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ELECTRIC AND QUANTUM EFFECTS IN SELF GRAVITATING STATIC PLASMA CONFIGURATIONS

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Abstract. We derive dimensionless numbers that define relative significance of electrostatic, gravitational and quantum forces in self gravitating astrophysical plasmas and show that quantum effects have practically no influence on static configurations of self gravitating plasmas. The inclusion of quantum forces, which can be frequently found in the literature in recent years, is therefore not justified for realistic astrophysical conditions. Moreover, our estimates show that inclusion of quantum effects into the hydrodynamic treatment of plasma is essentially meaningless.

1. INTRODUCTION

Self gravitating multicomponent plasma configurations are commonly found in nature on various characteristic scales of physical quantities (Verhest 2000). Among problems of interest in this field that are frequently dealt with in literature are properties of quasi-static distributions of charged and massive particle components of plasmas subject to particle electrostatic and gravitational interactions (Verhest 2000, Mayer 2005, Chavanis 2002). In recent years, also the contribution of quantum forces is included into studies of various hydrostatic equilibrium states and their perturbations resulting into varieties of plasma modes and possible instabilities (Sahu et al 2007, Shukla et al. 2006).

In this paper, we introduce dimensionless numbers into equations for a hydrostatic equilibrium of a two component electron-ion (e-i) self gravitating quantum plasma, which determine the significance of electrostatic and quantum effect terms in a broad domain of characteristic plasma parameters. Such an analysis is necessary in particular in those cases when quantum effects are added (Sahu et al. 2007) in standard approaches of finding static plasma configurations, and studies of waves and instabilities.

2. HYDROSTATIC EQUILIBRIUM

The considered plasma is assumed a perfect polytropic two component gas with a pressure-density relation for electrons and ions given by $p_{e,i} \sim n_{e,i}^{\kappa}$.

The hydrostatic equilibrium of a two component e-i self gravitating quantum plasma is now described by the following set of equations (Sahu et al 2007, Shukla et al. 2006):

1. Balance of forces for the electron and ion component respectively:

$$
\begin{aligned}
Z_i e\nabla\phi + m_i\nabla\psi + \frac{1}{n_i}\nabla p_i - \frac{\hbar^2}{2m_i}\nabla\left[\frac{\nabla^2\sqrt{n_i}}{\sqrt{n_i}}\right] = 0,\\
e\nabla\phi - m_e\nabla\psi - \frac{1}{n_e}\nabla p_e + \frac{\hbar^2}{2m_e}\nabla\left[\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right] = 0.
\end{aligned}
\tag{1}
$$

2. Poisson's equations for the electrostatic and gravitational potentials:

$$
\nabla^2\phi = \frac{4\pi e}{\epsilon_0}(n_e - Z_i n_i), \qquad \nabla^2\psi = 4\pi G(m_i n_i + m_e n_e). \tag{2}
$$

3. Polytropic electron and ion components:

$$
\frac{p_i}{p_{i0}} = \left(\frac{n_i}{n_{i0}}\right)^{\kappa_i}, \qquad \frac{p_e}{p_{e0}} = \left(\frac{n_e}{n_{e0}}\right)^{\kappa_e} \tag{3}
$$

where: $p_{0e,0i} = n_{0e,0i}kT_{e,i}$ with $p_{0e,0i}, n_{0e,0i}$=const and k=1.38x10^{-23}kg m^{2}s$^{-2}K^{-1}$ is the Boltzmann constant.

In what follows, we shall take $T_e = T_i$ and $Z_i = 1$ meaning we are dealing with an electron-proton plasma.

3. SCALING AND DIMENSIONLESS NUMBERS

Equations Eq. (1)-(3) can be cast in a dimensionless form by introducing typical scaling quantities r_0, T_0, and n_0 for the length, temperature and particle number densities respectively.

In this case, Eq. (1)-(3) can be written as:

$$
\begin{aligned}
&K_Q\frac{m_i}{m_e}\tilde{\nabla}^2\left[\frac{\tilde{\nabla}^2\sqrt{n_i}}{\sqrt{n_i}}\right] - \kappa\tilde{n}_i^{\kappa-2}\tilde{\nabla}^2\tilde{n}_i - \kappa(\kappa-2)\tilde{n}_i^{\kappa-3}(\tilde{\nabla}\tilde{n}_i)^2\\
&\quad -K_E(\tilde{n}_e - \tilde{n}_i) - K_G(\frac{m_i}{m_e}\tilde{n}_i + \tilde{n}_e) = 0,\\
&K_Q\tilde{\nabla}^2\left[\frac{\tilde{\nabla}^2\sqrt{n_e}}{\sqrt{n_e}}\right] - \kappa\tilde{n}_e^{\kappa-2}\tilde{\nabla}^2\tilde{n}_e - \kappa(\kappa-2)\tilde{n}_e^{\kappa-3}(\tilde{\nabla}\tilde{n}_e)^2\\
&\quad +K_E(\tilde{n}_e - \tilde{n}_i) - K_G(\tilde{n}_i + \frac{m_e}{m_i}\tilde{n}_e) = 0.
\end{aligned}
\tag{4}
$$

where tilde designates the corresponding dimensionless quantities:

$$
\tilde{n}_{e,i} \equiv \frac{n_{e,i}}{n_0} \quad \text{and} \quad \tilde{\nabla} \equiv r_0\nabla.
$$

