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THE STRUCTURE OF SPACE

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Abstract. In the paper we consider the continuity and discreteness of space. The problem of microworld space in non-Archimedean theories is specially treated. The possibility of existence of an empty space is also considered. The basic properties of Riemann's differentially-metrical approach in the development of geometry, as a science dealing with the space structure, and of Klein's one, based on the theory of groups, are emphasized. One presents the basic properties of space in Newton's mechanics, quantum mechanics, relativity theory and quantum cosmology.

1. INTRODUCTION

The structure of space concerns what does the space consist of. A structure means a multiple form consisting of elements comprised by a relationship. A structure is more than a simple set of elements forming it. The consideration of the structure of space involves the case of finding simultaneously solutions for the problems of geometry expressing this structure. Although historically still not reliably established, it seems that, as early as in the times of Newton, there was an influential standpoint claiming that the subject of geometry was the structure of space rather than the spatial properties of material bodies (Nejgel, 1974). The model of a continuous three-dimensional space was generally accepted longer than any other and it is alternatively referred to as the continual model or continuum. It was also advocated by Aristotle (Aristotle Phys.). The purely discrete concept of space is as old as the continuity one. Its fundaments can be found in the ideas of ancient Greek atomists, especially in the works of Democritus (Kuraev, 1972). The study of the microworld space resulted in developing of a discrete quantum space in the non-Archimedean theories. Every theory concerning the space structure also involves the ontological aspects of space. In such a way the substantial concept views the space as a set of sites for all material objects. This concept was developed by Democritus (Eejnshtejn, 1957) and Newton (Andreev, 1969). The absolute space continuum exists per se and does not depend on the properties of material objects situated within it. Shoemaker (1969) proved in his thought experiment the possibility of existence of an empty space and empty time. The relational concept of space views it as a structure of relations among material objects. The main representatives of this concept are Aristotle and Leibniz (Andreev, 1969). They thought that without any objects there would have been neither space no time.

2. CONTINUITY AND DISCRETENESS OF SPACE

As the human knowledge and praxis developed the ideas concerning the continuous structure of space passed an incessant progress. After Aristotle the continuity of space was advocated by Descartes, Newton, Leibniz and others. According to Leibniz (Panhenko, 1988) nature made no jumps, whereas according to Penrose (Penrouz, 1972) all phenomena can be modelled in the framework of a continuous differentiable multiplicity called space-time continuum. A continuum is, first of all, an ideal, mental, mathematical construction. Using the construction of continuity for the purpose of describing the structure of space is justified due to the properties of the macroscopic space. The space-time continuum is an idealization also in the domains where it becomes a physical reality.

The ideas of spatial continuity play an important role in modern physics and are widely used in the apparatus of physical theories. Both in classical physics and relativity theory the topology of space and time is formed in the framework of continuum hypotheses.

Although the concept of space continuity has been thought more adequate, many modern physicists point out that the concept of infinitely differentiable space-time multiplicity does not reflect the reality of the microworld. The question of the microworld-space structure is contained in establishing the changes suffered by some space notions having arisen on the basis of the microworld experimental data and of the consideration of the microworld proper geometry. The notion of point, as the basic one in mathematics, obtains a different meaning after passing into the microworld. In geometry a point is an abstraction of an infinitely small space region where infinitely small does not mean something finally given, but this is an endless process of diminishing. In the microworld the process of endless diminishing for a particle is not realizable because its mass becomes infinite. A point in the microworld is no limit for the endless diminishing process. In the theory of the microworld a point loses its obvious geometric interpretation which it has in the Euclidean geometry and Newtonian mechanics. A line is another basic geometric notion expressing in the macroworld the boundary of figures and bodies or the geometric presentation of a trajectory of motion. In the microworld, within atoms or elementary particles, obvious ideas on bodies and figures become meaningless and it is impossible to talk about their boundary. In addition, the modern interpretation of quantum mechanics does not involve the notion of the classical particle trajectory. Other fundamental notions of geometry, such as position, distance, angle etc., are subject to significant changes in the microworld.

The question of the microworld space has been specially studied in the non-Archimedean theories. The fields of mathematics used in these theories today are "pads" analysis and Robinson's non-standard analysis. The hypothesis of discrete space has been developed by Ivanenko, Foradori, Ambartsumyan, March, etc (Andreev, 1969). The basic idea of this hypothesis is to introduce an elementary length in the microworld geometry. March (Andreev, 1969) started from the problem of measuring.

