

SPACE PROJECT MIMOSA

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Abstract. The aim of the project MIMOSA is to improve the knowledge of the close Earth artificial satellites' dynamics, moving at the heights of 200 to 1000 km. The usage of these kind of bodies is steadily growing, esp., for the purposes of remote sensing, telecommunications, meteorology, military goals and for the fundamental scientific research of the solid Earth and its neighbourhood. The influence of the upper atmosphere is not negligible and the effects of the non-gravitational forces, primarily of the radiation origin - direct solar radiation pressure, reflected radiation, infrared radiation are equally important when tracking such satellites and forecasting their orbital and rotational motion. To collect the relevant data the best way is to measure the respective tiny dynamical effects by accelerometers. The project MIMOSA was designed especially to meet such demands. A special small satellite will carry the only scientific instrument - a super sensitive microaccelerometer (10^{-12} g) working under the ambient temperature. The satellite is now ready for the launch which should be performed by the EUROCKOT company in April 2003. The principal scientific rationale and technical properties of the satellite and the instrument are described in the paper.

1. RATIONALE OF THE EXPERIMENT

Recent decade was marked by a growing interest in the low-Earth orbits satellites due to their very promising practical and even scientific usage. This is remarkable in telecommunications, meteorology, remote sensing and in basic science, esp., in the detailed studies of the solid Earth, modelling of its gravitational field, and of course, modelling of the upper atmosphere itself. As an example we can quote here the satellite CHAMP or the recently launched twin satellites GRACE. However, the atmosphere has a very negative effects on satellites forcing them to approach the Earth surface and finally to end their lifetimes. As an example, Fig. 1 shows the evolution of perigee and apogee heights in case of our MIMOSA experiment. The contraction of satellite orbits may be in the cases of low-Earth satellites considered as one of the most important parameters for the preliminary mission analysis of a specific satellite.

To improve the knowledge of the dynamical properties of the close Earth satellites and to comply with the growing demand on the accuracy of preliminary predictions of the orbital and rotational motions of the satellite bodies, we decided to perform an experiment which has the only aim of the measurements of the disturbing forces, which are manifested by the accelerations. We call the project (and the respective satellite) "MIMOSA", as the acronym of "**MI**cro**ME**asurements **O**f **S**atellite **A**cceleration" and

its ultimate goal is to investigate the details of the dynamics of the close, or near-Earth, or Low-Earth, artificial satellites.

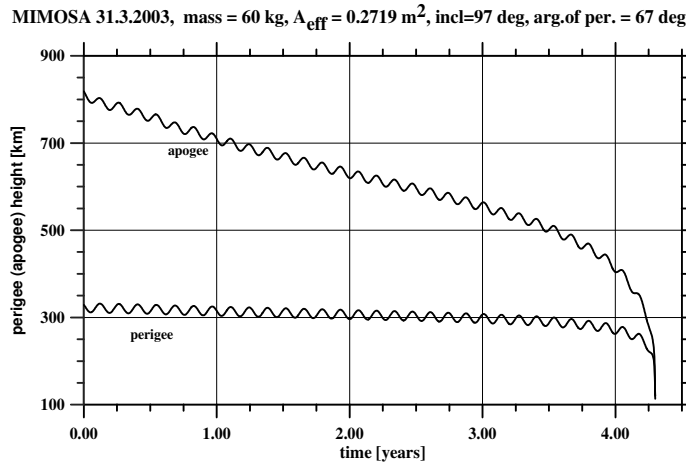


Figure 1: Lifetime of the MIMOSA satellite.

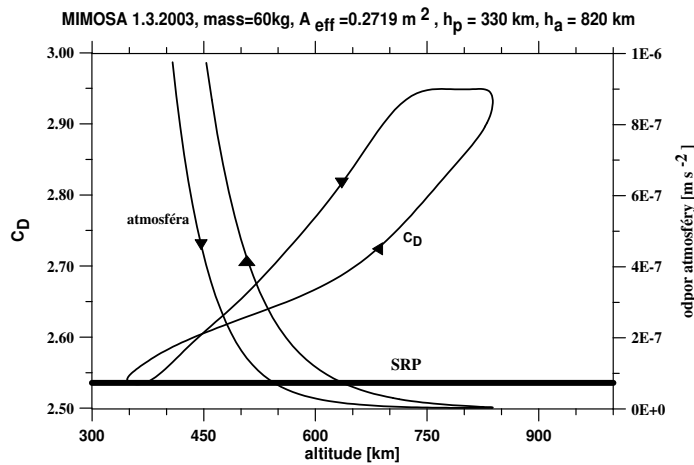


Figure 2: Changes of drag, C_D coefficient and radiation pressure during the first revolution.

