STABILITY OF MOTION IN EXTRASOLAR PLANETARY SYSTEMS

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Abstract. The discovery of extrasolar planets was and is a big challenge for astronomers because of the very different structure of these systems compared to our Solar System. In some of the extrasolar planetary systems (EPS) we can observe Jupiter-like planets very close to the central star – even inside Mercury's orbit around the Sun. Many of them – up to now 147 – host planets on high eccentric orbits and have masses up to several masses of Jupiter. In this lecture we concentrate on the dynamical state of extrasolar planetary systems and report how planets in multiplanetary systems may stay on stable orbits although their orbits cross. The major part of this review is devoted to the results of investigations with regard to the possibility of EPSs to host terrestrial like planets (TP) in the habitable zone (HZ) around a star. For these – still fictitious – planets one can distinguish three principal types: 1^{st} when the giant planet (GP) moves close to the star – then a TP may move on stable orbits inside the HZ; 2^{nd} when the GP moves far from the star, then we have the same situation like in our Solar System; 3^{rd} when the GP itself moves inside the HZ, then a terrestrial satellite or a trojan planet may exist. Future space missions will show whether we may observe such interesting systems with terrestrial planets.

1. INTRODUCTION

More than ten years ago the first planet around another star – in fact around a pulsar (Mayor and Queloz, 1995) – was observed. Ever since we have more and more new discoveries of planets in extrasolar planetary systems.

Basic investigations concerning extrasolar systems are to study their dynamical stability when several planets are involved. The literature hereto is quite large and different groups are working on this subject using different methods. Today the stability of multiple planetary systems (e.g. Kiseleva-Eggleton et al., 2002; Beaugé and Michtchenko, 2003) and of planets in double stars (e.g. Pilat-Lohinger and Dvorak, 2002; Dvorak et al., 2003a; Ferraz-Mello et al., 2005) seems to be quite well established. One of the most interesting question for astronomers is the dynamical stability of additional – fictitious – planets inside the habitable zone¹ and the EPSs were already investigated in this respect (e.g. Gehman et al., 1996; (Noble et al., 2002;

 $^{^{1}}$ The zone in which liquid water is possible on the surface of a terrestrial planet. For a detailed discussion we refer to Kasting et al. (1993)



Dvorak et al., 2004; Asghari et al., 2004; Menou and Tabachnik, 2003; Goździewski, 2003).

Figure 1: Eccentricities of extrasolar planets (courtesy of Jean Schneider (http://vo.obspm.fr/exoplanetes/encyclo/catalog.php).



Figure 2: Semimajor axes of extrasolar planets (courtesy of Jean Schneider (http://vo.obspm.fr/exoplanetes/encyclo/catalog.php).

The question of terrestrial planets (=TP) in EPSs has been a hot topic since the first discovery of planets outside our Solar System. One of the first studies hereto was by Jones et al. (2001), where they investigated the stability of terrestrial planets in 4 known systems (according to them "Rho Vrb" and "47 Uma" could host additional TP, "Gliese 876" and "Ups And" were found to be unlikely to have TP in HZ) with the aid of long term numerical integrations. Turnbull and Tarter (2003) applied Hill's definition of stability and compiled a catalogue of habitable systems. An extensive investigation was undertaken by Menou and Tabachnik (2003) where the authors

