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DISCRETE AND SINGULAR FUNCTIONS APPROXIMATED BY AMPLITUDE MULTIPLICATORS (SHADOWING EFFECTS BY SPECIAL FUNCTIONS)

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Abstract. General requirements and a review of existing approximations of discrete functions are given. Some AMMCO (Amplitude Multiplicators of Multi-Composing of Ordinates) are applied to selected theoretical examples and the results are compared with those obtained by using classical models. Using the shadow function we demonstrate the usability of these approximations in the construction of the so-called model of a planet atmosphere. A method of AMMCO choice for the variations in the solar-radiation on the artificial celestial bodies in the eclipse positions is developed.

1. INTRODUCTION

The short-period perturbations of orbits of artificial satellites due to radiation pressure in one orbital period are influenced by Earth's shadow. For the semi-analytical theories it is necessary to calculate great number of coefficients. Using computer algebra we have computed them for some of semi-analytical theories.

2. PERTURBING FUNCTION AND EARTH'S SHADOWING EFFECTS

The perturbing acceleration due to the radiation pressure is given by equation

$$\vec{f} = \vec{\kappa} P_0 \frac{S}{m}, \tag{1}$$

where the vector $\vec{\kappa}$ directed from the Sun to the Earth, S is cross-section of the satellite, m is the mass of the satellite and P_0 is the numerical value of the solar pressure to the unit surface.

If the eccentric anomaly E is taken as independent variable and if Lagrange perturbing equations are used for the variation of constants, after some algebraic transformations, the general formula for the arbitrary element ρ can be obtained as

$$\frac{d\rho}{dE} = \frac{f_p^\sigma}{1-\cos E}[(1-e^2)^{s_p^c}\sum_{q=0}^{3} c_{pq}^\sigma \cos^q E + (1-e^2)^{d_p^c}$$

$$\sum_{q=0}^{2} g_{pq}^\sigma \cos^q E \sin E] \qquad (2)$$

where f_p^σ is function of f and some orbital elements, c_{pq}^σ, g_{pq}^σ are functions of orbital elements and s_p^c, d_p^c are some constants.

The powers of trigonometrical functions of E can be transformed into functions of multiple of argument; the method was used by Sehnal and Mills (1966) and Šegan (1987) for computation of short-periodic perturbations due to atmospheric drag. Some parts of Eq. (2) are developed into infinite power series which are convergent for $e < 1$.

3. DISTURBING OF RADIATION PRESSURE PERTURBING FUNCTION BY EARTH'S SHADOW

Until Ferraz-Mello (1964) has introduced a special "shadow function", which has zero value if the satellite is in shadow and equals one in the opposite case, the shadowing effect was usually taken into account numerically (Kozai, 1961). Later, Lala and Sehnal (1969) and Vaškovjak (1974) have introduced different forms of the shadow function with a peculiar series expansion. General lines of these shadow functions lies from the geometry given by Fig 1.

We shall assume that the Earth is spherical. The angular distance of a satellite from an axis passing through the centers of the Sun and the Earth is denoted by λ and angular semi-diameter of position of crossing point of momentary orbit with shadow is given by Φ. The shadow function is

$$\Psi = \begin{cases} 0, & -\phi < \lambda < \Phi \\ 1, & -\pi < \lambda < -\Phi, \ \Phi < \lambda < \pi \end{cases} \qquad (3)$$

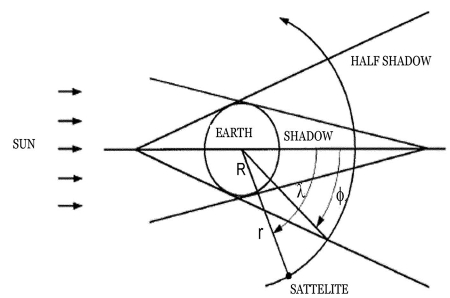

Figure 1: Geometry of shadow function.