2. DISTURBING FORCES

The closeness means here the fact that the influence of the upper atmosphere on the motion of these bodies is not negligible and that the effects of further non-gravitational forces are essential for their tracking and, esp., for the predictions of their orbital and rotational motion. Besides the atmosphere drag we cannot neglect here the effects

of different kinds of radiation origin - direct solar radiation pressure, the terrestrial albedo radiation, infrared radiation of the Earth etc. (Sehna 1979, Sehna 1981). Also, we have to take into consideration the proper thermal radiation of the satellite or the electromagnetic effects on the motion of an electrically charged satellite body of the Earth magnetic field (Sehna 1969). All those forces are of a dissipative character, changing with time, esp. with respect to the changing character of their force fields. The magnitudes of the accelerations raised by those forces may be theoretically estimated, however, we have to know the properties of the forces' sources. To fix our ideas we have to state precisely that the atmosphere is here considered to be between the altitudes of 100 and 1000 km.

Table 1:

Measured forces	ms⁻²
Atmospheric drag	10 ⁻⁴ to 10 ⁻⁹
Solar radiation pressure	2.9 · 10 ⁻⁸
Earth's albedo	10 ⁻⁸ to 10 ⁻⁹
Earth's infrared radiation	4 · 10 ⁻⁹
Parasitic forces	ms⁻²
Centrifugal forces	1 · 10 ⁻⁸
Irregular distribution of masses inside the satellite	7 · 10 ⁻⁹
Effects of electrically charged proof mass and cavity	4 · 10 ⁻⁸
Thermal effects	2.3 · 10 ⁻⁹
Internal molecular and magnetic forces	negligible

It is relatively easy to estimate the respective magnitudes of the accelerations if the physical parameters of the satellite are known. In Table 1, we summarize the most important ones. We see that the radiation forces are practically of the same magnitudes and the atmosphere drag decreases to similar values from appr. 700 km on. Therefore, to distinguish between the individual disturbing effects will be one of the most difficult tasks of the data analysis. We can get a help from the theory since many of the parameters are known (solar constant, albedo distribution, drag coefficient) and, also, the fact that the position of the satellite will be relatively (with respect to the desired precision) well known from the GPS measurements. E.g., drag will prevail till 500 km height whereas above 600 km the direct solar radiation pressure will be the most important disturbing force, see Fig. 2. If the satellite will move at those higher altitudes in the shadow, the Earth infrared radiation will be the most important perturbation, when satellite will move from shadow to illuminated space the Earth albedo will play a substantial role.

3. DRAG DETERMINATION

The magnitudes of the disturbing acceleration show that drag will be the most important effect, so that we shall concentrate our efforts when analyzing the measured data on the description of the distribution and variations of the atmosphere density. Drag D can be described by a relatively simple formula

$$D = \frac{1}{2} C_D \frac{A}{M} \rho V^2 \quad (1)$$

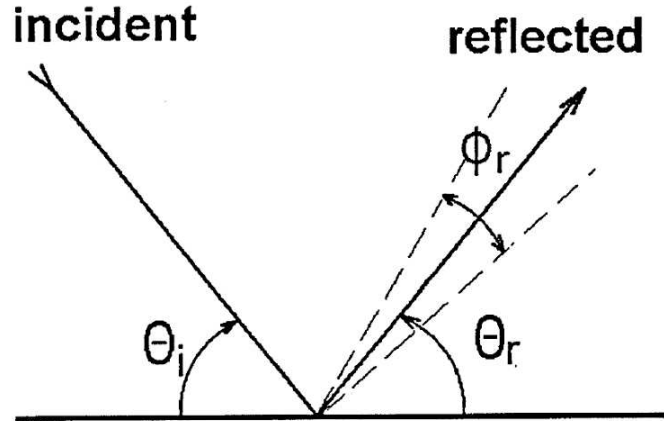


Figure 3: Incident and reflected particles vs. satellite surface.

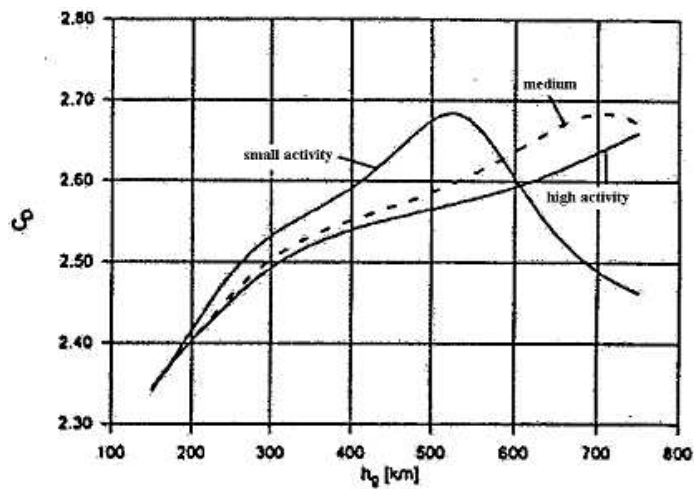


Figure 4: Changes of the drag coefficient C_D in terms of height for different values of solar activity.

