text{ctg } (\phi - \epsilon) - \text{ctg } (\phi + \epsilon))$, $\Delta \Delta_2 = L (\text{ctg } (\phi + \epsilon) - \text{ctg } (\phi - \epsilon))$.
 Разлика тангенса, па фрексим садржаном, можемо - као што знамо - написати у облику аугментације аренамо садржаном
 $\Delta \Delta_e = L \sin(\phi - \epsilon) \sec \phi \sec \epsilon$,
 $\Delta \Delta_1 = L \sin(\phi - \epsilon) \sec(\phi - \epsilon) \sec(\phi + \epsilon)$,
 $\Delta \Delta_2 = L \sin(\phi - \epsilon) \sec(\phi + \epsilon) \sec(\phi - \epsilon)$.

Са садржаном иб заграда и израчунавања косинуса саће:

$g_1 = +45^\circ 17' 53''$,	$g_2 = +45^\circ 18'$,	$g_3 - \epsilon = 21^\circ 51'$,	$g_4 + \epsilon = 68^\circ 45''$,
$g_5 = +44^\circ 18' 33''$,	$\epsilon = 23^\circ 27'$,	$g_6 - \epsilon = 20^\circ 52'$,	$g_7 + \epsilon = 67^\circ 46''$,
$g_8 - g_9 = 0^\circ 59' 20''$,	$g_5 = +44^\circ 19'$,		
$[\sin(\phi - \epsilon)] = 8.23701$,	$[\sin(\phi - \epsilon)] = 8.23701$,	$[\sin(g_6 - \epsilon)] = 8.23701$,	
$[\sec \phi] = 0.15278$,	$[\sec(\phi - \epsilon)] = 0.23237$,	$[\sec(\phi + \epsilon)] = 0.44078$,	
$[\sec \phi] = 0.14234$,	$[\sec(\phi - \epsilon)] = 0.23240$,	$[\sec(\phi + \epsilon)] = 0.42197$,	
$\frac{R}{s} = \frac{8.05013}{1.90000}$,	$\frac{R}{s} = \frac{8.30778}{2.90000}$,	$\frac{R}{s} = \frac{9.09076}{1.90000}$,	
$\Delta \Delta_e = 0.3440$,	$\Delta \Delta_1 = 2.9000$,	$\Delta \Delta_2 = 1.2600$,	

- 1) the value for constant k in the expression for refraction $R = k \tan z$,
- 2) the values for constants A and B in the expression for refraction $R = A \tan z - B \tan z$ if it is known that the declination of the star was $\delta = +62^\circ 07' 20''$.

A problem from Elements of Theory of Motion for Planets and Comets. Here Mišковић gives the theoretical apparatus, including the mathematical apparatus, for solving problems from this part of astronomy, for instance Kepler's laws, law of universal gravitation, as well as the more correct form of Kepler's third law. Kepler's equation is presented, together with the procedures for its solving, to be followed by expanding functions in series of true, mean and eccentric anomalies and radius vector of a planet. The number of solved problems is 36. As an example Problem 158 is given: *Solve Kepler's equation if $e = 0.147$ and $M = 136^\circ 25' 32''$* . 4.

A problem from Apparent Annual Motion of the Sun. In the theoretical part of this section of the manuscript

Figure 3: Mišковић's solution of the Problem: Apparent Annual Motion of the Sun.

one describes the apparent motion of the Sun, its apparent annual orbit across the celestial sphere, details concerning the apparent diurnal motion of the Sun, as well as the inequalities in the duration of the daylight and night and in that of twilights. The total of treated problems is 74. The example is No 213: *What is the difference in the lengths of the meridian shadows in the horizontal plane of a vertical pillar, 10 m high,*

on the days of equinoxes and solstices at topographical features along the meridian of Belgrade, of which one is 55 km north and the other one is south of Belgrade by the same distance?

Both parts of the Collection are characterized by an exceptional visibility and a systematic way of presentation. A larger part of the second part was written by hand, but the handwriting is very nice and readable. This was the reason to publish the original form of the second part of the Collection in the electronic form.

6. CONCLUSION

The collection of solved problems of general astronomy by Vojislav Mišković has, no doubt, had an important influence on the process of teaching astronomy at the Belgrade University. The first part was published and for this reason it has been available to the students. The second one has never been published, it has remained in the form of a manuscript only. Its straightforward use in the process of teaching began only when Prof. Simovljević gave the original copy to the author. Bearing in mind the fact that problems of this kind have been missing in the textbook literature in Serbian, my decision has been that the complete collection should be published in an electronic form. The digitized version can be found in the Virtual Library of the Faculty of Mathematics at <http://elib.matf.bg.ac.yu:8080/virlib/>.

References

- Milogradov-Turin, J.: 2006, Vojislav V. Mišković u knjizi Biografski leksikon SRBI KOJI SU OBELEŽILI XX VEK, Beograd, str. 333.
Mišković, V.: 1956, Zbirka rešenih zadataka iz opšte astronomije, Naučna knjiga, Beograd.
Simovljević, J.: 1980, Trideset godina Prirodno-Matematičkog fakulteta Univerziteta u Beogradu, PMF, Beograd.