studied the stability of orbits of TPs in all EPSs again via extensive via numerical integrations. Their results have shown that half of the known systems possibly host TP in HZs. Recently Jones et al. (2005) examined whether hypothetical TPs could stay long enough in the HZ to be able to develop a biosphere. Using the results of a detailed study of 7 of these systems as models 50 percents out of 111 systems investigated by them were found to possibly host additional TPs. There also exist detailed studies of specific EPSs e.g. 47 Ursae Majoris (Ji et al., 2005). Other work on individual systems has been performed e.g. by Erdi et al. (2004), Pál and Sándor (2002), Dvorak et al. (2003a), Dvorak et al. (2003b) and Asghari et al. (2004). Investigations of planets in binaries were made long before the first exoplanet was discovered (e.g. Rabl and Dvorak, 1988). Other, more recent theoretical studies of double star systems covered both models: S-type (e.g. Holman and Wiegert, 1999; Pilat-Lohinger and Dvorak, 2002) and P-type orbits (where S-type planets move around one component of the double stars while P-type planets move around both primaries). Additional investigations for the P-type orbits on inclined orbits in mode4l systems were undertaken by Pilat-Lohinger et al. (2003). Also many real exosolar planetary systems, like γ Cephei, HD 12661, HD 38529, HD 37124 and HD 160691 (Érdi et al., 2004) and HD 74156 (Dvorak et al., 2003b) were investigated.

2. BASICS OF THE KNOWN EXTRA SOLAR PLANETARY SYSTEMS

There are different detection methods for planets around other stars. The mostly used ones are: radial velocity curves (due to the presence of a massive planet the central star sometimes move towards the Earth and sometimes away which causes a shift of the spectral lines) and transit methods (like a Venus transit in front of the disk of the Sun); this latter asks for a very large inclination of the observed system with respect to the line of observations.

According to the catalogue by Jean Schneider² by now (January 2006) in 147 planetary systems we observe 170 planets; 17 of them are multiple planetary systems which means that they host even two or more planets. The respective histograms in Fig. 1 and Fig. 2 show the basic differences between our Solar System and the EPSs we observe up to now in our Solar neighborhood (up to 30 pc only!). In Fig. 1, the histogram of the eccentricities, one can see that more than half of the planets have eccentricities larger than 0.25 and 10 percents larger than $e > 0.5^3$! The distribution of semimajor- axes of all exoplanets, shows that they are located from 0.038 AU to 5.257 AU (Fig. 2). Within these limits to the host star there is also the region of the HZ of a typical main sequence star (for spectraltypes from K to F). These HZs range typically from ~ 0.5 AU to ~ 1.2 AU for K-stars and from ~ 1.4 AU to ~ 2.8 AU for F-stars.

²http://vo.obspm.fr/exoplanetes/encyclo/catalog.php

 $^{^3{\}rm which}$ means that the ratio of the apoastron to to the periastron distance is 3



Figure 3: Schema of the habitable zone (courtesy of Kasting (1991).

3. THE HABITABLE ZONE

The HZ is defined as the range of the distance from a planet to the central star where liquid water can exist on the surface of a TP (Kasting, 1991). The inner and outer border of this region is primarely defined by the temperature on the surface of a – fictitious – terrestrial planet, and depends on the characteristics of the system: the spectraltype, the mass and the age of the central star and the orbit of the TP, its mass and its atmosphere (Fig. 3). All this taken into account limits the possible HZ for many stars to a small ring; in our SS it is only the Earth which moves constantly in the HZ, Venus is inside and Mars is outside. Because of the aging of the Sun the HZ of our SS will in future (some 10^9 years) be pushed to Mars and even further out. In Fig. 3 this is well visible for the present configuration of our Solar System, and it also shows how the HZ depends on the spectral type of a star.

4. THE IMPORTANCE OF RESONANCES

From our Solar System we know that the mean motion resonances (=MMR) are quite important for the orbits of the planets but especially for the minor bodies. The inner planets do not show low order resonant motion with respect to MMR⁴, but the outer planets do: we have close MMR between Jupiter and Saturn (5:2), Saturn and Uranus (3:1), Uranus and Neptune (2:1) and Neptune and Pluto (3:2); the numbers should be read e.g. $5.n_{Saturn} - 2.n_{Jupiter} \sim 0$. (n is the mean motion of the planet). None of these couples suffer from very large perturbations (for Jupiter-Saturn the amplitude in mean longitude is about 1°). Recent studies have shown

 $^{^4\}mathrm{there}$ is an important 8:13 MMR between the Earth and Venus which acts in a protecting way for their orbits



