Publ. Astron. Obs. Belgrade No. 86 (2009), 185 - 190 *Contributed paper*

ADELIC MODELING OF THE UNIVERSE

B. DRAGOVICH

Institute of Physics, Pregrevica 118, 11080 Zemun, Belgrade, Serbia
E–mail: dragovich@phy.bg.ac.yu

Abstract. We consider possible adelic structure of the Universe. Theoretically this is mainly motivated by developments in p-adic mathematical physics, and especially in p-adic and adelic string theory. Phenomenological motivation is related to accelerated expansion of the Universe, dark matter and dark energy. The main direction of this research is investigation of the Universe as an adelic physical system and exploration of possible p-adic effects. We present a brief overview of recent developments in p-adic and adelic cosmology as well as some new results.

1. INTRODUCTION

It is well known that results of experimental and observational measurements are some rational numbers. On the field $\mathbb{Q}$ of rational numbers any non-trivial norm is equivalent either to the usual absolute value $|\cdot|_\infty$ or to a p-adic absolute value $|\cdot|_p$ (Ostrowski theorem). For a rational number $x = p^\nu \, \frac{a}{b}$, where integers a and b are not divisible by prime number p, by definition p-adic absolute value is $|x|_p = p^{-\nu}$ and $|0|_p = 0$. This p-adic norm is a non-Archimedean (ultrametric) one, because $|x + y|_p \leq \max\{|x|_p \, , |y|_p\}$. As completion of $\mathbb{Q}$ gives the field $\mathbb{Q}_\infty \equiv \mathbb{R}$ of real numbers with respect to the $|\cdot|_\infty$, by the same procedure one can construct the fields $\mathbb{Q}_p$ of p-adic numbers ($p = 2, \, 3 \, , 5 \, \cdots$) using $|\cdot|_p$. Any number $x \in \mathbb{Q}_p$ has a unique canonical representation

$$x = p^{\nu(x)} \sum_{n=0}^{+\infty} x_n \, p^n \, , \quad \nu(x) \in \mathbb{Z} \, , \quad x_n \in \{0, \, 1, \, \cdots, \, p-1\}. \tag{1}$$

Real and p-adic numbers, as completions of rational numbers, are unified by adeles. An adele α is an infinite sequence

$$\alpha = (\alpha_\infty, \, \alpha_2, \, \alpha_3, \, \cdots, \, \alpha_p \, , \, \cdots) \, , \quad \alpha_\infty \in \mathbb{R} \, , \ \alpha_p \in \mathbb{Q}_p \, , \tag{2}$$

where for all but a finite set $\mathcal{P}$ of primes p one has that $\alpha_p \in \mathbb{Z}_p = \{x \in \mathbb{Q}_p \, : |x|_p \leq 1\}$. $\mathbb{Z}_p$ is the ring of p-adic integers. The set $\mathbb{A}$ of all adeles can be presented as

$$\mathbb{A} = \bigcup_{\mathcal{P}} A(\mathcal{P}) \, , \quad A(\mathcal{P}) = \mathbb{R} \times \prod_{p \in \mathcal{P}} \mathbb{Q}_p \times \prod_{p \notin \mathcal{P}} \mathbb{Z}_p \, . \tag{3}$$

Endowed with componentwise addition and multiplication $\mathbb{A}$ is the adele ring. For a book on p-adic numbers, adeles and their functions, one can refer to Gel'fand et al. (1969).

Since 1998, there is a lot of evidence that the Universe is now in the stage of an accelerated expansion which began about five billion years ago. To explain this acceleration many models have been proposed but no one of them was generally accepted. A very natural and the most attractive is the approach with dark energy, which is a matter with a negative pressure and uniformly distributed in the space. About 72% of all energy content of the Universe is related to the dark energy, while about 23% belongs to the dark matter and only 4.6% is made of baryons (protons and neutrons).