The introduced dimensionless numbers in Eq. (4) are defined as:

$$
\begin{aligned}
K_Q &\equiv \frac{1}{2}\frac{\hbar^2}{m_e kT r_0^2} = \frac{1}{2}\frac{\lambda\!\!\!^{-2}}{r_0^2},\\
K_G &\equiv 4\pi\frac{G m_i m_e n_{i0}}{kT}r_0^2 = 4\pi\frac{r_0^2}{\lambda_G^2},\\
K_E &\equiv 4\pi\frac{n_{i0}e^2 r_0^2}{\epsilon_0 kT} = 4\pi\frac{r_0^2}{\lambda_D^2}.
\end{aligned}
\tag{5}
$$

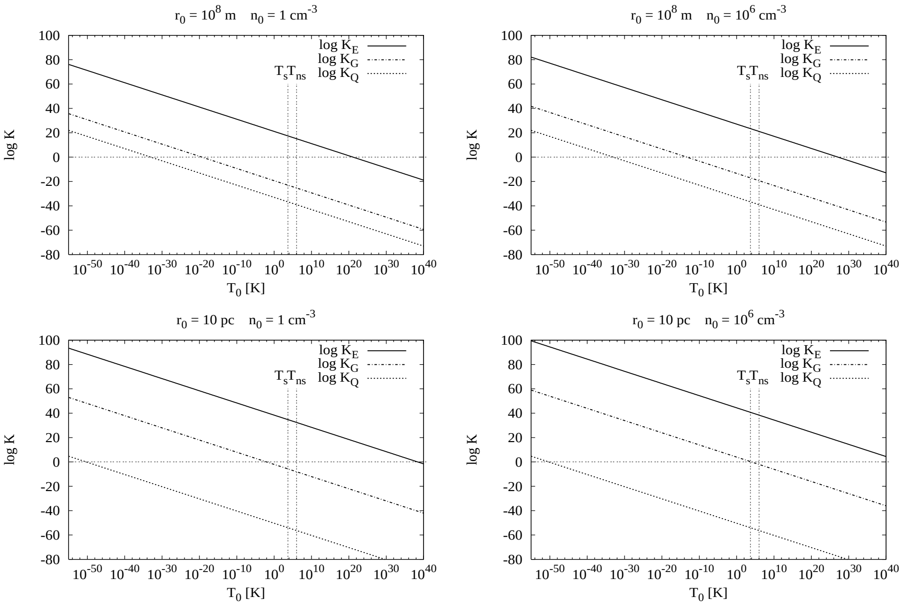

Figure 1: Temperature dependence of dimensionless numbers for two characteristic densities $n_0 = (1,\, 10^6)$ cm^{-3} and for r_0=10^8m, and $n_0 = (1,\, 10^6)$ cm^{-3} and for r_0=10 pc are shown in the upper i.e. lower two plots respectively. As a reference, two temperatures T_{ns} and T_s related to a neutron star and the Sun are indicated in the plots.

where: $\lambda\!\!\!^{-}$ and λ_D are the electron de Broglie and Debye length respectively while λ_G is a sort of gravitational equivalent to the Debye length. It sets a typical distance at which kinetic energy of thermal motions matches the gravitational potential energy of particles.

4. CONCLUSIONS

The obtained characteristic dimensionless numbers Eq. (5) depend on scaling parameters r_0 and n_0, and also on temperature T as shown in Figs. 1 and 2.

Domains where $K_E \gg 1$ indicate strong electrostatic forces and practically no local charge separation as can be concluded from Eq. (4). Namely, if $K_E \to \infty$ then $|\tilde{n}_e - \tilde{n}_i| \to 0$ which further makes the product $K_E(\tilde{n}_e - \tilde{n}_i)$ finite and of the same order of magnitude as the remaining leading terms in Eq. (4).

Large K_G indicates strong gravitational effects resulting in a pronounced plasma density stratification while the number K_Q measures the contribution of quantum effects in the plasma. As seen in Figs. 1 and 2, this number is extremely small for physical conditions relevant to realistic astrophysical systems. Inclusion of quantum effects into processes typical for astrophysical plasmas is therefore of purely academic significance and it induces no new phenomena to those already widely investigated in classical plasmas. In addition, one can easily see from Figs. 1 and 2 that $\lambda\!\!\!^{-2} \ll \lambda_D^2$

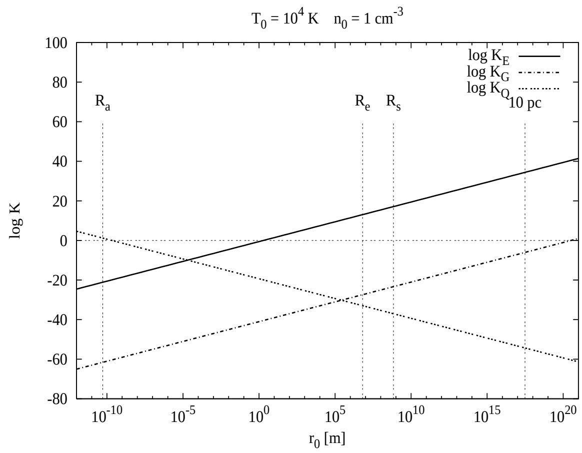

Figure 2: Dimensionless numbers as a function of the typical length r_0 for T_0=10^4K and n_0=1cm. As a reference several characteristic sizes are indicated in the plot: radii of an atom (R_a), Earth (R_E), Sun (R_S) and 10 pc a size typical of an HII region.

which clearly violates the very implicit assumption of a fluid description of plasmas as frequently found in the literature on quantum plasmas.

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