Figure 4: The restricted three body problem in a rotating coordinate systems with the two primaries on fixed position at the x-axis. The two stable equilibrium points L_4 and L_5 form triangles with the primaries.

that the Jupiter-Saturn couple is close to a chaotic region (Michtchenko and Ferraz-Mello, 2001). Neptune and Pluto are in an exact, but 'protected' MMR: due to the large eccentricity of Pluto's orbit its perihelion position is even smaller than the one of Neptune and consequently their orbits cross each other. They have stable orbits because of different initial phases. This effect also plays an important role when a multiplanetary system has crossing orbits in a MMR (e.g. Ferraz-Mello et al., 2005; Psychoyos and Hadjidemetriou, 2005). It is also interesting to note that since about 15 years we have had evidence for a whole group of asteroids in the 3:2 resonance with Neptune namely the Plutinos. They are part of the Kuiperbelt of bodies, which move in the outer planetary system, and have also a dynamical structure which is primarily determined by resonances.

Of special importance in Solar System Dynamics is the 1:1 MMR. Taking into account only 2 primary masses (the Sun and Jupiter) – which is quite a good approximation for the dynamics of small bodies like comets and asteroids – the 1:1 MMR is of special importance⁵. 60 degrees ahead and 60 degrees behind Jupiter there exist two stable equilibrium points which are named after the mathematician and astronomer Joseph-Louis Lagrange (1736-1813) and they are populated with the Trojan asteroids (compare Fig. 4). Today we know of about 1000 around L_4 (the preceding equilibrium point) and about 600 around L_5^6 . The 1:1 MMR seems to be important because it opens another possibility for TP in ESPs (see later chapter).

 $^{^5\}mathrm{This}$ dynamical model is the restricted three body problem where the primaries move on exact circles

 $^{^{6}\}mathrm{This}$ asymetry in the number of Trojans is still an unsolved riddle for astronomers



Figure 5: Possible orbits of the two planets in HD82943 in the 2:1 MMR where the periastron positions are aligned (left picture) or anti-aligned (right picture).



Figure 6: Schematic figure for the different possibilities where we may find additional, habitable planets in existing extrasolar systems: 1) C1: the GP moves inside the HZ; 2) C2 the GP moves outside the HZ; 3) C3 habitable moons 4) C4 habitable trojan planets.

5. MULTIPLE PLANETARY SYSTEMS

It is quite evident that the determination of the orbital parameters is quite a difficult task and when only observations from one technique are available they have relative large errors especially in the eccentricities. For single planetary systems this is not a problem, but when more than one planet has been observed this leads sometime to very surprising orbits with quite large eccentricities. In Table 1 one can see that the determined orbits may even cross because of the large eccentricities; we need an adequate explanation why these systems are still stable. A number of articles is devoted to this difficult question and the answer, which was found recently, is given in Fig. 5 for HD 82943, a system where the two planets are in 2:1 MMR. When the 2 periastron positions are either aligned or antialigned the configuration is such –according to the initial phase – that the two planets never suffer from a close encounter.



Figure 7: Results for Gl 777A: initial semimajor axes of a fictitious TP in the HZ versus different eccentricities of the large planet. Only the initial conditions 0.5 < a < 0.8 for the whole range of the eccentricities of the GG allow stable orbits with small enough eccentricities (e < 0.3) of the terrestrial planet; the maximum eccentricity is shown in different grey tones.