There are many candidate models for the dark energy. The simplest model is the cosmological constant with $w = -1 \quad (p = -\rho = const.)$. Some other dynamical scalar field models are: quintessence, braneworld, Chaplygin gas and phantom dark energy (with $w < -1$). In spite of many interesting results, neither these nor other so far proposed models offer satisfactory answers to all questions around the dark energy. In the equation of state $p = w\rho$, observational value of parameter w is $w = -1 \pm 0.2$ (for a recent review on dark energy and accelerating the Universe, see e.g. Frieman et al. 2008)

Dark matter was introduced in 1930's to explain dynamics of galaxies in clusters as well as dynamics inside individual galaxies. It is nonbaryonic and gravitationally clustered (attractive) matter without electromagnetic interaction. Many nonbaryonic particles (in particular, neutralino and axion) are considered as candidates for dark matter, but without satisfactory results (for a recent review on dark matter candidates, see e.g. Taoso et al. 2007).

The nature of dark energy and dark matter is still a big mystery, which presents one of the greatest problems and challenges of contemporary cosmology and theory of elementary particles.

Since 1987, p-adic numbers and adeles have been applied in many parts of modern mathematical physics, making so-called p-adic mathematical physics (for a review, see Brekke and Freund 1993, as well as Vladimirov et al. 1994). Here we continue to consider p-adic and adelic aspects of cosmology. Some minisuperspace quantum models are presented in Djordjević et al. (2002). p-Adic origin of dark energy and dark matter was conjectured by Dragovich (2006). In this article we present some results based on this conjecture.

2. SOME SIMPLE EXAMPLES OF ADELIC LIKE CONTENT OF MATTER IN THE UNIVERSE

Taking string theory seriously there exist not only real but also p-adic strings. Real and p-adic matter consist of real and p-adic strings, respectively. Even without string theory, using p-adic quantum-mechanical models one can argue possible existence of p-adic matter. We start with the exact Lagrangian for the effective field of p-adic tachyon string, then extend prime number p to arbitrary natural number n, undertake various summations of such Lagrangians over all n and obtain some scalar field models with the operator valued Riemann zeta function.

The exact tree-level Lagrangian of effective scalar field φ for open p-adic string tachyon is

$$\mathcal{L}_p = \frac{m_p^4}{g_p^2}\,\frac{p^2}{p-1}\Big[-\frac{1}{2}\,\varphi\, p^{-\frac{\Box}{2m_p^2}}\,\varphi + \frac{1}{p+1}\,\varphi^{p+1}\Big]\,, \tag{4}$$

where p is any prime number, $\Box = -\partial_t^2 + \nabla^2$ is the 4-dimensional d'Alembertian. An infinite number of spacetime derivatives follows from the expansion

$$p^{-\frac{\Box}{2}} = \exp\Big(-\frac{1}{2}\ln p\,\Box\Big) = \sum_{k\geq 0}\Big(-\frac{\ln p}{2}\Big)^k\,\frac{1}{k!}\,\Box^k\,.$$

The equation of motion for (4) is

$$p^{-\frac{\Box}{2m_p^2}}\,\varphi = \varphi^p\,, \tag{5}$$

and its properties have been studied by many authors (see, e.g. Barnaby et al. 2008 and references therein).

Prime number p in (4) and (5) can be replaced by any natural number $n \geq 2$ and such expressions also make sense. Moreover, taking $p = 1 + \varepsilon \to 1$ there is the limit of (4)

$$\mathcal{L}_0 = \frac{m^4}{g^2}\,\Big[\frac{1}{2}\,\varphi\,\frac{\Box}{m^2}\,\varphi + \frac{\varphi^2}{2}\,(\ln\varphi^2 - 1)\Big] \tag{6}$$

which corresponds to the ordinary bosonic string in the boundary string field theory.