where A/M is the effective satellite cross-section vs. mass. If the satellite is nearly symmetrical and its mass is constant, this ratio can be determined already in the laboratory. V is the simply determined satellite's velocity with respect to the rotating atmosphere, ρ (density) should be determined from the measurements of D and C_D is the s.c. drag coefficient which is to be computed from the theory. This number depends on the interaction of the satellite surface with the incoming molecules, which

means that its value changes with altitude in accordance with the composition of the atmospheric layers. We developed our own theory which we tested on suitably measured orbital data (Sehnal 1994). Acc. to Fig. 3, the particles incoming under the angle Θ_i toward the satellite surface are reflected into a cone with a solid angle Θ_r the axis of which deviates from the surface plane by the angle Θ_r . The coefficient C_D is then given by the relation

$$C_D = 2 [1 - (1 - \alpha)^{\frac{1}{2}} \Phi_\rho \sin(\Theta_i + \Theta_r)] \quad (2)$$

where α is the s.c. coefficient of the thermal accomodation which shows how the kinetic energy of the incoming particles accomodates to the thermal energy of the satellite's surface. Its value depends on the atomic mass of the incoming molecules and on the character of the satellite surface. Figure 4 shows how the C_D coefficient increases with the altitude in case of satellite MIMOSA. The change from 2.5 till 3.0 is certainly important for the determination of the atmosphere density. Since the satellite has a regular shape and its surface properties can be measured in the laboratory we believe that the accelerometric data will contribute to the improvement of the values of the drag coefficient C_D .

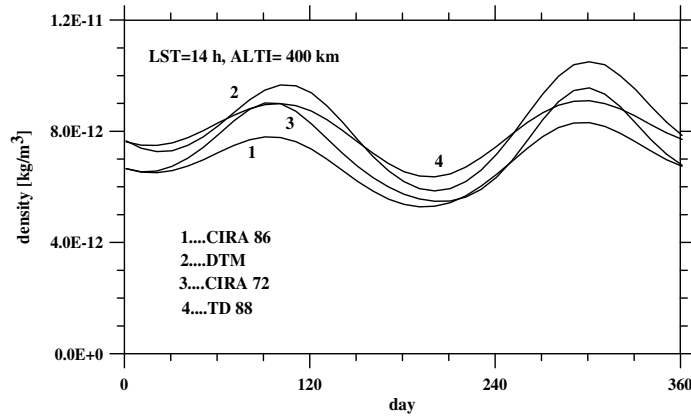


Figure 5: Annual variations of density as implied by different atmosphere models.

The satellite velocity V in Eq. (1) is taken from the orbital parameters of a satellite moving in the rotating atmosphere. From the long-term observations of the upper atmosphere rotation it is known that the rotation rate is somehow higher than that of the solid Earth between the heights of app. 250 – 500 km (King-Hele 1992). This phenomenon is due to the irregular heating of the upper atmosphere layers and is not yet accurately described, so that we have again one question to be solved by the satellite accelerometer. The accelerometer is a three-axial measuring device and the resulting acceleration is given as a vector sum of three components. Another detail of the satellite dynamics is the influence of cloudiness which is actually changing the effective values of the terrestrial albedo (Vokrouhlický and Sehnal 1993).

The tasks which are prepared for our accelerometer consist then in the collection of

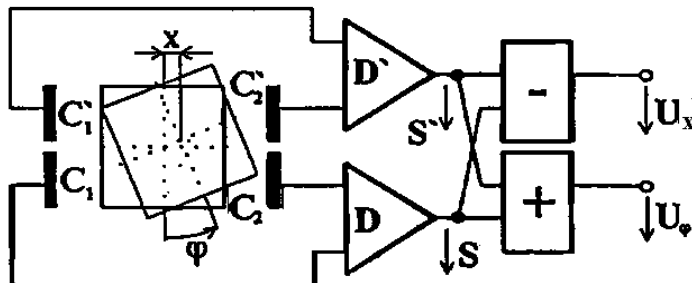


Figure 6: Principle of the accelerometer. The internal cubic proof-mass is moving freely between the electrodes C_1, C_2 and C_1', C_2' which measure the capacitance with respect to the walls of the external cubic cavity. Capacitance is changing according to translation (X) as well as rotation (ϕ) and is detected (D, D'). After mixing (S, S'), capacitance is transformed into tension U, U_ϕ which positions the proof-mass into its original location.

