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STRUCTURES AND CORRELATIONS IN THE LARGE SCALE UNIVERSE

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Abstract. One of the main assumptions to explain structure formation in an expanding universe is that a dominant mass component in the form of non-baryonic dark matter plays a central role. However, although dark matter is supposed to provide with more than 0.9 of the total fraction of the mass-energy in the universe, its amount and properties can only be defined a posteriori. In this context, a crucial point concerns the identification of a possible clear feature of dark matter fields which is expected to hold on very general theoretical conditions. This property, in standard cosmological models, is represented by super-homogeneity, i.e. a very fine tuned balance between negative and positive (weak) correlations of density fluctuations, which must be imprinted both in the anisotropies of the Cosmic Microwave Background Radiation and in the large scale distribution of galaxies. We discuss the fact that the complex clumpiness characterizing galaxy structures corresponds to power-law correlations, i.e. a regime where the main feature is represented by scale-invariance of galaxy clustering. The transition between such a regime and the super-homogeneous one has not been detected yet. This would represent the main observational verification of standard models of galaxy formation, linking the early universe physics, through the statistical properties of the initial density field, to the non-linear structures observed today in galaxy structures.

1. INTRODUCTION

The attempt to construct a cosmological model has been the common effort of all civilizations. Modern cosmology essentially poses the same questions as ever but it stands on more solid grounds than ever. Indeed, in recent times there has been an impressive growth of observational data, and the development of cosmological models has become a sophisticated activity at the boundaries between different fields in physics and astronomy. The standard scenario, known as Big-Bang cosmology, is made of different pieces, each of them elaborated to explain a number of observations. The foundations are provided with General Relativity, which allows one to formulate an explanation for the Hubble's law that galaxies distances are linearly proportional to their redshifts; in particular the Friedmann-Robertson-Wlaker (FRW) solutions of Einstein's field equations, which assume the matter distribution to be a perfectly spatially uniform fluid. In order to explain the high isotropy observed in the Cosmic Microwave Background Radiation (CMBR) and the apparent flatness of space in 1982 the theory of inflation (Baryshev 1994) (see Disney 2007 for a recent critical discussion of these issues) was introduced.

The high isotropy of the CMBR has given rise to the need of cosmological dark matter in the form of a non-baryonic type. While there are several astrophysical observations that in galaxies and galaxy clusters, cosmological arguments related to the growth of structures in an expanding universe pose strong constraints on the amount and type of dark matter. In fact, in contemporary cosmological models the structures observed today at large scales in the distribution of galaxies in the universe are explained by the dynamical evolution of purely self-gravitating matter (dark matter) from an initial state with low amplitude density fluctuations, the latter strongly constrained by satellite observations of the fluctuations in the temperature of the cosmic microwave background radiation (e.g. the satellites COBE (Bennett et al. 1994) and WMAP Spergel 2003). Despite the apparent simplicity of the scheme, fundamental theoretical problems remain open and the overall picture is based on the assumption that the main mass component is not only dark but also with very specific properties.

In this theoretical framework one crucial element is represented by the initial conditions (IC) of the matter density field. Models of the early universe (Padmanabhan 1993) predict certain primordial fluctuations in the matter density field, defining their correlation properties and their relation to the present day matter distribution. When gravity start to dominate the dynamical evolution of density fluctuations, which can generally be described by the Vlasov or "collision-less Boltzmann" equations coupled with the Poisson equation, perturbations are still of very low amplitude. One of the most basic results (see e.g., Peebles 1980) about self-gravitating systems, treated using perturbative approaches to the problem (i.e. the fluid limit), is that the amplitude of small fluctuations grows monotonically in time, in a way which is independent of the scale. This linearized treatment breaks down at any given scale when the relative fluctuation at the same scale becomes of order unity, signaling the onset of the "non-linear" phase of gravitational collapse of the mass in regions of the corresponding size. If the initial velocity dispersion of particles is small, non-linear structures start to develop at small scales first and then the evolution becomes "hierarchical", i.e., structures build up at successively larger scales. Given the finite time from the IC to the present day, the development of non-linear structures is limited in space, i.e., they can not be more extended than the scale at which the linear approach predicts that the density contrast becomes of order unity at the present time. This scale, estimated in current models to be of the order of several Mpc (Padmanabhan 1993), is fixed by the amplitude of initial fluctuations, constrained by the CMBR, by the hypothesized nature of the dominating dark matter component and its correlation properties.