Now we are going to introduce a Lagrangian which incorporates all the above Lagrangians (4), with p replaced by $n \in \mathbb{N}$, and $\mathcal{L}_0$ (6). To this end, we take the sum of all Lagrangians $\mathcal{L}_n$ in the form

$$L = \sum_{n=0}^{+\infty} C_n\,\mathcal{L}_n = C_0\,\mathcal{L}_0 + \sum_{n=1}^{+\infty} C_n\,\frac{m_n^4}{g_n^2}\,\frac{n^2}{n-1}\Big[-\frac{1}{2}\,\phi\, n^{-\frac{\Box}{2m_n^2}}\,\phi + \frac{1}{n+1}\,\phi^{n+1}\Big]\,, \tag{7}$$

whose explicit realization depends on particular choice of coefficients C_n, string masses m_n and coupling constants g_n. To avoid a divergence in $1/(n-1)$ when $n = 1$ one has to take that $C_n\, m_n^D/g_n^2$ is proportional to $n-1$. Here we shall consider some cases when coefficients C_n are proportional to $n-1$, while masses m_n as well as coupling constants g_n do not depend on n, i.e. $m_n = m$, $g_n = g$. Since this is an attempt towards effective Lagrangian of an adelic string it seems natural to take mass and coupling constant independent on particular p or n. To differ this new field from a particular p-adic one, we use notation ϕ instead of φ. Potential of the above Lagrangian (7) is $V = -L$ at $\Box = 0$.

We have considered three cases, which depend on the choice of coefficients C_n.

Case 1: $C_n = \frac{n-1}{n^{2+h}}\,, \quad n \geq 1\,.$

This case is introduced and considered by Dragovich (2008a). The corresponding Lagrangian is

$$L_h = C_0\,\mathcal{L}_0 + \frac{m^4}{g^2}\Big[-\frac{1}{2}\,\phi\,\zeta\Big(\frac{\Box}{2\,m^2} + h\Big)\,\phi + \mathcal{AC}\sum_{n=1}^{+\infty}\frac{n^{-h}}{n+1}\,\phi^{n+1}\Big]\,, \tag{8}$$

where $\mathcal{AC}$ denotes analytic continuation. We see that nonlocality is controlled by the Riemann zeta function. The equation of motion is

$$\zeta\Big(\frac{\Box}{2\,m^2}+h\Big)\,\phi-\mathcal{AC}\sum_{n=1}^{+\infty}n^{-h}\,\phi^n-C_0\,\frac{\Box}{m^2}\,\phi-C_0\,\phi\,\ln\phi^2=0\,. \qquad (9)$$

Case 2: $C_n=\frac{n^2-1}{n^2}\,,\quad n\geq 1\,.$

The Lagrangian and equation of motion are:

$$L=C_0\,\mathcal{L}_0+\frac{m^4}{g^2}\Big[-\frac{1}{2}\,\phi\,\Big\{\zeta\Big(\frac{\Box}{2\,m^2}-1\Big)+\zeta\Big(\frac{\Box}{2\,m^2}\Big)\Big\}\,\phi+\frac{\phi^2}{1-\phi}\Big]\,, \qquad (10)$$

$$\Big[\zeta\Big(\frac{\Box}{2\,m^2}-1\Big)+\zeta\Big(\frac{\Box}{2\,m^2}\Big)\Big]\,\phi-\frac{\phi((\phi-1)^2+1)}{(\phi-1)^2}-C_0\,\frac{\Box}{m^2}\,\phi-C_0\,\phi\,\ln\phi^2=0\,. \qquad (11)$$

Some classical field properties of this Lagrangian are analyzed by Dragovich (2008b).