6. THE TERRESTRIAL PLANETS?

In Fig. 6 we see a schematic view of possibile configurations of terrestrial planets to move in the HZ. Five different classes of such orbits in the presence of a Jupiter like GP like Jupiter can be distinguished:

- 1. C1: the GP is very close to the star there could exist stable orbits outside for time scales long enough to develop a biosphere
- 2. C2: the GP moves far away from the central star (like Jupiter) then stable orbits for additional planets can exist inside
- 3. C3: the GP itself moves in the habitable region a terrestrial satellite (like e.g. Titan in the system of Saturn) could be in a stable orbit
- 4. C4: when the GP itself moves in the habitable region, a Trojan like terrestrial planet may move on a stable orbit around the Lagrangean equilibrium points L_4 or L_5

Name	M_{star}	Spec.Type	a_{Pl}	e_{Pl}	M_{Pl}	Mean motion
	$[M_{Sun}]$		[AU]		$[M_{Jup}]$	Resonance
v And	1.3	F8 V	-	-	-	
b	-	-	0.059	0.012	0.69	
с	-	-	0.829	0.28	1.89	
d	-	-	2.53	0.27	3.75	
55 Cnc	0.95	G8 V	-	-	-	
b	-	-	0.115	0.02	0.84	3:1
с	-	-	0.24	0.34	0.21	
d	-	-	5.9	0.15	4.05	
HD 74156	1.27	G0	-	-	-	
b	-	-	0.294	0.636	1.86	
с	-	-	3.4	0.583	6.17	
HD 12661	1.07	G6 V	-	-	-	
b	-	-	0.82	0.33	2.3	
с	-	-	2.6	0.2	1.5	
HD 82943	1.15	G0	-	-	-	
b	-	-	0.746	0.359	2.01	2:1
с	-	-	1.19	0.219	1.75	

Table 1: Selected multiplanetary systems: note that the first two systems have three large planets with a secondary star almost 10^3 AU away; the last column indicates the mean motion resonance of two planets.

5. C5: the GP itself moves in the habitable region a TP may be on a stable exchange orbit (see below) with the large planet.

It is evident that the C1 group needs no further dynamical investigation if we want to establish zones of stable motion for a TP; the very close large planet – known as hot Jupiter – on an almost circular orbit allows motion of an additional planet in the HZ. For the C3 group we have no special results; for this case the limit of a terrestrial satellite orbiting a giant planet in the HZ can be estimated even analytically via the Hill's radius.

The C2 group has to be investigated carefully because sometimes the GPs have large eccentricities which do not allow stable orbits in the HZ. For this case many investigations have been undertaken which we cited already in the introduction. We show in the appendix for 5 selected systems a schematic view; all these cases were investigated with the aid of long term numerical integrations of fictitious TPs in the HZs. As one example we show the respective results for HD Gl 777A (Fig. 7), where we plotted the maximum eccentricity of fictious planets in different distances from the host star, where we also took into account the uncertainties in the determination of the eccentricity of the large planet.

The other groups of possible TPs (C4 and C5) will be discussed separately.



Figure 8: Example of different orbits of trojan asteroids in the restricted three body problem in a rotating coordinate system with the two primaries on fixed position at the x-axis.



Figure 9: Stable region around L_4 for HD 23079. For a fine grid of initial conditions in the angular distance to the Lagrange point versus the semimajor axis we computed the eccentricities of the fictitiuos TP. z-axis is its maximum value during the integration time; three different eccentricities of the large planet were taken as initial model: e=0.05, e=0.1 and e=0.15.



Figure 10: Stable regions for exchange orbits for 4 examples of two equally massive TPs: Earth, 5 Earth masses, Uranus and Saturn; the initial semimajor axis of the two planets is plotted versus the maximum eccentricity (logscale in y).



Figure 11: Exchange orbits for Earth and Saturn for 300 years in a distance of 1 AU; every 40 years the close encounter leads to an exchange of the orbits. The semimajor axes (y-axis) is plotted with respect to the time; the thick line close to a=1 AU is a_{Saturn} .

6. 1. TROJAN ORBITS

The Trojan asteroids have orbits which are always in the vicinity of the two equilibrium points L_4 or L_5 . According to their initial conditions they librate around the circle connecting the two primaries Sun and Jupiter with different libration angles (see Fig. 8).

