Publ. Astron. Obs. Belgrade No. 84 (2008), 181 - 184 *Contributed Paper*

BACKSCATTERING OF FAST ELECTRONS FROM SOLIDS WITHIN A MULTIPLE COLLISION MODEL

J. VUKANIĆ and D. M. DAVIDOVIĆ
Vinča Institute of Nuclear Sciences, P.O. Box 522, 11001 Belgrade, Serbia
E–mail: vukanic@vin.bg.ac.yu
E–mail: davidd@vin.bg.ac.yu

Abstract. Reflection of electrons from solids is treated by the approximate analytic solution of the linearized transport equation. Scattering of electrons on target atoms is determined by the screened Coulomb interaction and the energy loss due to interaction with target electrons is defined by Bethe- Bloch formula. The anisotropic P_3 approximation of the collision integral is utilized and the Bolzmann transport equation is Laplace transformed in relative path length and solved by applying the DP0 technique. The approach is applicable in a wide range of electron energy –from several tens of keV to several MeV- and for materials where the mean number of collisions of an electron with target atoms during slowing down is large. Analytic expressions for energy distribution of backscattered electrons as well as for the particle and energy reflection coefficients were derived. Comparison of our results with data of the computational bipartition model is presented.

1. THE BASIC PHYSICAL MODEL

The scattering and slowing down of fast electrons penetrating a solid targe can be desribed by the following expressions:

(a) The transport cross section for a screened Coulomb interaction between a fast electron and target atom can be approximated by the modified Rutherford formula

$$\sigma_1(T) = 2\pi \frac{Z(Z+1)r_e^2}{T^2} \frac{(T+1)^2}{(T+2)^2} L_1(T). \tag{1}$$

with

$$L_1(T) \approx L_1(T_0) = \ln(1 + \frac{1}{\eta}) - \frac{1}{1+\eta}. \tag{2}$$

The screeing constant η is determined by the formula of Moliere (see Moliere 1947). Furtermore, $T = E/(m_0c^2)$ is the kinetic energy of the electron expressed in units $m_0c^2 = 510.7keV,$ Z is the atomic number of target atom, and r_e is the classic radius of the electron. The cross section (1) incudes relativistic corrections and deflections caused by inelastic collisions between electrons.

(b) For energies $E > 10keV$, specific energy loss of primary electron caused by interaction with target elecrons can be well described by Bethe- Bloch formula (see Berger et al. 1982)

$$\frac{dT}{d\tau} = -NS(T) = -N4\pi Z r_e^2 \frac{(T+1)^2}{T(T+2)} L_{ion}(T) \tag{3}$$

where τ is the path length traveled, and the term $L_{ion}(T)$ varies very slowly with the energy and can be regarded as a constant i.e. $L_{ion}(T) \approx L_{ion}(T_0)$. Then, the total path length has the form

$$\tau_0 = \frac{1}{4\pi N Z r_e^2 L_{ion}(T_0)} \frac{T_0^2}{(1+T_0)}. \tag{4}$$

We treat the case of fast electrons moving in a solid target when the total path length τ_0 is much greater than the transport mean free path $\lambda_1(E_0) = [N\sigma_1(E_0)]^{-1}$. The dimensionless parameter ν

$$\nu = \frac{\tau_0(E_0)}{\lambda_1(E_0)} = N\sigma_1(E_0)\tau_0 \tag{5}$$

gives the mean number of wide angle collisions for the electron before slowing down to rest. According to Eqs. (1), (4) and (5) the parameter ν becomes

$$\nu = \frac{Z+1}{2} \frac{1+T_0}{(2+T_0)^2} \frac{L_1(T_0)}{L_{ion}(T_0)} \tag{6}$$

Eq. (6) gives the mean number of large angle collisions of electrons penetrating different targets, as a function of initial electron energy. It turns out that - except for very light targets-electrons undergo several deflections before coming to rest.

2. SOLUTIONS OF ELECTRON TRANSPORT EQUATION

The basic simplifying assumption entering the model is that the large number of wide angle deflections produce a nearly isotropic distribution of backscattered electrons. This model of backscattering enabled us to use the convenient approximation of the collision integral of the electron transport equation. We expanded the Laplace transformed electron distribution function in series of double Legendre polynomials over the angular variable. In the lowest order of approximation (DP0 method) we obtained Laplace transformed reflection function.

