KINETIC THEORY OF QPESIC WAVES IN ISOTHERMAL IONOSPHERIC PLASMAS WITH RESONANT NEGATIVE IONS

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Abstract. The process of spontaneous excitation of quasi-perpendicular electrostatic ion-cyclotron (QPESIC) waves by an electron drift of sufficient magnitude, parallel to the external magnetic field, in a weakly ionized Maxwellian plasma containing one species of light positive ions and one species of heavy negative ones is analyzed from the standpoint of kinetic theory combined with the linear theory of perturbation. Attention is focused on the mode with the modal frequency close to the first ion-cyclotron harmonic of the negative (heavy) ions. It is found that the increase in the light-ions (not resonant to the wave analyzed) number density lowers both the maximum deviation of modal frequency from that of the ion-cyclotron harmonic and the threshold drift for spontaneous excitation of the mode.

In the last two decades or so, one may notice a growing interest in the ion-cyclotron waves in weakly ionized multi-ion plasmas containing, apart from neutrals and electrons, also various species of ions, both positive and negative. Plasmas with negative ions attract much attention in that context, as they are frequently encountered both in laboratory devices (e.g. in Q-machines (An et al. 1993; Song et al. 1991)) and in astrophysical situations (Ganguli et al. 1992; Kindel and Kennel 1971). Particular attention is generally given to electrostatic (potential) ion-cyclotron waves \( B = 0 \) propagated almost at right angles with respect to the external, static and homogeneous, magnetic field \( B_0 \). These quasi-perpendicular electrostatic ion-cyclotron (QPESIC) waves are studied herewith using kinetic equation with BGK model collision integrals. The QPESIC waves are rather slow, and have phase velocities lying between ion and electron thermal velocities \( (v_{T_i} \ll w/k \ll v_{Te}) \). Their assumed quasi-perpendicular character leads to the conditions \( \omega, |\omega - \omega_B| > k ||_v v_{Ts}, \nu_s \) valid for any \( s \) (index \( s \) labelling the ionic species present). Furthermore, it has long been known that the QPESIC instabilities set in first at long waves, with wavelengths significantly larger than the electron mean-free path \( (\nu_e \gg k ||_v v_{Te}) \), provided that the condition \( \omega \nu_e \ll k^2 v_{Te}^2 \), keeping the phase velocities below \( v_{Te} \), is met.

In the present paper a model of infinite and uniform, weakly ionized, low-temperature and isothermal plasma placed in parallel external fields \( E_0 \) and \( B_0 \) is adopted.
This combination of external fields causes the charged particles to drift along magnetic lines of force, so that only the effects due to the electronic drift velocity should be considered, the magnitudes of ionic drifts being much smaller. Furthermore, it is assumed that only two singly-charged ion species are present, one with positive and the other with negative electrical charge, and that the negative ions have considerably larger mass. Using the indices \( l \) (for light) and \( h \) (for heavy) ions, one thus has \( m_l < m_h \), i.e. \( M = m_h/m_l > 1 \), and the condition of macroscopic quasi-neutrality becomes

\[
n_e + n_h = n_l,
\]

where \( n_\alpha (\alpha = e, h, l) \) are the number densities of the charged constituents present. It will be convenient to introduce the dimensionless quantities

\[
\delta = \frac{n_l}{n_e}, \quad \epsilon = \frac{n_h}{n_l} = 1 - \frac{1}{\delta},
\]

noticing that \( \delta > 1 \).

The dispersion equation for the QPESIC modes in the plasmas of the type considered here reads

\[
\delta \varepsilon_e + \delta \varepsilon_h + \delta \varepsilon_l = 0,
\]

where \( \delta \varepsilon_\alpha (\alpha = e, l, h) \) are the corresponding contributions. The ionic contributions are (Milic and Brajuskovic 1983):

\[
\delta \varepsilon_s = \frac{\omega_{ps}^2}{k^2 v_{Ts}^2} \left( 1 - \sum_{n=-\infty}^{\infty} \frac{\omega A_n(\mu_s)}{\omega - n\omega_{Bs}} - \frac{\nu_s \omega A_n(\mu_s)}{(\omega - n\omega_{Bs})^2} \right), \quad (s = l, h),
\]