data for the atmosphere density, its rotation and interaction with the satellite surface. Actually, the data will serve for the improvement of our own model of the density distribution and variation. This model describes the relations which are valid for the atmosphere as a whole without decomposing it into layers of individual constituents. Consequently, this concept is of advantage for the studies of the dynamics of the objects moving in it. The model is called *TD88* (Total Density) and is now widely used (Sehnal 1988). Besides the relative simplicity it enables the analytical solution of the equations of motion which is important above all for fast computation of the drag effects without using very short integration steps (Sehnal 1990). The basic equation of the model is

$$\rho = f_x(F_{10.7}) f_m(F_{10.7}) k_o(K_p) \sum_{n=1}^7 g_n(t) [K_{n,0} + \sum_{j=1}^3 \exp(120 - h)/H_j] \quad (3)$$

where $f_x(F_{10.7})$ depends on the instantaneous value of the index of the solar activity whereas $f_m(F_{10.7})$ is its three-month average and $k_o(K_p)$ depends on the index of the geomagnetic activity. The altitude dependence is given by the expression within the square bracket and the periodical variations (diurnal, annual and latitudinal changes) are given by the terms $g_n(t)$. Basic property of the model TD88 is its linear dependence on the coefficients $K_{n,j}$ which allows the development according to those coefficients and consequently the creation of the theories of first, second and even higher orders. If we want to determine the lifetime of the satellite orbit from the evolution of the semi-major axis and the eccentricity, we have to integrate both respective equations simultaneously (Sehnal 1990). First, we determine the changes over one revolution and then extend the computation to higher order terms. This means we have to develop the right sides of the equations of motion to get finally, e.g., for the second order perturbations of the semi-major axis a the formula

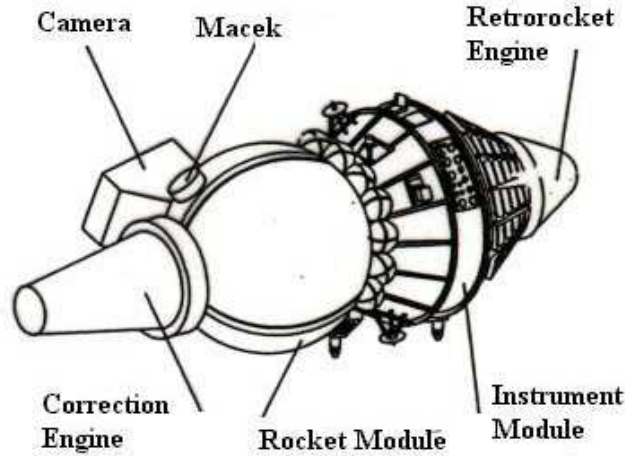


Figure 7: Position of the microaccelerometer on the RESOURCE 1F capsule.

$$a = a_0 + \sum_{n,j} [K_{n,j} \Delta a_0 N + \frac{1}{2} K_{n,j}^2 (\frac{\partial \Delta a_0}{\partial a} + \frac{\partial \Delta e_0}{\partial e}) N^2] \quad (4)$$

and similarly for the eccentricity e . N is the number of revolutions and Δa_0 and Δe_0 are the changes of the semi-major axis and of the eccentricity over one revolution with the initial elements a_0 and e_0 . When using these formulae we have to keep in mind that both elements undergo changes caused by the Earth oblateness.

The analytical determination of the individual terms of the Eq. (4) is, of course, very cumbersome and laborious. The best way is probably to use the methods of computer solution of the algebraic systems which was first successfully done by S. Šegan (1987, 1988). Actually, it is not necessary to know the analytical form of the

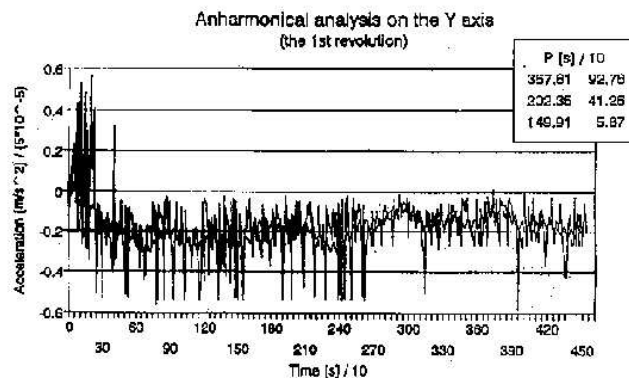


Figure 8: Data from the RESOURCE F1 experiment.

