AN ALGORITHM FOR GENERATING
AMPLITUDE MULTIPLICATORS

BRANKICA ŠURLAN and STEVO ŠEGAN

Department of Astronomy, Faculty of Mathematics, University of Belgrade
Studentski trg 16, 11000 Belgrade, Serbia and Montenegro
E–mail: brankica74@matf.bg.ac.yu
E–mail: ssegan@matf.bg.ac.yu

Abstract. It has been shown in selecting of the methods for the separating of periodic components that a very efficient use of the method of ordinate transformation is possible. Here an algorithm for generating almost arbitrary amplitude multipliers and their complexes is presented. The applicability limits are interactively variable. The algorithms are associated preprocess and postprocess graphical interfaces.

1. INTRODUCTION

The aim of this paper is to present a general algorithm for generating arbitrary **amplitude multiplicators** (AM). These multiplicators are a special result of Labroust's transformations, and have been discussed in detail in (Šegan et al., 2003). Program code is given in standard FORTRAN. The main is the user interface for choosing the correspondent selectivity curve which is convenient either for spectral analysis or discrete functions approximations.

2. AMPLITUDE MULTIPLICATORS

Functions given by

$$\rho_m = c_0 + c_1 \alpha_1 + ... + c_m \alpha_m \tag{1}$$

or

$$\rho_m = c_0 + \sum c_j \alpha_j, \tag{2}$$

where $m = 1, 2, ...$ and α_m is in form dependent of the type of transformation, are called **amplitude multiplicators** (AM). A set of amplitude multiplicators $\rho = \rho(\omega)$, where ω is the circular frequency, is *selectivity curve*.

A FORTRAN program was written for generating the main sequence of AM. By combination of several AM (that is **order** or **power**) you will define different transformations:

  – collective or additive, which summarize the AM
  – distinctive or subtractive, which subtract the AM,
from which we can construct arbitrary selectivity curve.

  The forms of AM are different. Ordinary additive transformations are of the following:

$$\alpha_m = 2\cos(\frac{2m\pi}{T}) \tag{3}$$

By choosing 9 as the degree of transformation we obtain the selectivity curve as shown at Fig. 1.

  Ordinary subtractive transformations generally are in the form

$$\beta_m = 2\sin(\frac{2m\pi}{T}) \tag{4}$$

For example, a selectivity curve of the order 4 is given with Fig. 2.

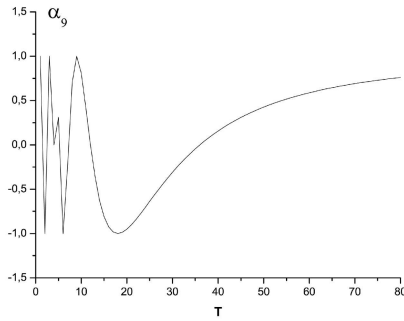Figure 1: Selectivity curve for the transformation α_9.
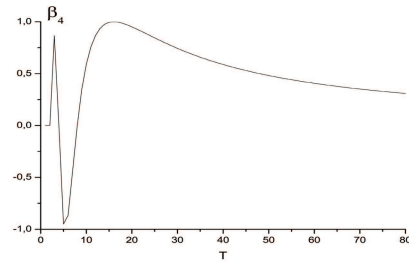
Figure 2: Selectivity curve of the transformation β_4.

For the sum of AM with ordinary additive transformations we have the expression

$$\sigma_m = 1 + \alpha_1 + \alpha_2 + ... + \alpha_m = \frac{sin\frac{(2m+1)\pi}{T}}{sin\frac{\pi}{T}} \tag{5}$$

With order 4 selectivity curve is given by Fig. 3.

  AM of the sum of the ordinary transformations has next form

$$\tau_m = 1 + \beta_1 + \beta_2 + ... + \beta_m = 2\frac{sin\frac{(m+1)\pi}{T}sin\frac{m\pi}{T}}{sin\frac{\pi}{T}} \tag{6}$$

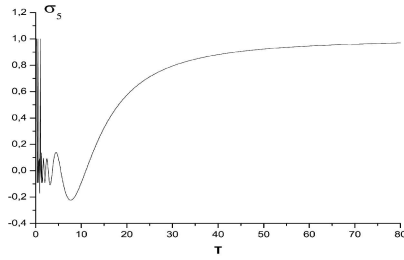With order 5 selectivity curve is given with Fig. 4.