Thus in order to explain the existence of contemporary visible structures, such as galaxies and clusters, which otherwise would never condense within the expanding universe, one must make the hypothesis that the "seeds" from which such structures have grown would be made of non-baryonic dark matter, i.e. a type of dark matter that has a very weak coupling with radiation (the so-called cold dark matter — CDM). This is so because dark matter could collapse into the structures we see today, thereafter dragging the much lesser amounts of ordinary matter in afterwards, without leaving large traces into the CMBR radiation, compatible with CMBR anisotropies of order of 10^{-5} (Bennett et al. 1994, Spergel 2003.

Although the very speculative and ad-hoc hypotheses to explain the growth of structures, observations of large scale galaxy distributions provide important tests for these models. On the one hand the first question concerns the extension of the regime of non-linear clustering and the intrinsic properties of galaxy structures. On the other hand according to the standard scenario, at some large scales where fluctuations are still of small amplitude, the imprints of primordial correlations should be preserved and their detection represents a key observation for the validation of the model.

2. DARK MATTER

More than twenty years ago it was surprisingly discovered that galaxy velocity rotation curves remain flat at large distances from the galaxy center while the density profile of luminous matters rapidly decays (e.g. Rubin 1980). This is one of the strongest indications of the need from dynamically dominant dark matter in the universe. Most attention has been focused on the fact that these bound gravitational systems contain large quantities of unseen matter and an intricate paradigm has been developed in which non-baryonic dark matter plays a central role not only in accounting for the dynamical mass of galaxies and galaxy clusters (Faber and Gallagher 1979) but also for providing the initial seeds which have given rise to the formation of structure via gravitational collapse (Peebles 1980).

In current standard cosmological models, different forms of dark matter are needed to explain a number of different phenomena. In fact, the results of several observations, such as the scale size of fluctuations of the CMBR (e.g. Spergel 2003), the measurements of clustering mass on large scales (e.g. Eisenstein 2005), the magnitude-redshift relation of type Ia supernovae (SNe Ia) (e.g. Perlmutter 1999), are interpreted to give consistent measurements of the amount of dark matter. In this framework, baryons (Ω_B), which can be detected in the form of, for example, luminous objects such as stars and galaxies, would only be the 5% of the total mass in the universe; the rest is made of entities about which very little is understood: dark matter and dark energy. More specifically dark matter, in form of non-baryonic elementary particles, would contribute to the $\sim$ 30% of the total mass of the universe ($\Omega_m \sim 0.3$). It is worth noticing that its direct detection in laboratory experiments is still lacking and that the standard model of particle physics does not predict the existence of candidate dark matter particles with the necessary properties from a cosmological point of view.

Several evidences, from supernovae and other observations, shows that the expansion of the universe, rather than slowing because of gravity, is increasingly rapid. Within the standard cosmological framework, this must be due to a substance, which has been termed dark energy, that behaves as if it had negative pressure. This is a mysterious form of energy which would cause the accelerating expansion of the universe and it should account for about 70% (i.e. $\Omega_\Lambda \sim 0.7$) of the mass-energy in the universe. It is thus not surprising that great observational and theoretical effort is devoted to the understanding of the nature and properties of dark matter and dark energy which, giving the main contribution to the mass-energy density of the universe, play a crucial role, for example, in the problem of structure formation.

The previous discussion enlightens the fact that we know very little about the nature of cosmological dark matter both from a fundamental and observational points of view. Although dark matter is so central in modern cosmology, its amount and properties can only be defined a posteriori. In this context a crucial question concerns a possible clear property of dark matter density fields which is not arbitrary, i.e. a property which has to be satisfied by dark matter fluctuations under some very general theoretical conditions. In fact, from the above discussion it seems that much freedom is left for the choice of dark matter, its physical properties and its statistical distribution. However there is an important constraint which must be valid for any kind of initial matter density fluctuation field in the framework of FRW models and which represents a consistency condition to be satisfied by any fluctuation field compatible with the FRW metric. As we discuss below this must be imprinted both in the fluctuations of the CMBR and in the large scale distribution of galaxies.