Case 3: $C_n=\mu(n)\,\frac{n-1}{n^2}\,,\quad n\geq 1\,.$

Here $\mu(n)$ is the Möbius function, which is defined for all positive integers and has values $1, 0, -1$ depending on factorization of n into prime numbers p. It is defined as follows:

$$\mu(n)=\begin{cases}0, & n=p^2m\\ (-1)^k, & n=p_1p_2\cdots p_k,\ p_i\neq p_j\\ 1, & n=1,\ (k=0)\,.\end{cases} \qquad (12)$$

Recalling that the inverse Riemann zeta function can be defined by

$$\frac{1}{\zeta(s)}=\sum_{n=1}^{+\infty}\frac{\mu(n)}{n^s},\quad s=\sigma+i\tau,\quad \sigma>1, \qquad (13)$$

the corresponding Lagrangian is

$$L_\mu=C_0\,\mathcal{L}_0+\frac{m^4}{g^2}\Big[-\frac{1}{2}\,\phi\,\frac{1}{\zeta\Big(\frac{\Box}{2m^2}\Big)}\,\phi+\int_0^\phi\mathcal{M}(\phi)\,d\phi\Big], \qquad (14)$$

where $\mathcal{M}(\phi)=\sum_{n=1}^{+\infty}\mu(n)\,\phi^n=\phi-\phi^2-\phi^3-\phi^5+\phi^6-\phi^7+\phi^{10}-\phi^{11}-\ldots$. The equation of motion is

$$\frac{1}{\zeta\Big(\frac{\Box}{2m^2}\Big)}\,\phi-\mathcal{M}(\phi)-C_0\,\frac{\Box}{m^2}\,\phi-C_0\,\phi\,\ln\phi^2=0\,. \qquad (15)$$

This case is introduced recently by Dragovich (2008c).

The above Lagrangians, except $\mathcal{L}_0$, belong to the class of nonlinear nonlocal Lagrangians. During last few years relevance for cosmology of some of them has been considered with growing interest. In the paper of Barnaby et al. (2007), Lagrangian

(4) is investigated as candidate for cosmic inflation. Approximate inflationary solutions are found, which are compatible with Cosmic Microwave Background (CMB) observations if $g_p/\sqrt{p} \sim 10^{-7}$ and $m_p < 10^{-6} M_P$, where M_P is the Planck mass. It seems that there is a generic feature of nonlocal models that they support inflation even for the very steep potential. Also nonlocal inflation predicts large non-gaussianity in the CMB (see Barnaby and Cline 2008). For a recent discussion of nonlocal cosmic acceleration, which includes also string field theory nonlocality, one can refer to Aref'eva (2007).

For cosmology with Friedmann-Robertson-Walker metric it is important to know homogeneous time dependent solutions $\phi(t)$. Note that the corresponding nonlocal equations of motion contain an infinite number of time derivatives and there is a problem of initial conditions. Investigation of the initial value problem for dynamics with infinitely many time derivatives is presented in Moeller and Zwiebach (2002), and Barnaby and Kamran (2008).

To consider the Friedmann equations one has to know the time dependent energy-momentum tensor $T^{(p)}_{\mu\nu}$ for the matter content. For p-adic Lagrangian (4), space-time dependent $T^{(p)}_{\mu\nu}$ was constructed by Moeller and Zwiebach (2002), and Yang (2002). Yang (2002) has constructed $T_{\mu\nu}$, which can be rewritten in the following form:

$$\begin{aligned} T^{(p)}_{\mu\nu} &= g_{\mu\nu}\, \frac{m_p^4}{g_p^2}\frac{p^2}{p-1}\Big[\frac{1}{2}\,\varphi\, p^{-\frac{\Box}{2m_p^2}}\,\varphi - \frac{1}{p+1}\,\varphi^{p+1} \\ &\quad + \frac{\ln p}{4m_p^2}\int_0^1 d\tau\, \Big(\Box p^{-\frac{\tau\Box}{2m_p^2}}\varphi\Big)\Big(p^{-\frac{(1-\tau)\Box}{2m_p^2}}\varphi\Big) \\ &\quad + \frac{\ln p}{4m_p^2}\int_0^1 d\tau\, \Big(\partial_\alpha p^{-\frac{\tau\Box}{2m_p^2}}\varphi\Big)\Big(\partial^\alpha p^{-\frac{(1-\tau)\Box}{2m_p^2}}\varphi\Big)\Big] \\ &\quad - \frac{m_p^2 \ln p}{2g_p^2}\frac{p^2}{p-1}\int_0^1 d\tau\, \Big(\partial_\mu p^{-\frac{\tau\Box}{2m_p^2}}\varphi\Big)\Big(\partial_\nu p^{-\frac{(1-\tau)\Box}{2m_p^2}}\varphi\Big), \end{aligned} \tag{16}$$

