Publ. Astron. Obs. Belgrade No. 80 (2006), 53 - 56 *Contributed paper*

POSSIBLE EFFECTS OF SUBPHOTOSPHERIC FLOWS ON HELIO-SEISMIC g-MODES

VLADIMIR M. ČADEŽ

Astronomical Observatory, Volgina 7, 11160 Belgrade 74, Serbia and Montenegro
E–mail: vcadez@aob.bg.ac.yu

Abstract. Non-uniform subphotospheric horizontal flows may arise from differential rotation of the sun's interior as well as from a global meridional plasma circulation. The fluid dynamics equations for a model of solar interior with the convective zone reveal a second order singularity at locations where a non-uniform flow and eigen-modes of solar global oscillations are in resonance. This indicates a possibility for a local resonant amplification of internal g-modes which makes them easier for observational detection.

1. INTRODUCTION

It is well known that various types of bulk oscillations exist and are being observed on the Sun. These the so called helioseismic modes are equivalent to seismic modes of the Earth and they are known to be of three characteristic typec of eigen modes: A set of pressure driven acoustic p-modes, a single incompressible surface f-mode and a set of gravity driven g-modes. The p-modes are being observed on the Sun and some of their frequencies are precisely determined with relative errors of only 10^{-5}.

All these modes are spatially localized and have perturbation amplitude distributions with one or more peaks located in the solar interior. An thorough overview on helioseismology and its application in stellar diagnostics can be found in Gough and Toomre (1991); Christensen-Dalsgaard et al. (1998) and Christensen-Dalsgaard (1998).

2. MODEL AND EQUATIONS

In this work, we focus our attention on patterns of spatially localized horizontal flows and their influence on stellar global seismic modes. The macroscopic flow velocity $\vec{v}$ is taken horizontal with the speed $U_0(z)$ and all other physical parameters of the basic state varying in the vertical direction z only. The applicability of the Cartesian geometry in our analysis, it is valid only for perturbations of a sufficiently small wavelength λ ($R_* \gg \lambda$) when the effects of stellar sphericity are unimportant.

The medium is an ideal plasma initially in a stationary equilibrium:

$$\nabla p_0 + \rho_0(\vec{v}_0 \cdot \nabla)\vec{v}_0 = \rho_0 \vec{g} \tag{1}$$

with the $z-$axis oriented toward the solar interior. The basic state temperature and density profiles, $T_0(z)$ and $\rho_0(z)$ respectively, are assumed known and prescribed according to the model applied. As $\rho_0(\vec{v}_0 \cdot \nabla)\vec{v}_0 = 0$ for horizontal $z-$dependent motions, the flow patterns we consider do not affect the initial hydrostatic balance for any flow profile $U_0(z)$ initially specified.

A suitable model function describing such a flow pattern can be taken in the following form (Čadež and Vanlommel, 2005):

$$U_0(z) = \frac{cU_0\rho_c}{\rho_0(z)} \left[\cos\frac{n\pi z}{W}\right]^{2p+1} . \tag{2}$$

The parameters $p = 1, 2, ...$ and $c = 0, 1, 2, ...$ determine respectively: the flow concentration at horizontal cell boundaries, and scaling of some reference flow speed U_0 at $z = 0$.

The described basic state is subject to isentropic linear harmonic perturbations having frequency ω and the horizontal wavenumber k which can also be expressed in terms of the degree l by $k^2 = l(l+1)/R_*^2$ ($R_*^2 \equiv R_\odot = 696$ Mm - the Solar radius). To remain within the applicability of the Cartesian geometry, the wavelength $\lambda \equiv 2\pi/k$ must be sufficiently small as to satisfy the condition $R_\odot \gg \lambda$ which is equivalent to $l \gg 2\pi \approx 6$. In this case, the degree l and the horizontal wavenumber k are mutually related as $l \approx kR_\odot$. These perturbations are governed by standard equations of fluid dynamics which reduce to two linear equations for the vertical displacement ξ_z and the perturbed pressure P (Čadež and Vanlommel, 2005):

$$D\frac{d\xi_z}{dz} = C_1\xi_z - C_2 P, \qquad D\frac{dP}{dz} = C_3\xi_z - C_1 P. \tag{3}$$

The coefficients D, C_1, C_2 and C_3 are given by

$$\begin{aligned} &D(z) = \rho_0(z)v_s^2(z)\Omega^2(z), \quad C_1(z) = -\rho_0(z)g\Omega^2(z), \\ &C_2(z) = \Omega^2(z) - \omega_s^2(z), \quad C_3(z) = \rho_0^2(z)\Omega^2(z)v_s^2(z)\left[\Omega^2(z) - \omega_{BV}^2(z)\right]. \end{aligned} \tag{4}$$

where Ω, ω_s and ω_{BV} are the Doppler shifted wave frequency ω, the sound frequency and the Brunt-Väisälä frequency respectively, defined as:

$$\Omega \equiv \omega - kU_0(z), \quad \omega_s^2 \equiv v_s^2 k^2, \quad \omega_{BV}^2(z \equiv g\left[(\gamma-1)\frac{g}{v_s^2(z)} - \frac{d}{dz}\ln T_0(z)\right]. \tag{5}$$

The system is locally stable against the convective instability if $\omega_{BV}^2(z) > 0$. In the basic state we consider, this is assumed true in the solar corona and solar interior while the unstable convection zone tends to establish approximately the adiabatic temperature gradient yielding $\omega_{BV}^2(z) = 0$.

