Publ. Astron. Obs. Belgrade No. 80 (2006), 197 - 202 *Contributed paper*

GRAVITATIONAL INFLUENCE OF ASTEROIDS AND DETERMINATION OF THEIR MASSES

(Phd Thesis)

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Abstract. The masses of the seven largest asteroids: (1) Ceres, (2) Pallas, (4) Vesta, (10) Hygiea, (52) Europa, (511) Davida and (704) Interamnia were determined from gravitational perturbations exerted on selected asteroids. These masses were calculated by means of the modified method. The procedure of the mass determination is based on calculation of perturbed orbit using observations which cover pre or post encounter part of the orbit. Then the pre and post encounter parts of perturbed orbit has been linked by some chosen value for the mass of a perturbing body. The procedure is an iterative one. The resulting mass of the perturbing asteroid is obtained as one which provides the minimum O-C residuals in the RMS sense.

1. INTRODUCTION

The largest reservoir of asteroids in the inner Solar system is the main belt, located between the orbits of Mars and Jupiter. Much of what we see in the asteroid belt is a consequence of past collisions, which shaped the size-frequency distribution of asteroids and led to their heavily bombarded surfaces. Although knowledge of the masses and bulk densities of asteroids is critical in assessing their composition, to determine these quantities is a difficult task. There are two groups of methods for asteroid mass determination: astrophysical and dynamical. In the dynamical methods the mass of an asteroid has to be estimated from its perturbation of the motion of some other body. At present, masses more than 20 asteroids have been determined by using asteroid-asteroid close approaches, masses of 3 asteroids were determined using spacecraft trajectories and masses of about 10 bodies were calculated using revolution period of binary asteroids. Use of close encounter requires a precise determination of the orbit of perturbed asteroid before and after the event. Classical least squares method has been used by many authors in order to determine asteroids masses. According to this method, correction of the mass of the perturbing asteroid is computed along with the corrections of six osculating elements of the perturbed asteroid. In our work we tried to find out is it possible to determine correction of perturbing mass separately from corrections of six osculating elements of perturbed asteroid. As a consequence we introduce the modified method of asteroid mass determination.

2. PROCEDURE OF MASS DETERMINATION USING MODIFIED METHOD

There are several modification of least squares method introduced by different authors (Sitarski and Todorovic-Juchniewicz, 1992; Michalak, 2000). The idea of our modification is to separate preencounter and postencounter sets of observations (parts of orbit) of perturbed asteroid. During this process it is not necessary to know the mass of the perturbing asteroid, because its perturbing effects are negligible. These two orbits are separated by an impulsive change due to the close encounter and have to be connected by properly accounted gravitational effects of the perturbing body. If the pre and post encounter orbits are accurately determined, the same mass of the perturbing body will give the best representation of the postencounter observations with the preencounter orbit and vice-versa. Similarly to classical least squares method, system of linear equations of modified method can be expressed in the matrix space as:

$$A\Delta m = B\,, \tag{1}$$

where the matrix A depends on the partial derivatives of the coordinates of postencounter observations (right ascensions and declinations) of the perturbed asteroid with respect to the perturbing mass. Δm is the correction of the perturbing mass and B is the matrix depending on $(O-C)$ residuals in postencounter coordinates of the perturbed body. Elements of matrices A, B were computed for each epoch of observation. The procedure of solving the system (1) is an iterative one. At the first iteration, elements of matrices A and B were calculated using previously selected observations of perturbed bodies (based on 3σ criterion). Obtained correction for the perturbing mass produced a new solution which was used as initial condition for the next iteration. Only two iterations were necessary until convergence. The formal error of calculated mass can be described as follows:

$$\sigma_m = \frac{\sigma_0}{\sqrt{\sum_{i=1}^{n} \frac{\partial c_i}{\partial m}}}\,, \tag{2}$$

n is the number of observations and σ_0:

$$\sigma_0 = \sqrt{\frac{(O-C)^2}{2n-1}}\,, \tag{3}$$

We calculated masses of 7 largest asteroids using modified method and standard least squares method. Both methods were applied to close approaches used by other authors as well as to newly found ones. Bearing in mind that some other minor planets could perturb the motion of a chosen perturbed asteroid, 9 largest asteroids have been included in the dynamical model, besides all major planets.