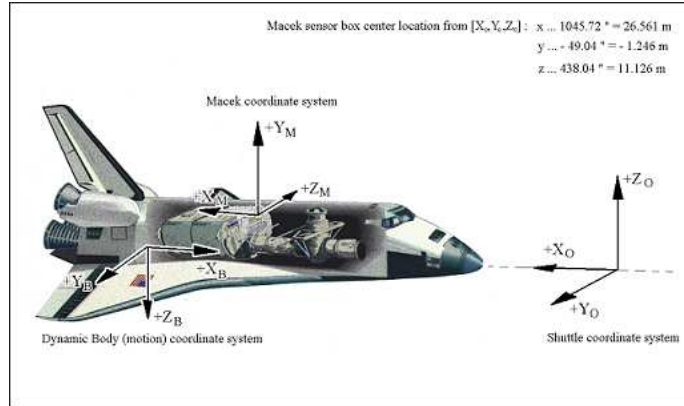


Figure 9: Emplacement of the microaccelerometer in the Space Shuttle Atlantis.

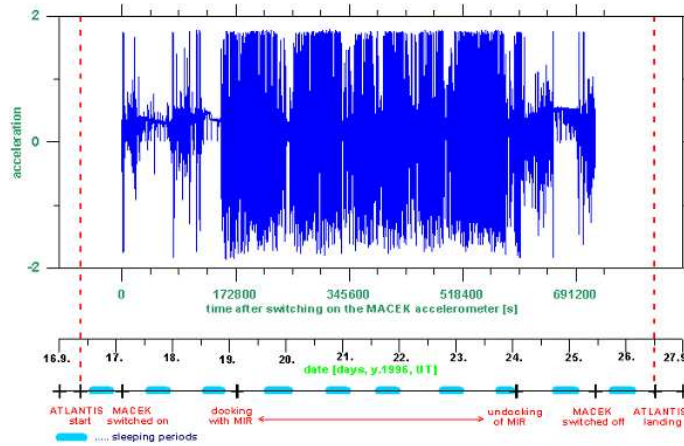


Figure 10: Data from the Space Shuttle experiment.

individual terms; we used the program MAPLE and got the results very rapidly and effectively.

The height definition of the model TD88 ranges from 150 to 700km and its comparison with the aeronomical models shows a good agreement (Fig. 5). Further improvement of the accuracy of the model depends now on the drag data collected on orbit by the accelerometer. The accelerometric measurements have against the regular density determination by the measurements of long-term orbital changes the preference not only in a higher accuracy but also in a good time and position definition which means a very substantial advantage. Moreover, since the TD88 model is given by a linear sum of coefficients the compilation of the coefficients means the same procedure as in case of the determination of the Earth gravity field. The method of least squares would be used for simultaneous treatment of big amount of data to compute several tens of data. Also, the models of albedo distribution (Sehnal 1979) and of the Earth infrared radiation field (Sehnal 1981) are given by similar pattern

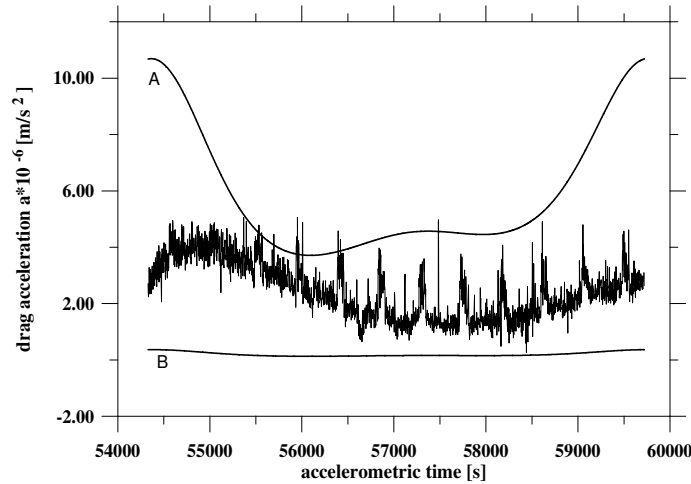


Figure 11: Data taken during one revolution of Space Shuttle.

of formulas so that the system of data treatment might be standardized. The accelerometric data with their relatively precise specification of position and time are well suited for the determination of finer properties of the disturbing forces, like the effects of cloudiness (Vokrouhlický and Sehnal 1993) on the orbital changes. Thus, all our requirements for the establishment of good models of the disturbing force fields show that the measurements by accelerometers on an properly chosen orbit will be needed.