3. RESULTS AND CONCLUSIONS

We see that Eqs (3)-(4) have a second order Doppler-type singularity at $z = z_r$ where $D(z_r) = 0$. At this location, the phase matching, or a resonant interaction between the global mode with frequency ω and the flow, takes place: $\omega = kU_0(z_r)$. Consequently, an instability develops causing the modal amplitude to increase in time by gaining energy from the flow.

As the linear amplitudes grow, the effects of nonlinearities and plasma dissipations become important and cannot be ignored anymore which eventually leads to a saturation of growth. The linear approach therefore only points out the fact that a resonance exists where the linear solutions diverge. A full analysis of eigensolutions in the vicinity of a resonance requires a nonlinear approach and taking dissipations into account. In our case, the divergent linear solutions can easily be obtained in an asymptotic form in the domain $z \approx z_r$ where Eqs (3)-(4) reduce to a single equation:

$$\frac{d}{dz}\left[(z-z_r)^2\frac{d\xi_z}{dz}\right] + \frac{\omega_{BV}^2(z_r)}{(U_0')^2}\xi_z = 0, \tag{6}$$

with $U_0' \equiv dU_0(z)/dz|_{z=z_r} \neq 0$ and $\omega_{BV}^2(z_r) \geq 0$ in the considered basic state.

Eq (6) has solutions in form of powers $\xi_z = \mathit{const} \times |z - z_r|^\alpha$ which, after substitution into Eq (6), yields the following characteristic equation for α: $\alpha^2 + \alpha + \omega_{BV}^2(z_r)\,(U_0')^{-2} = 0$ whose two solutions $\alpha_{1,2}$ are:

$$\alpha_{1,2} = -\frac{1}{2} \pm \mu; \qquad \mu = \frac{\left[(U_0')^2 - 4\omega_{BV}^2(z_r)\right]^{1/2}}{2|U_0'|}.$$

Depending on whether μ is real or not, i.e. depending on the sign of $|U_0'| - 2\omega_{BV}(z_r)$, there exist two distinct types of solution to Eq (6):

If μ is real, i.e. if $|U_0'| \geq 2\omega_{BV}(z_r)$, the general solution is

$$\xi_z = a_1|z-z_r|^{(\mu-1/2)} + \frac{a_2}{|z-z_r|^{(\mu+1/2)}} \tag{7}$$

where a_1 and a_2 are the integration constants.

If μ is imaginary: $\mu = i|\mu|$, i.e. if $|U_0'| \leq 2\omega_{BV}(z_r)$, the particular solution of Eq (7) takes the following form

$$\xi_z \sim |z-z_r|^\alpha = |z-z_r|^{(-1/2\pm i|\mu|)} = |z-z_r|^{(-1/2)}\mathrm{e}^{(\pm \mathrm{i}|\mu|\ln|\mathrm{z}-\mathrm{z_r}|)}$$

and the general solution to Eq (7) becomes

$$\xi_z = a_1\frac{\cos(|\mu|\ln|z-z_r|)}{|z-z_r|^{1/2}} + a_2\frac{\sin(|\mu|\ln|z-z_r|)}{|z-z_r|^{1/2}}. \tag{8}$$

In either case, the solution to the linear vertical displacement ξ_z diverges at $z = z_r$ if dissipations and nonlinearities are ignored. In this paper, however, we are not interested in exact numerical values for computed perturbation amplitudes in the resonance domain. It is only to draw attention to the existence of the Doppler resonance which can act as a localized amplitude amplification mechanism for solar global modes. For a model of solar interior discussed in Čadež and Vanlommel (2005), we show that the this resonance can occur only for global modes whose wavelengths do not exceed 1% of the Solar radius $R_{\odot}$. Consequently, such a resonant wave amplification can affect also the low frequency $g-$modes and thus lessen the difficulties of their observational detection at the solar surface.

Acknowledgements. This work has been carried out at the Astronomical Observatory in Belgrade within the project No. 1951 "The Solar Spectral Irradiance Variation" supported by the Ministry of Science and Environmental Protection of the Republic of Serbia.

References

Christensen-Dalsgaard, J., Däppen, W., Dziembowski, W.A. and Guzik, J.A.: 1998, Variable Stars as Essential Astrophysical Tools. Proc. NATO Advanced Study Institute, ed. C. Ibanogher, Kluwer Acad. Publ., 59.

Christensen-Dalsgaard, J.: 1998, Lecture Notes on Stellar Oscillations, Fourth Edition, Institute of Physics and Astronomy, University of Arhus, Arhus, Denmark.

Čadež, V.M. and Vanlommel, P.: 2005, *Serb. Astron. J.*, **171**, in press.

Gough, D. O. and Toomre, J.: 1991, *Annu. Rev. Astron. Astrophys.*, **29**, 627.