where $\sin\Phi = r(t_0)/R_\oplus$ and $r(t_0)$ is radius vector at moment of intersection with shadow boundary. By using this definition Ferraz-Mello expanded Ψ in Fourier's series:

$$\Psi = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos k\lambda \qquad (4)$$

where

$$a_k = \frac{2}{\pi}\int_0^\pi \Psi \cos k\lambda d\lambda.$$

After dividing integration interval on $[0,\Phi]$ and $(\Phi,\pi]$ we have

$$a_0 = \frac{2}{\pi}(\pi - \Phi), \quad a_k = -\frac{2}{k\pi}\sin k\Phi.$$

By introducing this in (3), we have

$$\Psi = 1 - \frac{\Phi}{\pi} - \sum_{k=1}^{\infty} \frac{2}{k\pi} \sin k\Phi \cos k\lambda. \tag{5}$$

The computation of the coefficients of this expansion is not so complicated, but after multiplying perturbation Eq. (2) for an arbitrary orbital element by (5) integration will be very complicated.

Lala and Sehnal (1969) started from the fact that a discontinuity in the shadow function can be represented as

$$\Psi = \frac{1}{2}(1 + \frac{\sin x}{+\sqrt{1-\cos^2 x}}) \tag{6}$$

where is $x = \lambda - \Phi$, which leads to

$$\Psi = \frac{1}{2}\{1 + \sum_{j=0}^{\infty}\sum_{k=j}^{\infty} D_{jk} \sin^{2j+1}(\lambda - \Phi)\} \tag{7}$$

$$\Psi = \frac{1}{2}\{1 + \sum_{j=0}^{\infty}\sum_{k=0}^{\infty} B_{jk} \sin^{j}\Phi \cos^{k}\lambda\} \tag{8}$$

with

$$D_{jk} = (-1)^j \frac{(2k-1)!!}{(2k)!!} C_{jk}$$

where C_{jk} are binomial coefficients.

Comparison of Ferraz-Mello and Lala-Sehnal approximation of shadow function is given by Fig. 2. for upper summation limit of 20.

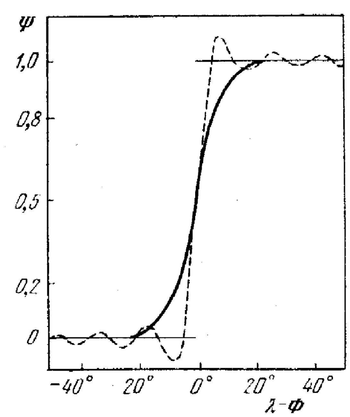

Figure 2: Comparison of Ferraz-Mello and Lala-Sehnal approximation.

S.N. Vashkoviak has expanded shadow function into series of Legendre polynomials

$$\Psi = \sum_{k=0}^{\infty} c_k P_k(\cos\lambda). \qquad (9)$$

By using $P_k(-1) = (-1)^k$ and formula

$$c_k = \frac{2k+1}{2} \int_0^{\pi} \Psi P_k(\cos\lambda) \sin\lambda d\lambda$$

for calculation of the coefficients, we have

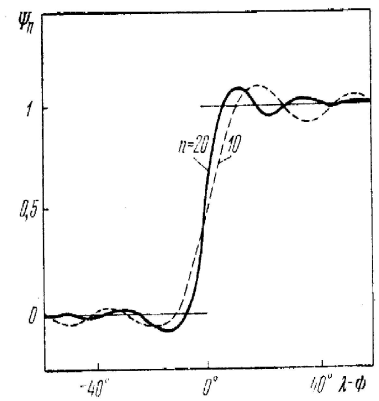

Figure 3: Vashkoviak's shadow function for upper limits of 10 and 20.

$$\Psi = \frac{1}{2}(1+\cos\Phi) + \frac{1}{2}\sum_{k=1}^{\infty} P_k(\cos\lambda) P_{k+1}(\cos\Phi)$$
$$-\frac{1}{2}\sum_{k=1}^{\infty} P_k(\cos\lambda) P_{k-1}(\cos\Phi). \qquad (10)$$

For upper limits of 10 and 20 Vashkoviak's shadow function is shown on Fig. 3.