3. THE SIGNATURE OF DARK MATTER

The most prominent feature of the initial conditions in the early universe, in standard theoretical models, derived from inflationary mechanisms, is that matter density field presents on large scale super-homogeneous features (Gabrielli et al. 2002). This means the following. If one considers the paradigm of uniform distributions, the Poisson process where particles are placed completely randomly in space, the mass fluctuations in a sphere of radius R grows as R^3, i.e. like the volume of the sphere. A super-homogeneous distribution is a system where the average density is well defined (i.e. it is uniform) and where fluctuations in a sphere grow slower than in the Poisson case, e.g. like R^2: in this case there are the so-called surface fluctuations to differentiate them from Poisson-like volume fluctuations.

Note that a uniform system with positive correlations presents fluctuations which grow faster than Poisson. For example a perfect cubic lattice of particle is a super-homogeneous system. A well known system in statistical physics systems of this kind is the one component plasma (Gabrielli et al. 2003) (OCP) which is characterized by a dynamics which at thermal equilibrium gives rise to such configurations. The OCP is simply a system of charged point particles interacting through a repulsive $1/r$ potential, in a uniform background which gives overall charge neutrality. Simple modifications of the OCP can produce equilibrium correlations of the kind assumed in the cosmological context (Gabrielli et al. 2003). In the cosmological context inflationary models predict a spectrum of fluctuations of this type. In terms of the normalized mass variance

$$\sigma^2(R) = \frac{\langle M(R)^2 \rangle - \langle M(R) \rangle^2}{\langle M(R) \rangle^2} \,, \tag{1}$$

where $\langle M(R) \rangle$ is the average mass in a sphere of radius R and $\langle M(R)^2 \rangle$ is the average of the square mass in the same volume. Thus for a Poisson distribution, where there are no correlation between particles (or density fluctuations) at all, one simply has

$$\sigma^2(R) \sim R^{-3} \; . \tag{2}$$

For an ordered system characterized by small-scale anti-correlation the variance behaves as

$$\sigma^2(R) \sim R^{-4} \; , \tag{3}$$

which is the fastest possible decay for discrete or continuous distributions (Gabrielli et al. 2002). By comparing Eq. 2 with Eq. 3 one may notice that the long-range order of the distribution corresponds to the presence of mass fluctuations which are depressed with respect to the uncorrelated Poisson case.

The reason for this peculiar behavior of primordial density fluctuations is the following. In a FRW cosmology there is a fundamental characteristic scale length, the horizon scale $R_H(t)$. It is simply the distance light can travel from the Big Bang singularity $t = 0$ until any given time t in the evolution of the universe, and it grows linearly with time. The Harrison-Zeldovich (H-Z) criterion can be written as

$$\sigma_M^2(R = R_H(t)) = \text{constant}. \tag{4}$$

This condition states that the mass variance at the horizon scale is constant: this can be expressed more conveniently in terms of the power spectrum (PS) of density fluctuations (Gabrielli et al. 2002)

$$P(\vec{k}) = \left\langle |\delta_\rho(\vec{k})|^2 \right\rangle \tag{5}$$

where $\delta_\rho(\vec{k})$ is the Fourier Transform of the normalized fluctuation field $(\rho(\vec{r}) - \rho_0)/\rho_0$, ρ_0 being the average density. It is possible to show that Eq. 4 is equivalent to assuming $P(k) \sim k$: in this situation matter distribution presents fluctuations of the type given by Eq. 3 (Gabrielli et al. 2002).

In order to illustrate more clearly the physical implications of this condition, one may consider gravitational potential fluctuations $\delta\phi(\vec{r})$ which are linked to the density fluctuations $\delta\rho(\vec{r})$ via the gravitational Poisson equation: $\nabla^2 \delta\phi(\vec{r}) = -4\pi G \delta\rho(\vec{r})$. From this, transformed to Fourier space, it follows that the PS of the potential $P_\phi(k) = \left\langle |\delta\hat{\phi}(\vec{k})|^2 \right\rangle$ is related to the density PS $P(k)$ as: $P_\phi(k) \sim \frac{P(k)}{k^4}$.