186

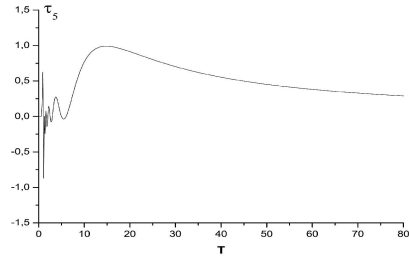Figure 3: Selectivity curve of the transformation σ_5.

Figure 4: Selectivity curve of the transformation τ_5.

AM of the sum of ordinary transformations with arbitrary lag has a form

$$(\sigma_m)_{\frac{N}{2}} = sign\left(\frac{cos\frac{(2m+1)N\pi}{2T}}{cos\frac{N\pi}{2T}}\right) \tag{7}$$

For even m the AM is positive, for the odd m the AM is negative. Let order of transformation is 5 and lag is 3; the selectivity curve is given with Fig. 5.

AM for the sum of the ordinary subtractive transformations with arbitrary lag has the form

$$(\tau_m)_{\frac{N}{2}} = sign\left(\frac{sin\frac{(m+1)N\pi}{2T}}{cos\frac{N\pi}{2T}}\right) \tag{8}$$

With $m = 4m' + 1$ the AM is positive; if $m = 4m' - 1$ the AM is negative. The m' is any ordinary number. For the order 9 and lag 3 selectivity curve is given with Fig. 6.
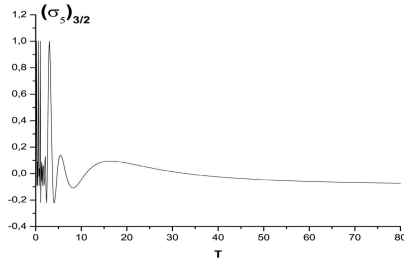

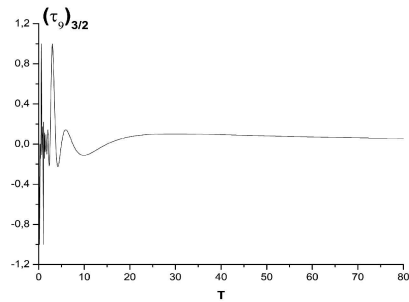Figure 5: Selectivity curve of the transformation $(\sigma_5)_{\frac{3}{2}}$.

Figure 6: Selectivity curve of the transformation $(\tau_9)_{\frac{3}{2}}$.

There are next few examples. AM of the complex transformation of ordinary additive transformations with arbitrary lag has a form

$$\Pi((\sigma)_{\frac{N}{2}}) = (\sigma_m)_{\frac{N}{2}}(\sigma_p)_{\frac{N}{2}}...(\sigma_r)_{\frac{N}{2}} \tag{9}$$

187

The complex transformation 3,5,6 with step 24 selectivity curve is given with Fig. 7.
The AMs of complex subtractive transformations with arbitrary step has a form

$$\Pi((\tau)_{\frac{N}{2}}) = (\tau_m)_{\frac{N}{2}}(\tau_p)_{\frac{N}{2}}...(\tau_r)_{\frac{N}{2}} \tag{10}$$

As an example of the transformation with terms 9,13,17 with step 20 on Fig. 8.
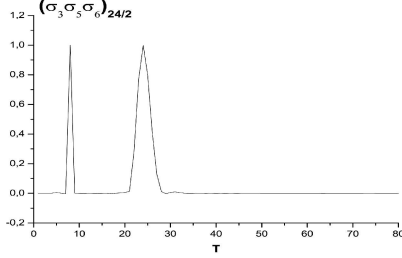

Figure 7: Selectivity curve of the transformation $(\sigma_3\sigma_5\sigma_6)_{\frac{24}{2}}$.
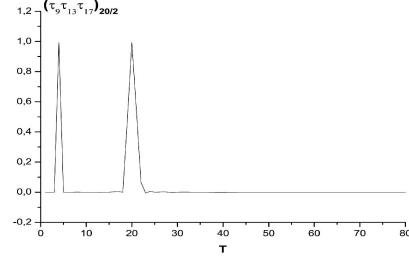

Figure 8: Selectivity curve of the transformation $(\tau_9\tau_{13}\tau_{17})_{\frac{20}{2}}$.