Table 1: Close encouters used for mass determinations: T is the total number of used close encounters, N is the number of newly found close encounters.

Perturbing asteroid	T	N
(1) Ceres	21	4
(2) Pallas	4	0
(4) Vesta	12	4
(10) Hygiea	8	1
(52) Europa	2	0
(511) Davida	3	1
(704) Interamnia	1	1

3. RESULTS

As can be seen from Table 1, we used 51 close encounters in order to determine masses of 7 largest asteroid in the main belt using modified and standard method. The largest asteroid in the main belt has the largest number of efficient close encounters (already known as well as newly found). The range of values for Ceres mass, determined by other authors, is $(4.6 - 5.0)\,10^{-10}M_{\odot}$. After applying modified and standard method we obtained results which are presented in Table 2. As it can be seen, both methods provided results which differ from each other by no more then 3σ (3σ is their own formal error). Also, they are within the historical range of determined masses of Ceres.The results for the Ceres mass based on newly found close encounters, that occured with asteroids: (2051) Chang, (6010) Lyzenga, (6594) Tasman and (34755) 2001QW120, are published elsewhere (Kovačević and Kuzmanoski, 2005). In addition, we found that weighted mean of the values of the Ceres mass obtained by the modified $(4.63 \pm 0.07)\,10^{-10}M_{\odot}$ and the standard method $(4.70 \pm 0.05)\,10^{-10}M_{\odot}$ satisfied 3σ criterion with respect to the adopted value of the mass of Ceres.

Because of the highly inclined and eccentric orbit of (2) Pallas, its close encounters with other asteroids are rare. In Table 3 we present the solutions for the mass of (2) Pallas with formal errors not greater than approximately half the mass of this minor planet. Its weighted mean value obtained using modified method is $(1.23 \pm 0.11)\,10^{-10}M_{\odot}$, while standard method produced $(0.95 \pm 0.08)\,10^{-10}M_{\odot}$.

Many authors emphasized the importance of reliable level of accuracy of the mass of (4) Vesta, since this asteroid is the second most massive body in the main belt. The results are listed in Table 4. The final values of the mass of Vesta, determined as a weighted mean are $(1.28 \pm 0.03)\,10^{-10}M_{\odot}$(using modified method) and $(1.35 \pm 0.18)\,10^{-10}M_{\odot}$ (using standard method). These values agree with all determinations made so far. Results based on newly found close encounters have been published in Kovačević (2005).

Table 2: Geometrical and kinematical parameters of close encounters: ρ is the minimum distance, V_r is relative velocity and θ is deflection angle of perturbed asteroid.Mass of Ceres obtained using standard and modified method is given in columns SM and MM.

Perturbed asteroid	date d.m.y	ρ [AU]	V_r [$\mathrm{km\,s^{-1}}$]	θ [arcsec]	SM [$10^{-10}\,M_\odot$]	MM [$10^{-10}\,M_\odot$]
(2) Pallas	16.05.1825	0.188	12.61	0.01	4.45 ± 0.05	4.22 ± 0.04
(32) Pomona	16.05.1825	0.188	12.61	0.01	5.32 ± 0.16	5.18 ± 0.05
(76) Freia	05.08.1957	0.212	4.08	0.05	4.27 ± 0.08	4.14 ± 0.06
(91) Aegina	13.09.1973	0.033	3.28	0.49	4.91 ± 0.04	5.00 ± 0.02
(203) Pompeja	22.08.1948	0.016	4.12	0.63	4.73 ± 0.04	4.79 ± 0.02
(348) May	02.09.1984	0.046	0.79	6.07	4.74 ± 0.05	4.77 ± 0.01
(347) Pariana	29.05.1943	0.078	1.48	1.02	4.80 ± 0.09	4.72 ± 0.05
(454) Mathesis	23.11.1971	0.021	2.93	0.97	4.48 ± 0.06	4.33 ± 0.01
(488) Kreussa	17.07.1963	0.282	3.00	0.07	4.64 ± 0.16	4.26 ± 0.11
(534) Nassovia	24.12.1975	0.023	2.75	1.00	4.83 ± 0.07	5.12 ± 0.04
(548) Kressida	13.07.1982	0.049	2.95	0.41	5.28 ± 0.24	4.89 ± 0.10
(621) Werdandi	01.05.1962	0.050	3.04	0.38	4.35 ± 0.15	4.56 ± 0.20
(792) Metkalfia	25.07.1950	0.013	5.78	0.40	5.81 ± 1.10	5.22 ± 0.35
(850) Altona	22.02.1970	0.026	3.84	0.45	4.91 ± 0.16	4.68 ± 0.11
(1642) Hill	25.11.1925	0.012	5.54	0.47	4.81 ± 0.06	4.81 ± 0.08
(1847) Stobbe	07.09.1958	0.094	1.76	0.60	3.94 ± 0.23	4.10 ± 0.17
(3344) Modena	27.09.1980	0.021	2.39	1.45	4.34 ± 0.38	4.36 ± 0.11