The H-Z condition corresponds therefore to $P_\phi(k) \propto k^{-3}$ so that gravitational potential fluctuations become constant as a function of scale. In fact, the variance in real space spheres of the gravitational potential fluctuations can be written as (Gabrielli et al. 2002) $\sigma_\phi^2(R) \approx \frac{1}{2} P_\phi(k) k^3|_{k=R^{-1}}$. The H-Z condition fixes this variance to be constant as a function of R. This is a consistency constraint in the framework of FRW cosmology. In fact the FRW is a cosmological solution for a homogeneous Universe, about which fluctuations represent an inhomogeneous perturbation: if density fluctuations obey a condition different from Eq.4, then the FRW description will always break down in the past or future, as the amplitude of the perturbations become arbitrarily large or small. For this reason the super-homogeneous nature of primordial density field is a fundamental property independently on the nature of dark matter. This is a very strong condition to impose, and it excludes even Poisson processes ($P(k)$ =const. for small k) (Gabrielli 2002).

Various models of primordial density fields differ for the behavior of the power spectrum at large wave-lengths, i.e. at relatively small scales. For example, for the case the Cold Dark Matter scenario (CDM), where elementary non-baryonic dark matter particles have a small velocity dispersion, the power spectrum decays as a power law $P(k) \sim k^{-2}$ at large k. For Hot Dark Matter (HDM) models, where the velocity dispersion is large, the power spectrum presents an exponential decay at large k. However at small k they both exhibit the H-Z tail $P(k) \sim k$ which is in fact the common feature of all density fluctuations compatible with FRW models.

4. SUPER-HOMOGENEITY IN LARGE SCALE GALAXY STRUCTURES

In terms of the correlation function $\xi(r)$ (the Fourier conjugate of the PS) CDM/HDM models present the following behavior: it is positive at small scales, it crosses zero at a certain scale and then it is negative approaching zero with a tail which goes as r^{-4} (in the region corresponding to $P(k) \sim k$) (Gabrielli et al. 2005). The super-homogeneity (or H-Z) condition says that the volume integral over all space of the correlation function is zero

$$\int_0^\infty d^3r\xi(r) = 0 \,. \tag{6}$$

This means that there is a fine tuned balance between small-scale positive correlations and large-scale negative anti-correlations (Gabrielli et al. 2002, Gabrielli et al. 2005). This is the behavior that one would like to detect in the data in order to confirm inflationary models. Up to now this search has been done through the analysis of the galaxy PS which has to go correspondingly as $P(k) \sim k$ at small k (large scales). However one should consider an additional complication.

In standard models of structure formation galaxies result from a *sampling* of the underlying CDM density field: for instance one selects (observationally) only the highest fluctuations of the field which would represent the locations where a galaxy will eventually form. It has been shown that sampling a super-homogeneous fluctuation field changes the nature of correlations (Durrer 2003). The reason can be found in the property of super-homogeneity of such a distribution: the sampling necessarily destroys the surface nature of the fluctuations, as it introduces a volume (Poisson-like) term in the mass fluctuations, giving rise to a Poisson-like PS on large scales $P(k) \sim$ constant. The "primordial" form of the PS is thus not apparent in what one would expect to measure from objects selected in this way. This conclusion should hold for any generic model of bias and its quantitative importance has to be established in any given model (Durrer 2003). On the other hand one may show (Durrer 2003) that the negative r^{-4} tail in the correlation function does not change under sampling: on scales large enough, where in these models (anti) correlations are small enough, the biased fluctuation field has a correlation function which is linearly amplified with respect to the underlying dark matter correlation function. For this reason the detection of such a negative tail would be the main confirmation of the super-homogeneous character of primordial density field (Gabrielli et al. 2005).

Up to now, no quantitative evidences in this respect have been reported as measurements of galaxy data are not extended enough to reach the region where $\xi(r) \sim -r^{-4}$. Future surveys, like the complete SDSS catalog (York et al. 2000), may sample this range of scales, but a precise study of the crossover to homogeneity, discretization effects, sampling effects and statistical noise is still needed (Sylos Labini 2007). At smaller scales instead, there have been found evidences for the presence of highly clustered structures which exhibit power law correlation, i.e. a situation which is extremely different from a super-homogeneous distribution. Let us make some comments on this issue.