The approximate solution is simple if one assumes inverse square scattering potential for the screened Coulomb interaction between a high energy electron and target atom. Similar procedure was already successfully used for low energy ion reflection from heavy targets(see Vukanić et al. 2001). The reason is that the situation $\nu >> 1$ apears during penetration of low energy light ions through solids.

The scattering cross section can be approximated by the expression

$$d\sigma(T_0, \mu) = \frac{1}{2\sqrt{2}} \frac{\sigma_1(T_0) d\hat{\mu}}{(1-\mu)^{3/2}} \quad . \tag{7}$$

Here, $\sigma_1(T_0)$ is the transport cross section of the first order, determined by the formula of Moliere , given by Eq. (1). By applying Laplace transform in relative path length, and utilizing further DPN method ,we have found the solution for the half space reflection function in the lowest order of approximation (DP0 approximation) in form of an exponential function.

3. RESULTS AND DISCUSSION

Applying Laplace inversion, we obtained the analytic expression for the path length and angular distribution of backscattered electrons

$$R(\mu_0, \mu, s)\, d\mu\, ds = \frac{h\nu}{\sqrt{\pi}} \frac{\mu}{(\nu s + g)^{3/2}} \exp[-\frac{h^2}{4(\nu s + g)}]\, d\mu\, ds \tag{8}$$

with $h = h(\mu_0) = 3.094\, \mu_0$ and g=g$(\mu_0) = -1.72 + 14.44\mu_0^2 - 24.2\mu_0^4 + 11.48\mu_0^6$

Thank to the simplicity of the expression (6) for $R(\mu_0, \mu, s)$, we obtained by simple integrations over all exit directions and all possible path lengths the particle reflection coefficient in the form

$$R_N(\mu_0, \nu) = erfc(\frac{3.094\mu_0}{2\sqrt{\nu + g}}) \tag{9}$$

One can see from Eq. (9) that the particle reflection coefficient appears to be a universal function of the parameter ν which represent the mean number of wide-angle collisions of the ion during slowing down. Fig. 1 shows the universal curve $R_N(1, \nu)$ for electrons incident normally on solid targets. Our results calculated from Eq. (9) are compared with the data obtained from the computational bipartition model (see Luo Zheng-Ming 1985). One can see that the reflection coefficient scales with the characteristic parameter ν. The agreement of the present results with the computational data is good.

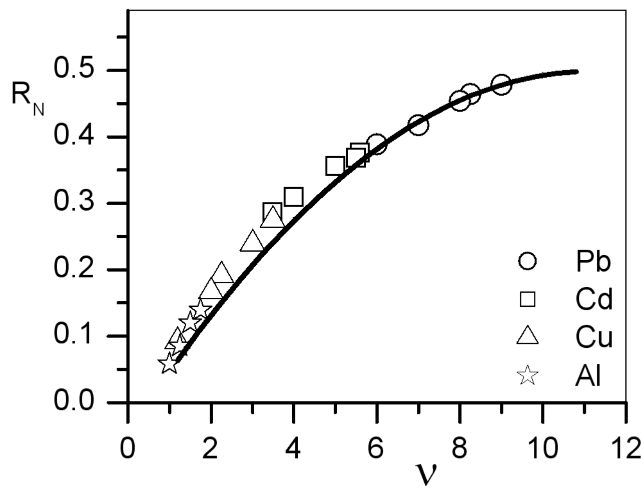

Figure 1: Particle reflection coefficient for electrons incident normally on solid targets. Our universal curve, calculated from Eq.(9) (solid line) is compared with computational data for different materials.

Good agreement of our theoretical estimates for the particle reflection coefficient with the result of computational bipartition model suggests that the obtained formulae describe correctly the backscattering of fast electrons from solids.

References

Berger, M. J., Seltzer, S. M.: 1982, Stopping Powers and Ranges of Electrons and Positrons, National Bureau of Standards Report NBSIR **82-2550**.

Moliere, G. Z.: 1947, *Naturforsch*, **24**, 133.

Vukanić, J., Simović, R.: 2001, *Nuclear Intruments and Methods in Physics Research* B, **179**, 497.

Zheng-Ming Luo: 1985, *Phys. Rev. B*, **32**, 824.