with \( \omega_{ps} = (q^2 s n_s/\varepsilon_0 m_s)^{1/2} \) being the ion plasma frequencies, \( v_{Ts} = (kT_s/m_s)^{1/2} \) denoting the ion thermal velocities, and \( \omega_{Bs} = q_s B_0/m_s \) standing for the ion gyrofrequencies (negative sign in the case of \( q_s < 0 \) indicates the opposite direction of cyclotron rotation for negative ions). Also, \( \mu_s = k^2 s v_{Ts}^2/\omega_{Bs}^2 \), and \( A_n(z) = I_n(z) \exp(-z) / [I_n(z) \text{ being the modified Bessel function of the } n\text{th order}] \). The electronic contribution in the longwave domain is of the form (Milic and Brajuskovic 1983):

\[
\delta \varepsilon_e = \frac{\omega_{pe}^2}{k^2 v_{Te}^2} \left( 1 + i \frac{\nu_e}{k||v_{Te}} \frac{\omega - k||u}{k||v_{Te}} \right),
\]

where \( u \) is the magnitude of the electron drift. One should point out that, according to the strict billiard-ball model adopted herewith for the collisions of the charged particles with the neutrals, one has \( \nu_e/\nu_s = v_{Te}/v_{Ts} \) (\( s = l, h \)); collisions among the charged particles are neglected altogether, in view of the assumed weak ionization.

Inserting (4) and (5) into (3), one arrives at the dispersion equation. To study the lowermost ion-cyclotron mode to which the heavy ions are resonant one should, bearing in mind that \( M \omega_{Bl} = -\omega_{Bl} \) (consequently, \( \mu_l = \mu_h/M \)), retain only the term \( n = 0 \) in the infinite sum (4) for the light ions, and the terms \( n = 0 \) and \( n = -1 \) for the heavy ones; it will be convenient to use \( |\omega_{Bh}| = -\omega_{Bh} \) further on. Equating the
real part of the thus formed dispersion equation to zero, one obtains the spectrum of the mode considered. Solved explicitly for the modal frequency, the result reads:

$$\omega = \left| \omega_{Bh} \right| \frac{1 - \epsilon + [1 - A_0(\mu_h/M)] + \epsilon [1 - A_0(\mu_h)]}{1 - \epsilon + [1 - A_0(\mu_h/M)] + \epsilon [1 - A_0(\mu_h) - A_1(\mu_h)]}. \quad (6)$$

Figure 1 shows this result graphically for three values of $M$, chosen to correspond roughly to the physical situation in ionosphere (e.g., for the ions $K^+$ and $SF_6$ one has $M = 3.5$). It can be seen that the deviation of $\omega$ od $|\omega_{Bh}|$ increases both with $M$ and with $n_h$. This trait agrees with the observation in Chow and Rosenberg (1996), although the QPESIC waves were analyzed on the ground of Vlasov equations (collisionless plasma) in that reference.

The magnitude of the critical electron drift required for the onset of the QPESIC instability (in form of waves of growing amplitude) follows from the condition of marginal instability, obtained by equating the imaginary part of the dispersion equation (3) to zero. The result is:

$$\frac{u}{v_{Th}} = \left( \frac{m_l}{m_e} \right)^{1/4} U, \quad (7)$$

with
Here, $W = (\omega - |\omega_{Bh}|)/\omega$ is the relative deviation of $\omega$ from $|\omega_{Bh}|$. The threshold drift $u^*$ corresponds to the minimum of the function on the right-hand side of (8), which had to be evaluated numerically. Figure 2 shows the results for the quantity $U^*$ which is related to $u^*$ by (7). It can be seen that increases in either $\epsilon$ or $M$ lead to increases in $u^*$. Numerical values for $u^*$ obtained herewith agree satisfactorily with the results of Kindel and Kennel (1971) and Portnyagin et al. (1992) which pertain to the astrophysical situations mentioned.

References