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THE CONCEPT OF A HIERARCHICAL COSMOS

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Abstract. The idea of a hierarchically structured cosmos can be traced back to the Presocratic Hellada. In the fifth century BC Anaxagoras from Clazomenae developed an idea of a sort of fractal material world, by introducing the concept of *seeds* (*spermata*), or *homoeomeries* as Aristotle dubbed it later (Grujić 2001). Anaxagoras ideas have been grossly neglected during the Middle Ages, to be invoked by a number of post-Renaissance thinkers, like Leibniz, Kant, *etc*, though neither of them referred to their Greek predecessor. But the real resurrections of the hierarchical paradigm started at the beginning of the last century, with Fournier and Charlier (Grujić 2002). Second half of the 20-ieth century witnessed an intensive development of the theoretical models based on the (multi)fractal paradigm, as well as a considerable body of the observational evidence in favour of the hierarchical cosmos (Saar 1988). We overview the state of the art of the cosmological fractal concept, both within the astrophysical (Sylos Labini *et al* 1998), methodological (Ribeiro 2001) and epistemological (Ribeiro and Videira 1998) context.

1. INTRODUCTION

The principal ingredient in almost all contemporary cosmological paradigms is the concept of a uniform global distribution of the cosmic matter. At least it was the situation until the second half of the last century, when a significant deviation from this picture on large cosmic scale began to emerge. Another dominant construct within our attempts to visualize the Universe as a whole was the assumption of a unique cosmos, whatever structure it is endowed with. This concept has been questioned in the last few decades too, and assumptions of many cosmoses, the so-called multi-verse, have gained a number of supporters among present day cosmologists.

Both paradigms are of no recent descent, however, and can be traced back to as early as to the Presocratic Hellada, when a number of bold cosmological hypotheses were developed by contemporary thinkers. In fact, both assumptions on an inhomogeneous and multiplied cosmos may be joined together in a unique cosmological paradigm, that of the so-called hierarchical cosmos. This concept we owe to Anaxagoras from Clazomenae, and it survived up to the present, when it re-emerged as the fractal paradigm in the last quarter of the last century.

Here we present an overview of how this concept evolved and what are theoretical and observational evidences that might support the idea of at least partial hierarchical cosmos. A more detailed account of this development may be found in Grujić (2001, 2002).

2. PRESOCRATIC GREECE

Presocratic Hellada has witnessed a very prolific activities concerning human picture of the world as a totality of a possible reality. Almost all modern cosmological paradigms can be found within the body of the philosophical researches into the extent and nature of the conceivable Universe. In particular Ionian thinkers, from Anaximander to Anaxagoras, offered a variety of cosmic models, that contained seeds of a wide class of the present day cosmological paradigms.

The main step forward from the mythological towards a much more general and rational picture was the assumption of an infinite universe, as conceived by Abderian school (Leucippus and Democritus) and Anaxagoras from Clazomenae. The first paradigm describes an indefinitely extended cosmos, containing an infinite amount of matter, with a multitude of various cosmoses, like ours, but possibly quite different as well. Anaxagoras' concept was more subtle and intriguing from the point of view of the modern cosmology.

2. 1. ANAXAGORAS' MULTIVERSE

Though his primary motive was an explanation of the apparent diversity of material objects around us, his solution to the problem has been recognized by some modern scholars as the first realization of the hierarchically structured cosmos. As a result, Clazomenian thinker was able to construct an infinitely multiplied cosmos with a finite amount of matter.

Similarly to Abderian construct of atoms, Anaxagoras defined $\sigma\pi\epsilon\rho\mu\alpha$ (seed) as the primitive construct, out of which he was able to derive objects with distinct properties. His starting point was the premise that "Everything contains a portion of everything". Every part of a matter contains replicas, no matter how small, of the rest. According to some readings of the extant passages of his treatise $\Pi\epsilon\rho\iota\ \phi\upsilon\sigma\epsilon\omega\varsigma$ (*About Nature*), he conceived other worlds like ours or otherwise, that might exist at every level, down to smallest dimensions imaginable. This picture can be extended towards larger dimensions, cosmos, mega cosmos *etc*, forming a hierarchical multiverse, in modern parlance.

3. (POST)RENAISSANCE EUROPE

Anaxagoras' teaching concerning cosmology was mostly neglected in Medieval Europe, but a number of thinkers and scholars did try to devise an infinite universe. Newton postulated it, and Leibniz put forward the idea of *monades*, as a psycho-idealistic counterparts of the Anaxagoras' $o\mu o\iota o\mu\epsilon\rho\eta$. In his epoch-making treatise *Universal Natural History and Theory of the Heavens* Kant explicitly stated the hierarchically designed multiverse, with galaxies, clusters, *etc* as (sub)units (Kant 1968). He boldly assumed that the nebulae seen in the sky might be galactic systems like our Milky Way, that proved a correct interpretation a century and half later.