5. SCALE INVARIANCE OF GALAXY STRUCTURES

In the past twenty years observations have provided several tridimensional maps of galaxy distribution, from which there is a growing evidence of large scale structures. This important discovery has been possible thanks to the advent of large redshift surveys: angular galaxy catalogs, considered in the past, are in fact essentially smooth and structureless. In the CfA2 catalog (De Lapparent 1989), which was one of the first maps surveying the local universe, one has surprisingly observed the giant "Great Wall", a filament linking several groups and clusters of galaxies of extension of about 200 Mpc/h[1] and whose size is limited by the sample boundaries. Recently the SDSS project (York et al. 2000) has revealed the existence of structures larger than the Great Wall, as for example the so-called "Sloan Great Wall" which is almost double longer than the Great Wall. Nowadays this is the most extended structure ever observed, covering about 400 Mpc/h, and whose size is again limited by the boundaries of the sample (Gott et al. 2005). The search for the "maximum" size of galaxy structures and voids, beyond which the distribution becomes essentially smooth, is still an open problem. Instead the fact that galaxy structures are strongly irregular and form complex patterns has become a well-established fact. From the theoretical point of view the understating of the statistical characterization of these structures represents the key element to be considered by a physical theory dealing with their formation. The primary questions that such a situation rises are therefore: (i) what is the nature of galaxy structures and (ii) which is the maximum size of structures ? A number of statistical concepts can be used to answer these questions: in general one wants to characterize n-point correlation properties which are able to capture the main elements of points distributions (Gabrielli et al. 2005). For example, (Hogg et al. 2005) have recently measured the conditional average density in a sample of Luminous Red Galaxies (LRG) from a data release of the SDSS. Such a statistics

[1] The typical mean separation between nearest galaxies is of about 0.1 Mpc. By local universe one means scales in the range $[1, 100]$ Mpc/h, where space geometry is basically Euclidean and dynamics is Newtonian, i.e. effects of General Relativity are negligible. The size of the universe, according to standard cosmological models is about 5000 Mpc/h, where 1 Mpc $\simeq 3 \times 10^{22}$ m; distances are given in units of h, a parameter which is in the range [0.5,0.75] reflecting the incertitude in the value of the Hubble constant ($H = 100$ h km/sec/Mpc) used to convert redshift z into distances $d \approx c/Hz$ (where c is the velocity of light).

is very useful to determine correlation properties in the regime of strong clustering and the spatial extension of strong fluctuations in a given sample. This was firstly introduced by Pietronero (1987) and then measured in many samples by Sylos Labini (1998). The conditional density gives the average density of points in a spherical volume (or a spherical shell) centered around a galaxy (see e.g. Gabrielli et al. 2005). The results obtained by Hogg et al. (2005) can be summarized as follows:

(i) A simple power-law scaling corresponding to a correlation exponent $\gamma \approx 1$ gives a very good fit to the data up to at least 20 Mpc/h, over approximately a decade in scale. The power-law behavior of the correlation function is an evidence for the self-similarity of galaxy structures, i.e. of their fractal nature (Gabrielli et al. 2005). We note that these results are in good agreement with those obtained by Sylos Labini (1998) through the analyses of many smaller samples and more recently by Vasilyev et al. (2006) in another galaxy survey, the 2dFGRS.

(ii) The second important result of Hogg et al. (2005) is that at larger scales (i.e. $r > 30$ Mpc/h) the conditional density continues to decrease, but less rapidly, until about ~ 70 Mpc/h, above which it seems to flatten up to the largest scale probed by the sample (100 Mpc/h). The transition between the two regimes is slow, in the sense that the conditional density at ~ 20 Mpc/h is about twice the asymptotic mean density. Note that Sylos Labini (1998) found evidences for a continuation of the small scale power-law to distances of order hundreds of Mpc/h, although with a weaker statistics, which seems to be not confirmed by Hogg et al. (2005).