Table 3: Mass of Pallas obtained using standard and modified method.

Perturbed asteroid	date d.m.y	ρ [AU]	V_r [$\mathrm{km\,s^{-1}}$]	θ [arcsec]	SM [$10^{-10}\,M_\odot$]	MM [$10^{-10}\,M_\odot$]
(1) Ceres	02.01.1830	0.188	12.61	0.01	1.32 ± 0.05	1.57 ± 0.03
(582) Olimpia	14.07.1936	0.033	3.19	0.12	0.90 ± 0.08	1.04 ± 0.21
(3131) Mason-Dixon	04.12.1984	0.012	10.84	0.03	1.66 ± 0.32	1.23 ± 0.22
(5930) Zhiganov	17.06.1977	0.015	12.01	0.02	1.17 ± 0.44	1.36 ± 0.17

Table 4: Mass of (4) Vesta obtained using standard and modified method.

Perturbed asteroid	date d.m.y	ρ [AU]	V_r [$\mathrm{km\,s^{-1}}$]	θ [arcsec]	SM [$10^{-10}\,M_\odot$]	MM [$10^{-10}\,M_\odot$]
(8) Flora	09.03.1963	0.2265	3.02	0.02	1.35 ± 0.05	1.64 ± 0.02
(17) Thetis	19.06.1996	0.0194	1.18	1.83	1.35 ± 0.02	1.288 ± 0.001
(56) Melete	14.11.1923	0.1122	4.62	0.02	1.34 ± 0.09	1.57 ± 0.04
(67) Asia	20.01.1991	0.0311	4.45	0.08	1.18 ± 0.07	1.21 ± 0.01
(77) Frigga	07.06.1955	0.0249	4.62	0.09	1.38 ± 0.06	1.29 ± 0.04
(109) Felicitas	15.04.1959	0.0191	8.16	0.04	1.52 ± 0.06	1.77 ± 0.03
(163) Erigone	24.04.1934	0.1960	5.64	0.01	1.16 ± 0.09	1.05 ± 0.06
(197) Arete	14.05.1885	0.0181	2.22	0.55	1.32 ± 0.02	1.34 ± 0.01

From 8 close encounters with (10) Hygiea we obtained results presented in Table 5. Difference between the weighted mean mass obtained for the modified method ($(4.72 \pm 0.16)\,10^{-11} M_\odot$) and standard method ($(4.68 \pm 0.24)\,10^{-11} M_\odot$) are not larger than 3σ.

Table 5: Mass of (10) Hygiea obtained using standard and modified method.