With the AM of the complex additive transformation in the form

$$\Pi(\alpha) = \alpha_m\alpha_p...\alpha_r \tag{11}$$

and with the terms 2,3,4,5 selectivity curve is given with Fig. 9.
The usual form of transformations with AM has the form:

$$\Pi(\sigma) = \sigma_m\sigma_p...\sigma_r \tag{12}$$

In accordance with it, the selectivity curve for the terms 4, 5, 6, 7 is given with Fig. 10.
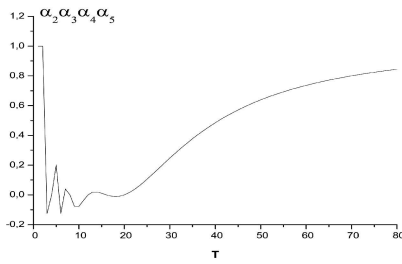

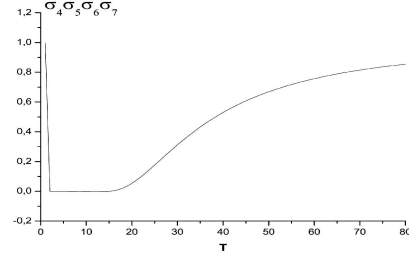Figure 9: Selectivity curve of the transformation $\alpha_2\alpha_3\alpha_4\alpha_5$.

Figure 10: Selectivity curve of the transformation $\sigma_4\sigma_5\sigma_6\sigma_7$.

Like that, the AM for the complex of the subtractive transformations has a form

$$\Pi(\beta) = \beta_m\beta_p...\beta_r \tag{13}$$

188

and with terms 3,4,4,5 selectivity curve is on Fig. 11.

In practice frequently you have transformations with AM in the form

$$\Pi(\tau) = \tau_m \tau_p ... \tau_r \tag{14}$$

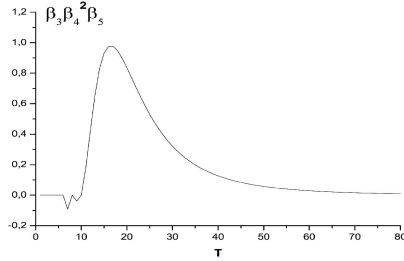For example, with terms 4, 5, 6, 7 selectivity curve is given with Fig. 12.

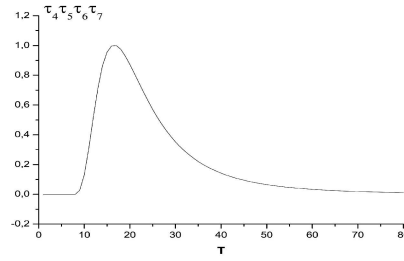Figure 11: Selectivity curve of the transformation $\beta_3 \beta_4^2 \beta_5$.

Figure 12: Selectivity curve of the transformation $\tau_4 \tau_5 \tau_6 \tau_7$.

3. CONCLUSIONS

By combination this algorithm with previous one (S. Šegan et al., 2003) we have developed very successful tool for analysis of complex transformations of the ordinates. It is coded in FORTRAN. Contrary to usual procedures, this algorithm is simple and easily generates any AM. Theoretically, unresolvable problems of spectral analysis as simultaneously time and frequency recognition, existence and separation real dumping and superposition and time to time harmonic influences, by this algorithm, with experienced user, have been undertaken. The limitation of this algorithm has been explained in (S. Šegan et al., 2003).

References

Djurović, D.: 1979, Matematička obrada astronomskih posmatranja, Ed. IŠRO "Privredno-finansijski vodič, Beograd, 280.

Labrouste, H. et I.: 1943, Analyse des graphiques resultant de la superposition de sinusoides, Ed. Les presses universitaires de France, Paris, **2**04, 1943.

Šegan, S., Damjanov, I., Šurlan, B.: 2003, Multi-composing of the ordinates as a spectral filter, *Serb. Astron. J.*, **167**, 53.

189