There exist a certain stability region around these points which can be determined via analytical methods for a simplified model (Jupiter has a circular orbit). In more realistic cases we need to establish the extension of these regions with the aid of extensive numerical intergrations. This task has been undertaken for the EPSs where the GP itself moves in the HZ. The respective results for the planetary system around HD 23079 (the GP has of more than 2 M_{jup} and the orbital period of approximately 2 years) are shown in Fig. 9. There we varied the eccentricity of the large planet according to the error bars given by the observers. We can see that the largeness of the zone diminishes with the eccentricity of the GP, but it still lets a large region for a TP to move there. The results were derived in the dynamical model of the former mentioned restricted three body problem (the TP without a mass) but test computations have shown that even large masses of the TP up to the mass of Uranus do not significatly change the size of the region.

As continuation of the libration orbits there exist orbits which surround both equilateral Lagrange points the horseshoe-orbits (see Fig. 8, right graph). In the full three body problem this configuration can lead to exchange orbits.

6. 2. EXCHANGE ORBITS

The exchange orbits of the general three body problem can be described as follows: two small but massive bodies are orbiting with nearly circular orbits a much more massive host with almost the same semimajor axes. Because of the 3^{rd} Keplerian law the one with the inner orbit is faster and approaches the outer body. Before they meet, the inner body is shifted to the orbit of the outer and vice-versa the former outer body moves to an orbit with a smaller semimajor axis: they have changed their orbits! This interesting interplay is stable for a very long time (e.g. Spirig and Waldvogel, 1985; Auner, 2001). In the satellite system of Saturn the two moons Janus and Epimetheus (the two circular orbits of these two moons differ only by 50 km (151472km and 151422 km and have diameter of more than 100 km) have exactly these kinds of orbits, so this may apply to EPSs too. Recent numerical integrations and analytical estimations show that these kind of orbits are stable up to a mass ratio where a TP is in exchange with a Saturn like planet.

In Fig. 10 we show the limits of stability for exchange orbits depending on their masses. It can be seen that for large masses involved the difference in semimajor axis can be larger: the respective results for two equally massive planets in exchange orbits with a mean semimajor axis a=1 AU are for 4 different masses (earth, 5 time the earth, uranus and saturn) $\delta a = 0.012, 0.020, 0.012$ and 0.034AU.

In Fig. 11 we depicted the change of the semimajor axes after every close encounter: the inner orbit is shifted outwards and the outer orbit is shifted inwards. The example shown in this figure is for Earth on exchange orbit with Saturn. It is also visible that the Earth makes bigger jumps than saturn, a consequence of the smaller mass of the Earth.

7. CONCLUSIONS

We can say up to now that we have a good knowledge of the dynamics of the EPS when more than one planet is orbiting the star. Even quite eccentric orbits with close semimajor axis are stable when the are in MMR resonances (due to the so-called alignment or antialignment of the pericenters of the two planets involved.) Many studies are now devoted to the interesting question of how terrestrial planets can be hosted by stars in habitable zones. We showed that besides the situation of a 'hot Jupiter' in a system (very close to the star) and the Solar System configuration with a Jupiter far outside the HZ there could survive TP in 1:1 resonance with a large planet: either as a large satellite, a Trojan planet or even as a TP in an exchange orbit with a large planet like Saturn.

Finally we emphasize that all results agree more or less that approximately half of the extrasolar planetary systems known up to now may harbor terrestrial like planets in stable orbits in regions favorable for a possible development of a biosphere. We should be aware that we have knowledge only of the close neighborhood of the Sun up to 30 pcs and new exciting discoveries will be made in the next decades concerning planets around other stars.



8. APPENDIX: ORBITAL STRUCTURE OF FIVE SELECTED EPSs

Figure 12: Example of 5 EPS: we show the planet's semimajor axis down to the the periastron distance (dark region left to the planet) on the x=axes, the habitable region (left to the planet light grey) and the location of the mean motion resonances of a fictitious TP with the large planet. Note that the last EPS 47 Uma has two large planets.

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