Perturbed asteroid	date d.m.y	ρ [AU]	V_r [$\mathrm{km\,s^{-1}}$]	θ [arcsec]	SM [$10^{-10}\,M_\odot$]	MM [$10^{-10}\,M_\odot$]
(7)Iris	18.01.1928	0.0724	4.26	1.31	5.86 ± 0.4	5.29 ± 0.40
(20)Massalia	05.11.1933	0.1499	2.15	2.48	4.69 ± 0.46	6.49 ± 0.5
(60)Echo	07.05.1867	0.2111	3.24	0.78	5.30 ± 1.00	5.08 ± 0.9
(69)Hesperia	05.09.1951	0.0862	5.23	0.73	5.76 ± 1.1	5.10 ± 0.8
(111) Ate	11.02.1878	0.0942	1.76	5.90	5.88 ± 0.6	5.60 ± 0.2
(209)Dido	09.05.1958	0.2463	2.22	1.42	4.30 ± 1.0	4.98 ± 0.7
(829)Academia	19.05.1927	0.0064	3.22	25.92	2.65 ± 1.0	2.49 ± 1.23
(3946) Shor	30.03.1998	0.0144	0.91	1.44	3.10 ± 0.4	2.52 ± 0.26

Only recently, the mass of (52) Europa was determined for the first time (Michalak 2001). Both our methods gave results for its mass only in the case of the close encounters with (306) Unitas and (1023) Thomana. We did not calculate the weighted mean value of the mass of (52) Europa, because we have only two cases.

Table 6: Mass of (52) Europa obtained using standard and modified method.

Perturbed asteroid	date d.m.y	ρ [AU]	V_r [$\mathrm{km\,s^{-1}}$]	θ [arcsec]	SM [$10^{-10}\,M_\odot$]	MM [$10^{-10}\,M_\odot$]
(306) Unitas	14.01.1945	0.0980	2.2	0.01	2.12 ± 0.56	2.76 ± 0.21
(1023) Thomana	31.05.1971	0.0066	3.76	0.04	0.78 ± 0.49	1.17 ± 0.70

The mass of (511) Davida was also estimated only recently (Michalak, 2001). As it can be seen from Table 7, we used 3 close encounters for its mass determination.Weighted mean values are:$(2.21 \pm 0.18)\,10^{-10} M_\odot$ obtained from the standard method and $(2.72 \pm 0.02)\,10^{-10} M_\odot$ based on the modified method.

Table 7: Mass of (511) Davida obtained using standard and modified method.

Perturbed asteroid	date d.m.y	ρ [AU]	V_r [$\mathrm{km\,s^{-1}}$]	θ [arcsec]	SM [$10^{-10}\,M_\odot$]	MM [$10^{-10}\,M_\odot$]
(89) Julia	27.10.1957	0.0389	9.90	0.001	2.09 ± 0.53	2.75 ± 0.02
(532) Herculina	14.04.1963	0.0307	4.24	0.01	2.20 ± 0.20	2.35 ± 0.07
(7191) 1993MA1	16.07.1969	0.0046	5.98	0.03	2.88 ± 1.08	2.47 ± 0.47

In the case of (704) Interamnia we have only one useful close encounter. As it can be seen from Table 8, formal errors of the masses of Interamnia are greater then 50% of the calculated values for the masses. Further observations of this asteroid are highly desirable to enable a more exact and reliable estimation of Interamnia's mass.

Table 8: Mass of (704) Interamnia obtained using standard and modified method.

Perturbed asteroid	date d.m.y	ρ [AU]	V_r [$\mathrm{km\,s^{-1}}$]	θ [arcsec]	SM [$10^{-10}\,M_\odot$]	MM [$10^{-10}\,M_\odot$]
(7461) Kachmokiam	31.05.1997	0.0075	5.29	0.02	2.23 ± 1.00	0.97 ± 0.52

4. CONCLUSION

We calculated the masses of 7 largest asteroids independently for all perturbed asteroids using standard and modified method. Some of these perturbed asteroids were never used before for this purpose, nevertheless giving quite good estimates of the mass of the massive minor planets. Generally, the masses we found agree with recent results of other authors and indicate that the mass of (1) Ceres appears to be equal to the adopted value as well as the mass of (4) Vesta. Results for the masses of other six asteroids are in good agreement with results obtained by other authors. However, most of the available observations used for their mass determination have high errors and uneven distribution. As it can be seen from numerical tests, modified method provided results which are in good agreement with standard method and adopted values of asteroid's masses. It can be used as a tool for asteroid mass determination. It is obvious that refining the dynamical model will improve the accuracy of the mass determination of massive asteroids.

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