4. ACCELEROMETER

Besides the theoretically substantiated reasons we have also an experience with the French CACTUS accelerometer which was in orbit from 1975 to 1979. However, the French accelerometer had a spherical proof-mass moving in a spherical cavity which means that that the a pure rotation of the device does not give any measurements since there is no change of the gap between the proof-mass and the cavity. Therefore, we decided to build an accelerometer with a cubic proof-mass moving in a cubic cavity.

The principle of such a device is very simple, see Fig. 6. The external perturbation sources force the cavity (fixed in the satellite body) to move with respect to the proof-mass which is protected from the external effects. We define the shift by the capacitance between the walls; capacitance is immediately transformed into the tension which forces the proof-mass to return back to the geometrical center (given by the electrodes) of the cavity. It is clear that the cubic accelerometer is also sensitive to a pure rotation which enhances its overall capability. In this way, the sensitivity of the instrument goes as high as $10^{-10} \text{ m s}^{-2}$ which ensures as the detection of the radiation forces acc. to Table 1.

As shown in Table 1, the accelerometer is subjected to several internal parasitic effects. One of the most important is the centrifugal force which can be avoided by a precise location of the device into the center of satellite mass. An inaccuracy of 0.1 mm by a rotation of 4 rev/min would mean a parasitic acceleration of the order

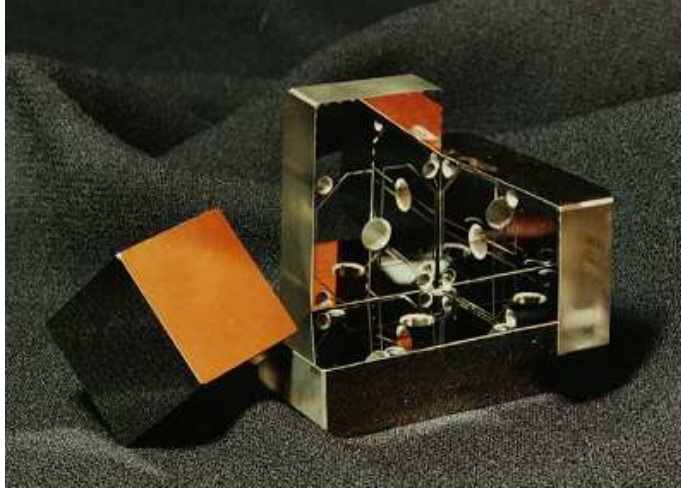


Figure 12: Satellite MIMOSA (artist's view).

of 10^{-6} ms^{-2} . To avoid such a nuisance we installed on board of the satellite a movable mass which can change the position of the center of mass according to the acceleration measurements taken on the individual electrodes. To take into account this fact is crucially important in case of an accelerometer which is fixed on a free rotating satellite. However, if we place the instrument on board of a spacecraft with a fixed or prescribed orientation we need not care about this effect. This was exactly the case when testing the accelerometer qualities on board of satellites otherwise designed for some other purposes.

First, we have flown our device on board of a Russian satellite *RESOURCE 1F* the aim of which was to take photos of the Earth surface for the intelligence usage. The accelerometer was fixed to the surface of the satellite and rotated with it (Fig. 7). We had at our disposal just two orbital revolutions but the results were so good that we have been convinced as to the functioning of the principle and technical construction of the device. Data of one of the two revolutions are seen on Fig. 8. and even if it was not possible to calibrate the device either a priori or in orbit we could see the device measures in prescribed intervals of the accelerations. The very important fact was that we have been able to recover the whole instrument after the landing of the spacecraft and to check the electronic as well as mechanic systems before and after the space experiment.

Based on the experiences of the first test flight we build a new accelerometer which was then flown on board of the Space Shuttle *Atlantis* on its mission *STS - 79* in 1996. It was a part of the instrumental package of the University of Alabama in Huntsville and was located in the s.c. Spacehab which is a room where all the research instruments are concentrated. (Fig. 9). The data were of interest to the people from the University since they experimented with the growth of crystals in weightlessness state. Our accelerometer was switched on by one of the astronauts (J. Blaha) 8 hours after the start and it provided us with the data till it was switched off again before the landing of the Space Shuttle, after 8 days, 8 hrs and 8 mins. of its flight. We