However one should consider that in the range of scales [~ 30,~ 100] Mpc/h there are evidences (Sylos Labini 2006) for systematic unaveraged fluctuations corresponding to the presence of large scale structures extending up to the boundaries of the present survey, which require a detailed analysis of the problems induced by finite volume effects on the determination of the conditional density. In addition there are evidences which suggest that in such range of scales the power-law index of the conditional density has a smaller value (Sylos Labini 2006). However future surveys will allow to distinguish between the two possibilities: that a crossover to homogeneity (corresponding to $\gamma = 0$ in the conditional density) occurs before 100 Mpc/h, or that correlations extend to scales of order 100 Mpc/h (with a smaller exponent $0 < \gamma < 1$). Thus the precise scale of homogeneity, if any, is not yet determined and a detailed analysis of the new coming data will clarify the situation in few years from now.

Finally we note that even if a transition toward a constant value of the conditional density will be finally detected this does not imply that the distribution becomes uncorrelated on larger scales. In fact, this means that structures, beyond the crossover scale, have small amplitude but they can be very well correlated on larger scales. It is then in this situation where the detection of anti-correlations, which as discussed above are predicted by all models of primordial density fields, become the relevant issue to be addressed.

6. SUPER-HOMOGENEITY IN THE COSMIC MICROWAVE BACKGROUND

Primordial density fluctuations have imprinted themselves on the patterns of radiation also, and those variations should be detectable in the CMBR. Three decades of observations have revealed fluctuations in the CMBR of amplitude of order 10^{-5} (Spergel 2003) [2]. It is in fact to make these measurements compatible with observed structures, that it is necessary to introduce non-baryonic dark matter which interacts with photons only gravitationally, and thus in a much weaker manner than ordinary baryonic matter. Thus in standard models of structure formation dark matter plays the dominant role of providing density fluctuation seeds which, from the one hand are compatible with observations of the CMBR and from the other hand they are large enough to allow the formation, through a complex non-linear dynamics, of galaxy structures we observe today.

In standard cosmological theories the CMBR represents a bridge between the very early universe and the universe as we observe today and in particular the galaxy structures. On the one hand the CMBR probes the very early hot universe at extreme energies through the theories proposed — notably "inflation" — to explain the origin of these perturbations. On the other hand the anisotropies reflect the local very small amplitude perturbations which give the initial conditions for the gravitational dynamics which should subsequently generate the galaxy structures observed today.

In the CMBR one measures fluctuations in temperature on the sky, i.e. on the celestial sphere. We will not enter here into the detail of the physical theory in standard models which link these temperature fluctuations to the mass density field (Padmanabhan 1993). It is useful however for what follows to give the precise relation between the two quantities. The temperature fluctuation field $\frac{T(\theta,\phi)-\langle T\rangle}{\langle T\rangle} \equiv \frac{\delta T}{T}(\theta,\phi)$, where θ , ϕ are the two angular coordinates, is conventionally decomposed in spherical harmonics on the sphere:

$$\frac{\delta T}{T}(\theta,\phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} a_{lm} Y_{lm}(\theta,\phi) \ . \tag{7}$$

The variance of these coefficients a_{lm} is then related to the matter power spectrum through

$$C_l \equiv \langle |a_{lm}|^2 \rangle = \frac{H_0^4}{2\pi} \int_0^\infty dk \frac{P(k)}{k^2} |j_l(k\eta)|^2 \tag{8}$$

where j_l is the spherical Bessel function and $\eta \simeq 2H_0^{-1}$ is a constant at fixed time (H_0 is the Hubble constant today). Note that the ensemble average contains no dependence on m because of the assumption of statistical isotropy.

Taking $P(k) \sim k$ in (8) we get that the $l > 2$ multi-poles are given by $C_l \sim (l(l+1))^{-1}$, so that the H-Z condition for the power spectrum $n = 1$ corresponds to a constant value of the quantity $l(l+1)C_l$. For this reason it is usually in terms of this combination of l and C_l that the data from the CMBR are represented.

[2] Recently evidences for an important non-cosmic contamination of the latest WMAP data (Spergel 2003) were reported by Copi (2004).

The WMAP team (Spergel 2003) has found that the two-point correlation function, the Fourier transform of the angular power spectrum C_l, nearly vanishes on scales greater than about 60 degrees, contrary to what the standard theories, where the super-homogeneous $P(k) \sim k$ is present, predict, and in agreement with the same finding obtained from COBE data about a decade earlier (Bennett et al. 1994). Note that the vanishing of power is much more apparent in real space than in multiple space (i.e. in C_l). In addition, particularly puzzling are the alignments of low multipoles with the solar system features as this implies that CMBR anisotropy should be correlated with our local habitat: the observed correlations seem to hint that there is contamination by a foreground (Copi 2004). Thus the observational situation for the CMBR is not able at the moment to determine whether fluctuations in the radiation and matter density fields really show the crucial super-homogeneous feature.