where $g_{\mu\nu}$ is metric tensor which can be taken flat with signature $(-,+,+,+)$. The complete $T_{\mu\nu}$ for the adelic like system (7) is

$$T_{\mu\nu} = \sum_{n=0}^{\infty} C_n\, T^{(n)}_{\mu\nu}\,, \tag{17}$$

where $T^{(p)}_{\mu\nu}$ is extrapolated to $T^{(n)}_{\mu\nu}$, $n \in \mathbb{N}$ replacing p by n, and $T^{(0)}_{\mu\nu}$ corresponds to Lagrangian $\mathcal{L}_0$. Instead of $\mathcal{L}_0$ in the above consideration one can take more realistic nonlocal tachyon Lagrangian from open string field theory (see e.g. Barnaby 2008 and references therein). Note that energy density and the pressure are, respectively: $\rho_\phi = -T_{00}$, $p_\phi = -T_{ii}$. Further investigation of cosmological aspects is in progress and will be presented elsewhere.

Acknowledgements

The work on this article was partially supported by the Ministry of Science and Technological Development, Serbia, under contract No 144032D.

References

Arefeva, I. Ya., Joukovskaya, L. V. and Vernov, S. Yu.: 2007, *Journal of High Energy Physics*, **07**, 087, (hep-th/0701184).

Barnaby, N.: 2008, arXiv:08110814v1.

Barnaby, N., Biswas, T. and Cline, J. M.: 2007, *Journal of Cosmology and Astroparticle Physics*, **0704**, 056, (hep-th/0612230).

Barnaby, N. and Cline, J. M.: 2008, *Journal of Cosmology and Astroparticle Physics*, **0806**, 030, (arXiv:0802.3218).

Barnaby, N. and Kamran, N.: 2008, *Journal of High Energy Physics*, **0802**, 008, (arXiv:0709.3968).

Barnaby, N., Mulryne, D. J., Nunes, N. J., Robinson, P.: 2008, arXiv:0811.0608v1.

Brekke, L., Freund, P. G. O.: 1993, *Phys. Rep.*, **233**, 1.

Djordjević, G. S, Dragovich, B., Nešić, Lj. D., Volovich, I. V.: 2002, *Int. J. Mod. Phys. A*, **17**, 1413.

Dragovich, B.: 2006, *AIP Conf. Proc.*, **826**, 25.

Dragovich, B.: 2008a, *Theor. Math. Phys.*, accepted (arXiv:0804.4114v1).

Dragovich, B.: 2008b, arXiv:0805.0403v1.

Dragovich, B.: 2008c, *Romanian Journal of Physics*, accepted (arXiv:0809.1601v1).

Frieman, J., Turner, M. S., Huterer, D.: 2008, *Annu. Rev. Astron. Astrophys.*, **46**, 385, (arXiv:0803.0982v1).

Gel'fand, I. M., Graev, M. I. and Pyatetski-Shapiro, I. I.: 1969, Representation Theory and Automorphic Functions, Saunders, Philadelphia.

Moeller, N. and Zwiebach, B.: 2002, *Journal of High Energy Physics*, **0210**, 034, (hep-th/0207107).

Taoso, M., Bertone, G., and Masiero, A.: 2008, *Journal of Cosmology and Astroparticle Physics*, **0803**, 022, (arXiv:0711.4996v1).

Vladimirov, V. S., Volovich, V. S., Zelenov, E. I.: 1994, p-Adic Analysis and Mathematical Physics, World Scientific, Singapore.

Yang, T.: 2002, *Journal of High Energy Physics*, **0211**, 007, (hep-th/0209197v3).