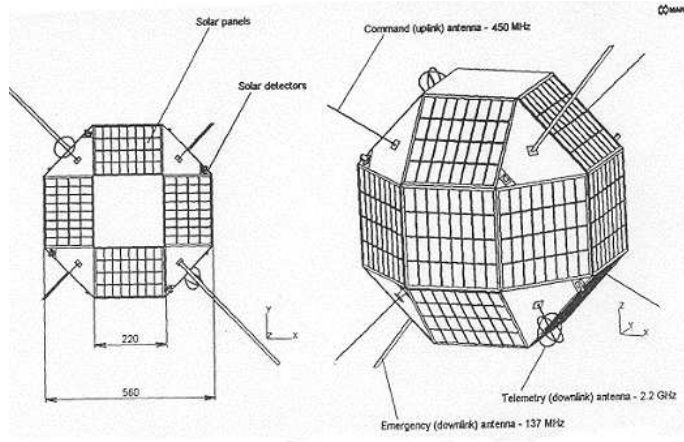


Figure 13: Dimensions of MIMOSA.

can see the data (Fig. 10) as measured on one of the accelerometer axes (most of the time in the direction of the velocity vector). We got altogether 360188 data on the Spacecraft accelerations. First, we can see that the Space Shuttle environment is not a favorable to a collection a data for a purely scientific analysis since the accelerations produced by the spacecraft itself are by far the most pronounced ones and exceed the disturbances caused by the external natural forces very conspicuously. Therefore, we analysed the data taken in times of the sleeping periods of the crew; the difference in the records is clearly seen. When reducing the data on the technical accelerations (e.g. on the thermal effects) we have been able to see the effects of the atmosphere (Fig. 11) as it changes its density from perigee to apogee of the orbit. This was the first and convincing proof of the capabilities of our instrument to sense the tiny natural accelerations.

5. SATELLITE MIMOSA

The two successful results of the accelerometer's testing convinced the Grant Agency of our republic to support the project of a special satellite carrying on board the accelerometer as the only scientific instrument and orbiting in a low-Earth orbit. The accelerometer was then again constructed as a new specimen and finally looks like we see in Fig. 12. The satellite is very similar to a sphere Fig. 13 (it can not be a spherical one due to the necessary solar panels to be placed on the surface of the satellite body). It should present a free flowing and free rotating body. The instrument itself can be unfolded which makes the construction works much easier. Inside the satellite we can trace the principal parts of the system which control the work of the whole satellite (Fig. 14.) We can identify 6 principal components of the system: (1) microaccelerometer with balancing system, (2) on-board computer with control electronics, (3) orientation and positioning system with a GPS receiver, magnetometer, solar sensors and charged coils, (4) power supply with solar panels and batteries, (5) telemetry system, (6) mechanical parts (body, thermal regulation).

The most important system, the microaccelerometer, has the properties summa-

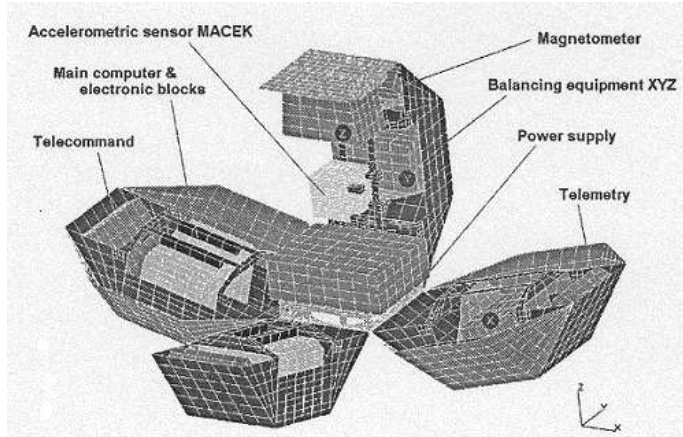


Figure 14: MIMOSA unfolded.

Table 2: Properties of the accelerometer

	Value SI	Value g's
Input Range	$\pm 4 \cdot 10^{-4} \text{ ms}^{-2}$	$\pm 40 \mu\text{g}$
Input Frequency	1 MHz – 100 MHz	
Scale Factor	25 kV ms^{-2}	$0.25 \text{ V } \mu\text{g}$
Scale Factor Error	Unknown	
Bias (nominal)	$< 1 \cdot 10^{-5} \text{ ms}^{-2}$	$< 1 \mu\text{g}$
Bias Temperature Coefficient	$2 \cdot 10^{-7} \text{ ms}^{-2}/^{\circ}\text{C}$	$20 \text{ ng}/^{\circ}\text{C}$
Resolution	10^{-10} ms^{-2}	0.01 ng
Axis Missalignment	$5 \mu \text{ rad}$	

rized in Table 2. The balancing system controlled from ground serves to ensure a precise coincidence of the center of mass of the proof mass and of the geometrical center of the satellite and, consequently, to diminish the centrifugal force. All the power needed comes from the solar panels which are of the same size and are placed on 12 walls of the satellite body. On board we have an internationally standardized telemetry system for the data transmission, and for the transmission of the commands from the ground station. The telemetry uses the bandwidth of 2.2 – 2.29 GHz, the command line is the same as used already previously with the MAGION satellites, 450.200 – 450.250 MHz. The satellite has two receiver and four transmission antennas. The central control unit is based on the controller Siemens SAB 80C166. The control program is held on the PROM chips. The satellite orientation is determined by solar sensors and a magnetometer; we can change the rotation rate with the electromagnetic coils. The satellite position is detected by the GPS system especially designed for the usage on-board of the artificial satellites. The details of the individual systems are described on the Internet address http://sunkl.asu.cas.cz/macek/mimosa_2.cz1250.html.