The idea of an infinite universe, though appealing to a rational mind, was burdened with paradoxes. Two of them, The Blazing Sky paradox and gravitational paradox, were formulated in the XIX century in a clear way, as Olbers' and Seeliger-Neumann's paradoxes, respectively. Both controversial points were known to Newton and his contemporaries, and with this sort of burden cosmology entered the XX century.

4. 20-IETH CENTURY COSMOLOGY

Modern cosmology began with Einstein's General Relativity, which provided mathematical tools for treating the universe as a whole in a quantitative manner. Einstein himself initiated quantitative cosmology, assuming the universe uniformly populated with matter. As is well known he tried to overcome the stability problem of a static universe by additional hypothesis (Λ cosmological constant), which spoiled the original selfconsistent approach. In 1992 Friedmann demonstrated that the stability problem is overcome by finding kinematical solution to the Einstein's equation, that described an ever expanding universe, starting from a point-like singularity in a finite past (Peebles 1993). It turned out that both Olbers' and Seeliger-Neumann's paradoxes were solved within this dynamical cosmological model.

Modern cosmological theory is heavily relying on General Relativity and Quantum Field theory, but apart from the assumed early epoch close to the cosmic singularity, Newtonian dynamics appears naturally applicable in describing the structure and evolution of the universe too. As with the relativistic approach, Newtonian cosmology started with static models, incorporating subsequently dynamical effects that contribute to the overall expansion of the observable universe. It is this scenario that the modern concept of a hierarchical cosmos began with, early in the last century.

4. 1. CHARLIER'S MODEL

Inspired by a science fiction book (Fournier d'Albe 1907), this astronomer from Lund Observatory conceived an ingenious scheme of a hierarchically arranged cosmos. Defining levels of complexity, he started with the zero-level with galaxies as elementary constituents, then at he next level a collection of spherical clusters of radius R_1, with N_1 galaxies, second level with spherical superclusters with radii R_2 and N_2 clusters within, etc. He found that if the relation $R_i/R_{i+1} > \sqrt{N_i}$ is satisfied, both Olbers' and Seeliger-Neumann's paradoxes disappear.

This remarkable result, however, was overshadowed by the newly proposed Friedmann's kinematic model, which explained the Olbers' paradox on account of the red shift and (predominantly) by the finite age of the universe, that the model implied. Further developments in 20-ieth century cosmology followed this Friedmannian paradigm and Charlier's schema fell into oblivion. But not quite.

4. 2. FRACTAL PARADIGM

About the time d'Albe published his scheme for multiverse a German mathematician Felix Hausdorff published a series of papers where a generalization of topological notions like metric, measure, dimension *etc* were proposed. In the course of time further developments led to the theory of fractals, mathematical objects that had Hausdorff dimension strictly larger than the topological one. It turned out that such curious objects abound in nature, from snowflakes to the coastlines (see, e.g. Mandelbrot 1983, Gouyet 1996). These mathematical ideas shed new light on the previous cosmological models and interest in the concept of a hierarchical universe reemerged.

Fractal systems appear characterized by two parameters: (i) fractal dimension D and the so-called *lacunarity* (see, e.g. Sylos Labini *et al* 1998). The first one is Hausdorff's dimension, whereas the second parameter determines exact geometrical structure of the system, within the class of objects that possess the same fractal dimension. The question arose as to the astronomical observational evidence that the universe is endowed with a large scale structure and the possibility that this structure is fractal (hierarchical) one.

5. FRACTAL COSMOS

5. 1. THEORETICAL AND METHODOLOGICAL CONSIDERATIONS

It is well known astronomical fact that the angular distribution of the galaxies appears uniform one. How this comply with possible three-dimensional nonuniform large-scale distribution? It turns out that if the universe is a fractal, and has $2 < D < 3$, its projection onto the celestial sphere would be uniform. Analogously to the projection of clouds, whose fractal dimension is $D \approx 2.5$, and which shed a compact shadow onto earth surface. Another important property of a fractal system is that it appears isotropic, if observed from any (occupied) point of the system. Hence, if the universe is fractal, it is inhomogeneous and isotropic. Further, an infinitely extended (in the ordinary, physical space) fractal system has zero average density. In particular, if our universe is fractal one, its mass density is zero. In fact, it is this property of a hierarchical cosmos that provided Charlier with clue for both paradoxes we mentioned above.

One particular system might be decomposed into two or more subsystems that are fractals, but with different fractal dimension D. Such systems are called *multifractals.* As we shall see immediately, if our cosmos is hierarchically structured, it is most probably multifractal.

Fractal systems possess no natural length. If a so-called homothetic (scaling) transformation is carried out, which expands (shrinks) all distances, the system remains unaffected, i.e., we have the selfsimilarity transformation. Since there is no intrinsic length, or a proper scale, the notion of global density looses sense. Starting from a (occupied) site within the system, local density diminishes as one deals with sphere of larger and larger radii, tending to zero in the limiting process. This property of fractals was overlooked by many astrophysicists who argued against the assumed fractal structuring of the large-scale cosmic matter (see, e.g. Sylos Labini *et al* 1998).