The elementary length $l_0 = 10^{-35}$ m (Planck's length) introduced by him is, at the same time, also unit of time. Planck's length is obtained from the formulae $l_0 = \sqrt{\frac{\hbar G}{c^3}}$, where $\hbar = \frac{h}{2\pi}$ is h-Planck's constant, G-gravitation constant, and c velocity of light. The reason for introducing the hypothesis concerning the possibility of existence of a space quantum, of the order of Planck's length, is also due to our awareness that at those distances all the types of physical interactions become equally effective and because of this their distinction is impossible. There are other indications that space-time at small distances possesses some unusual properties inexplicable in the framework of geometries constructed on the basis of the field of real numbers. At Planckian distances Archimedes' axiom of the standard Euclidean geometry is violated. According to Archimedes' axiom $\forall x, y \in N \;.\cdot. \left|x\right|_\infty < \left|y\right|_\infty$, there is a natural number n so that we have $\left|nx\right|_\infty > \left|y\right|_\infty$. Due to the given axiom it is possible to measure distances in the world of large dimensions. However, at super small distances, of order of l_0, there is a principle limitation to the measuring precision. According to this limitation it is impossible to measure distances shorter than l_0. For such distances the error exceeds the measuring amount. The consequence of this is that within such distances Archimedes' axiom is violated so that the corresponding geometry cannot be Euclidean. If measurements are carried out with an accuracy of l_0, the metrics has the following form $ds^2 = \sum\limits_{i=1}^{4} dx_i^2 \pm l_0$. A metrics of this type has a statistical character because when a measurement is repeated, each time an independent, accidental, result is obtained with excess or deficit of l_0.

3. GEOMETRY OF SPACE

In the intensive development of geometry after Lobachevskij, two research directions appeared: Riemann's differentially-metrical (Andreev, 1969) and Klein's approach based on the group theory (Andreev, 1969).

Following Riemann (Andreev, 1969), the ordinary space can be considered as a multiplicity of points in three dimensions, i. e. as a special case of n-dimensions multiplicity. His basic geometric notion is that of length determined by him by applying a differential form. The distance between two nearby points x_i and $x_i \div dx_i$ is determined by using the following formula $ds^2 = \sum\limits_{i,k} g_{ik} dx^i dx^k$ where ds^2 is a multiplicity invariant and g_{ik} is the metrics or fundamental tensor. The given distance form is, at the same time, the basic metrics form of space. Its use enables to calculate length, amounts of angles, areas and volumes. A Cartesian coordinate system can be also chosen and the distance between two infinitely close points can be measured in accordance with the theorem of Pythagoras, just as in the case of a plane and a space with elementary geometry. In the case of uneven multiplicities Riemann introduces the curvature as a special characteristic describing their deviating from even multiplicities. He formulated the fundaments of the theory of constant- and variable-curvature spaces. Constant-curvature spaces are homogeneous spaces. They contain no preferential points and directions. In these spaces bodies can be translated with no extensions and contractions. Variable-curvature spaces are inhomogeneous spaces.

In them bodies cannot be translated or rotated arbitrarily because the curvature does not remain the same in every point and in any direction. Riemann thought that there was a dependence of metrics relationships on real processes in which he came close to Leibniz's concept. Riemann's approach found its realization in the general relativity theory. The mutual interconnection between space and matter was proved in it.

The basic direction in Klein's research of the space geometry is the incidence axiom interpreted in the terms of mathematical group notion. Physical motion, considered analytically, is contained in varying or transforming the coordinates. If the motion of rigid bodies is considered, transformations of coordinates are considered as a motion of space. This motion can be one of the following four possibilities: translatory, rotational, helicoidally and mirror reflection. Any given motion can be also considered as a combination of the four space motions mentioned previously. The space properties studied in the Euclidean geometry are invariant with respect to the motion groups. The group concept is among the principal notions of modern mathematics and plays an important role in the development of geometry. Klein developed a theory concerning a space with properties invariant with respect to transformations of the corresponding group. Spaces determined by means of a group are referred to as Klein's spaces. If an M-multiplicity and G-group comprising its topological transformations onto itself are given, then the multiplicity M together with the group G gives rise to a Klein's space M_G. If substantial properties within a Klein's space are invariant with respect to the transformation of a group, all the bodies originated from a given one by means of this transformation are equivalent. A consequence is that there are no preferential places and that all Klein's spaces are homogeneous. In geometry, Klein's space appears as a generalization of the homogeneous space of the Euclidean geometry and of the Newtonian mechanics. The concept of homogeneous Klein's spaces is realized also in the special relativity theory because its space-time does not depend on its matter content.

4. CONCLUSION

Two basic models of space structure are the continuous and discrete ones. The former one found its application in the Newtonian physics, special and general relativity theories. The discrete model is applied in quantum cosmology. Riemann's differentially-metrical research direction of geometry (Andreev, 1969) has found its application in the general relativity theory, whereas Klein's one, based on group theory, finds its application in the Newtonian physics and special relativity theory.

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