7. CONCLUSIONS

Statistical properties of primordial density fields show interesting analogies with systems in statistical physics, like the one-component plasma, whose main characteristic is the ordered, or super-homogeneous, nature. In the FRW models the super-homogeneous (or Harrison-Zeldovich) condition arises as a kind of consistency constraint: other, more inhomogeneous, stochastic fluctuations, like the uncorrelated Poisson case, will always break down in the FRW models in the past or future as the amplitude of perturbations in the gravitational potential may become arbitrarily large. We have discussed that the observational detection of the super-homogeneous character of the matter density field, through the observation of galaxy distribution or of the CMBR anisotropies, is still lacking. However the main feature of galaxy two-point correlation function is represented by its power-law character in the strongly non-linear region. We stressed that a clear crossover to homogeneity is also not well established in the data, and thus the transition from the highly clustered phase to the highly uniform (super-homogeneous) one is among the main observational tests for theories of the early universe. For galaxies this should be evidenced as a negative correlation function behaving as $-r^{-4}$ at large scales. Future galaxy surveys, like the SDSS project, will provide with samples large enough to clarify this issue.

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References

Baryshev, Y. V., Sylos Labini, F., Montuori, M. and Pietronero, L.: 1994, *Vistas in Astron.*, **38**, 419.
Bennett, C. L. et al.: 1994, *Ap. J.*, **436**, 423.
Copi, C. J., Huterer, D. and Starkman, G. D.: 2004, *Phys. Rev. D*, **70**, 043515.
De Lapparent, V., Geller, M., Huchra, J.: 1989, *Ap. J.*, **343**, 1.
Disney, M.: 2007, *American Scientist*, **95**, 35.
`http://www.americanscientist.org/template/AssetDetail/assetid/55839`
Durrer, R., Gabrielli, A., Joyce, M., Sylos Labini, F.: 2003, *Ap. J.*, **585**, L1.

Eisenstein, D. J. et al.: 2005, *Ap. J.*, **633**, 560.
Faber, S. M., Gallagher, J. S.: 1979, *Astron. Astrophys. Ann. Rev.*, **17**, 135.
Gabrielli, A., Joyce, M., Sylos Labini, F.: 2002, *Phys. Rev.*, **D65**, 083523.
Gabrielli, A., Jancovici, B., Joyce, M., Lebowitz, J., Pietronero, L., Sylos Labini, F.: 2003, *Phys. Rev.*, **D67**, 043406.
Gabrielli, A., Sylos Labini, F., Joyce, M., Pietronero, L.: 2005, Statistical physics for cosmic structures, (Springer Verlag, Berlin.
Gott, J. R. III et al. : 2005, *Ap. J.*, **624**, 463.
Hogg, D. W. et al.: 2005, *Ap. J.*, **624**, 54.
Padmanabhan, T.: 1993, Structure formation in the universe, Cambridge University Press, Cambridge.
Peebles, P. J. E.: 1980, Large Scale Structure of the Universe, Princeton University Press, Princeton, New Jersey.
Perlmutter, S. et al.: 1999, *Astrophys. J.*, **517**, 565.
Pietronero, L.: 1987, *Physica A*, **144**, 257.
Rubin, V. C., Thonnard, N., Ford, W. K.: 1980, *Ap. J.*, **238**, 471.
Spergel, D. N. et al.: 2003, *Astrophys. J. Suppl. Ap. J. S.*, **148**, 175.
Sylos Labini, F., Vasilyev, N. L., Baryshev, Yu. V.: 2006, *A&A* in the press.
Sylos Labini F. and Vasilyev, N. L.: 2007, *A&A* in the press.
Sylos Labini, F., Montuori, M., Pietronero, L.: 1998, *Phys. Rep.*, **293**, 66.
Vasilyev, N. L., Baryshev, Yu. V., Sylos Labini, F.: 2006, *A&A*, **447**, 431.
York, D. et al.: 2000, *Astron. J.*, **120**, 1579.