An example for comparison is given on Fig. 4. That is approximation of shadow function by Fourier's series for satellite at the orbit around the Earth (R = 6378.14 km), at 1000 km altitude. The number of terms is 20. The comparable case is approximation of the shadow function with some amplitude multiplicators (Fig. 5)

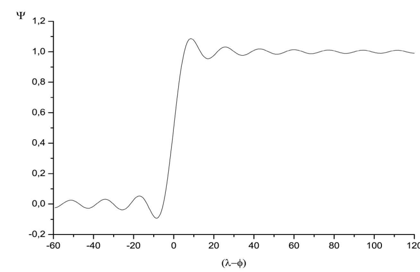

Figure 4: Fourier's series with 20 terms.

Let the first expression for AM be

$$\sigma_{17}\sigma_{18}\sigma_{19} = \frac{\sin\frac{35\pi}{T}\sin\frac{37\pi}{T}\sin\frac{39\pi}{T}}{\sin\frac{\pi}{T}\sin\frac{\pi}{T}\sin\frac{\pi}{T}}$$

After selection some of the above amplitude multiplicator and their combination with intermediary rotation (if that necessary) we have Fig. 6, Fig. 7, Fig. 8 and Fig. 9.

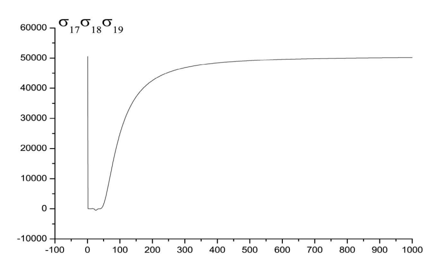

Figure 5: Amplitude multiplicators $\sigma_{17}\sigma_{18}\sigma_{19}$.

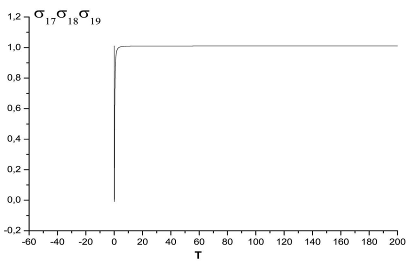

Figure 6: AM $\sigma_{17}\sigma_{18}\sigma_{19}$ after correcting x and y parameters.

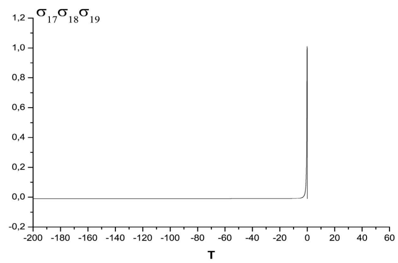

Figure 7: AM $\sigma_{17}\sigma_{18}\sigma_{19}$ after rotating.

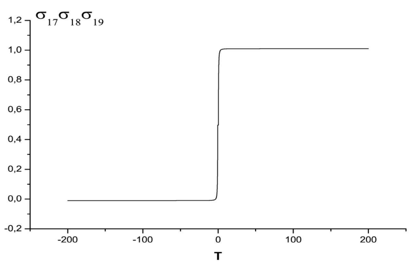

Figure 8: AM $\sigma_{17}\sigma_{18}\sigma_{19}$ combination of last two.

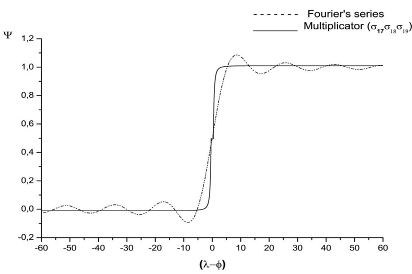

Figure 9: This is comparison Fig. 4 and Fig. 5; AM is $\sigma_{17}\sigma_{18}\sigma_{19}$.

4. CONCLUSION

We can conclude that the Amplitude multiplicators are very useful and effective method for approximation of the singular and discrete functions.

Simple structure of the Amplitude multiplicators are more convenient than Ferraz-Mello, Lala-Sehnal and Vashkoviak's approximations. There further transformations and exploitations is more simple then previous approximations.

It is possible to apply Amplitude multiplicators to many discrete structure in astronomy (like local systems, galaxies, interstellar medium, etc.)

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