5. 2. ASTRONOMICAL EVIDENCE

That the universe is far from being homogeneous at large scales became evident as early as 1934, when Shapley published his survey that revealed clusters of galaxies even larger than Virgo cluster. In 1958 Abell published the catalogue that indicated even larger concentration of clusters and superclusters, as Abell dubbed them. Further observations confirmed this hierarchical clustering, that now forms the basis of the fractal cosmology (Peebles 1993).

De Vaucouleurs in his review in 1970 made an attempt to identify a fractal pattern of galaxy clustering. His estimate of the fractal dimension was $D \approx 1.2$. Later catalogues provided much more material for inferring a hidden structuring in the sky. Two questions became prominent in these investigations into the large scale cosmic structuring. First, how reliable are data relating to the deep space observations, and second, what would be a proper methodology in interpreting the source material. A remarkable advance concerning the latter was made by Pietronero (1987), who showed that a biased data processing leads to wrong conclusions. In particular he showed that the correlation function, the principal mathematical tool used in ordinary statistical physics, appears inappropriate when fractal systems are investigated. He proposed a more general correlation function that may reveal a possible hierarchical clustering.

Another noticeable advance in understanding fractal structures was made by Mandelbrot, who introduced the notion of lacunarity, mentioned above, which took into account the presence and role of the cosmic voids, which determine the topology of fractal system beyond the fractal dimension D (Mandelbrot 1983). As for the dynamics of clustering formation, that goes beyond the descriptive level, not much advance has been made up to now. This is, however, understandable considering the complexity of the problem, which has not be satisfactorily resolved even at the galaxy formation stage. We mention here an early attempt by Haggerty (Haggerty 1971), and more recent calculations (see, e.g. Combes 1999), based on purely Newtonian dynamics. We mention also more speculative theoretical investigations by a number of authors, like those who deal with exotic cosmic (hypothetical) objects like the cosmic strings (see, e.g. Grujić 2002, for details).

Fractal paradigm turns out to be generic one and is capable of resisting many other paradigms, like the inflationary scenario (Winitzki 2002). There has been even a hint that the space-time manifold is not exactly four-dimensional, but deviates from 4 by a small amount $\epsilon \approx 3.5 \times 10^{-3}$ (Nakamura 2000). It is interesting that an early analysis made of the observational evidence (by the orthodox methodology, see above) indicates that the fractal dimension of the ordinary (physical) space should not deviate from 3 by more than $\epsilon \approx 10^{-3}$ (Peebles 1993).

The present efforts on the part of those who adhere to the fractal, or at least hierarchical, paradigm, are directed towards answering the following questions:

(i) Can one extract from the present day catalogues a discernable pattern of at least multifractal structure. (ii) If the answer to (i) is positive, up to which distance this pattern persists, or, equivalently, what is the so-called correlation length λ_0, beyond which a uniform matter distribution prevails? (iii) What would be the role of the hypothetical dark energy regarding the hierarchical structuring? (iv) If the universe is of a finite age, could one expect that it will become fractal in an infinite future and what would be the meaning of λ_0 in this case?

The present observational evidence, according to Sylos Labini *et al* (1998), renders an overall picture, which may be summarized as follows. With a wide span of distance $(0.5 - 1000 Mpc/h)$, with Hubble constant h in units $100\, Mpc/km/s$ (with measured value $h \approx 0.65$), data from the catalogues fit well the fractal pattern. It turns out that all surveys are mutually consistent and point to the $D = 2 \pm 0.2$ fractal dimension of the observable part of the Cosmos, with a clear fractal structure within $(0.5 - 150 Mpc/h)$ region.

6. CONCLUDING REMARKS

We have traced the concept of a hierarchical cosmos back to Pre-socratics and outlined its development up to the present time. From Anaxagoras to modern cosmology the concept has been defined in various terms, but the essential feature of a selfsimilar structure - filling the cosmic space with i finite amount of matter and constituting a multitude of worlds, possibly infinite in number, has remained a recurring task. As the observational evidence has accumulated during the last century, it has become clear that there is plenty of room for alternative cosmological paradigms, along with the orthodox ones, based on the ruling cosmological principles.

From a more formal, mathematical point of view, since the concept of fractal has emerged in the second half of the last century, hierarchical structuring has attained a more fundamental ground. In particular, the advance of the topology of spacetime, has enabled us to make a clear distinction between the standard matter distributions, which are described by well conceived analytical functions and fractal, nonanalytical ones. The concept of a Copernican principle has gained a new significance within the fractal pattern. Universe may be inhomogeneous, but still it will appear the same (isotropic) to every observer.

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