The orbit of the satellite should allow the satellite to penetrate the lower parts of

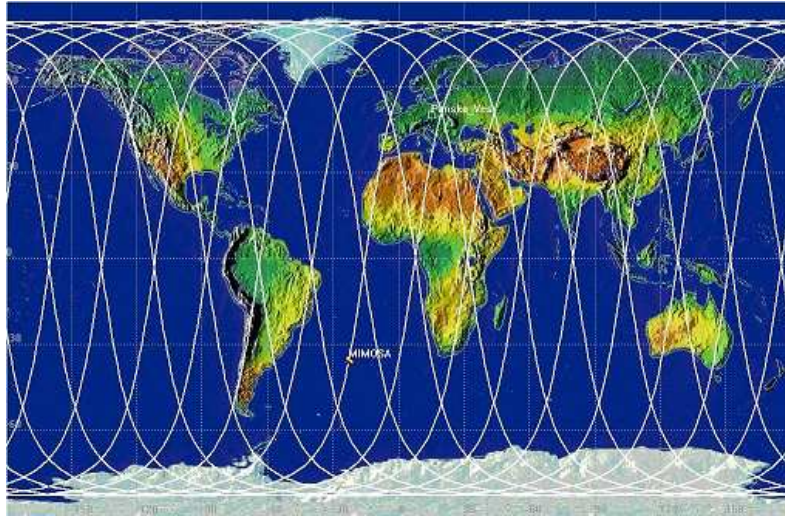


Figure 15: Projection of the initial one - day orbiting of MIMOSA on the Earth surfaces.

the atmosphere to collect the very interesting data but only so far as to ensure a sufficiently long lifetime of the satellite. The instruments are constructed to work reliably over a two-years term. Finally, we chose an orbit which assures us the satellite to be in-orbit for at least four years. The choice of the orbit was, of course, limited by the demands of the other participants of the "shared" launch - the Canadians with MOST for the stellar studies and Russians with their own satellite. All those satellites will be placed into orbits using the rocket called "ROKOT". Those launchers are actually the former Russian military rockets used now for peaceful purposes. Now, the German-Russian venture "EUROCKOT" (Bremen, Germany) is controlling the usage of those rockets. According to the most recent information, the start should be realized at the beginning of April 2003 from the launch site "Plesetsk" in north Russia near Archangelsk. The initial orbit of our satellite will have the perigee height of 330km , apogee of 820km , with an inclination of 97deg . The high inclination enables the collection of data over all of latitudes. The perigee height will be sufficient to cover the denser layers of the atmosphere and the apogee height enables us to calibrate the data by the measurements of the accelerations caused by the direct solar radiation pressure which, as we believe, is still of a constant magnitude and accurately predictable direction. The projection of the initial one-day-orbit on the Earth surface is shown on Fig. 15. There, we have the location of the ground station Panska Ves which was already used by our colleagues to track and telemeter the orbits of the MAGION satellites. Its instrumentation will be adjusted to the needs of MIMOSA observations and the data will be immediately transmitted to the main control center at the Ondrejov Observatory in central Bohemia.

Acknowledgment The project means the finalization of a long-time effort of a group of people of the Astronomical Institute of the Academy of Sciences of the Czech Republic where under the supervision of the author the research workers are engaged

with the problems of satellite dynamics already since 1965. Many colleagues from our country as well as from abroad like S. Šegan (Yugoslavia), M. Solarič (Croatia), A. Kohlhase (Germany), S. Fawzy (Egypt) are collaborating. The project was approved in 1996 and supported by the Grant Agency of the Czech Republic. The launch will be covered by the Academy of Sciences of the Czech Republic. The collaboration of domestic colleagues has been very important esp. of the project coordinator R. Peřestý, software specialist L. Pospíšilová, co-investigators D. Vokrouhlický, J. Kabeláč and A. Bezděk, who is now taking over the scientific exploitation of the project. The hardware (satellite and the instruments) has been delivered by the Prague firm "Space Devices, Ltd".

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