

## A PROGRAM FOR SOLVING MULTI-LEVEL NLTE RADIATIVE TRANSFER PROBLEMS

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**Abstract.** A program for solving multi-level NLTE radiative transfer problems in semi-infinite, plane-parallel one-dimensional atmospheres based on the Implicit Lambda Iteration method is being developed. In this paper the program is applied to the solution of a prototype hydrogen lines formation problem in the solar atmosphere.

### 1. INTRODUCTION

One of the main difficulties of NLTE (Non-Local Thermodynamic Equilibrium) radiative transfer problems arises from the nonlocal coupling between the radiation field and the excitation state of the gas. In the case of a multi-level atom NLTE line transfer problem, it is not possible to establish the line source function explicitly in terms of its radiation field and for the self-consistent solution of the radiative transfer and statistical equilibrium equations an iterative method is required to solve the problem. Such a method is a forth-and-back Implicit Lambda Iteration (ILI) developed and discussed by Atanacković-Vukmanović et al. (1997). Our objective is to develop a computer code based on this method that solves NLTE line formation problem. Here, we shall briefly describe its main features and a test application.

### 2. COMPUTER CODE FEATURES

The first part of the program contains information on the model atom and the model atmosphere to be studied along with data needed for computing the radiative and collisional transitions and background opacities as well. Therefore, at the beginning of the program the initiation is performed and its appropriate modification should be all that is required to treat the problem. LTE level populations are used as an initial estimate for the iterative procedure.

Within a forth-and-back approach we treat separately the natural two-stream representation of the radiation field along each straight line. Together with an implicit

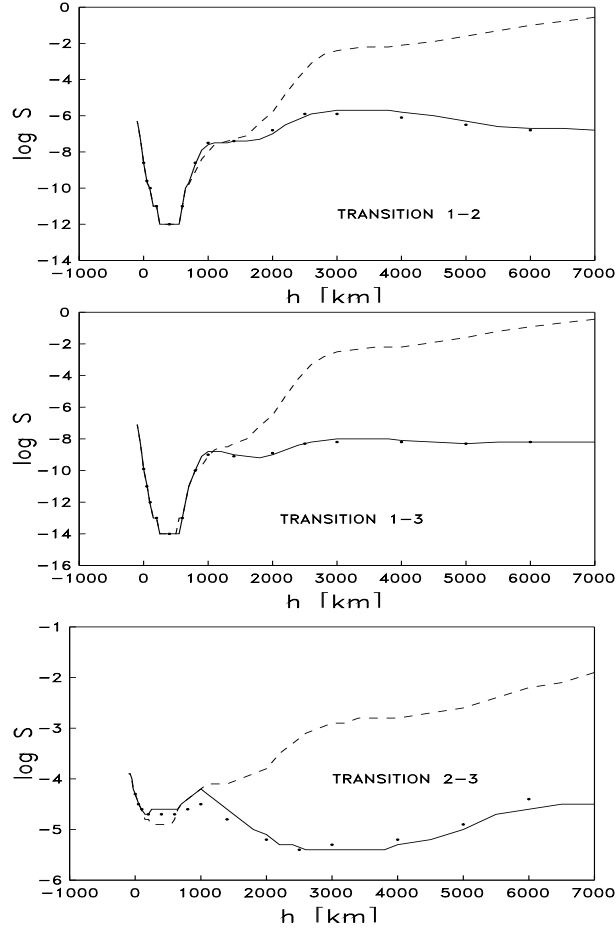


Figure 1: Source functions for the three H line transitions. Starting LTE values are given by dashed lines.

representation of the source function in the computation of the mean intensities of the radiation field such an approach can greatly accelerate the convergence of the iterative process while retaining the straightforwardness of classical Lambda iteration.

Using the known initial estimate of the source function and proceeding from the upper boundary condition we compute and store, at all optical depths, the coefficients of the linear relation representing implicitly the values of the down-going mean intensities. The aim is to obtain the coefficients of the linear relation

$$J(\tau) = \alpha(\tau) + \beta(\tau)S(\tau) ,$$

as this relation when substituted into the statistical equilibrium equations leads to a system that can be easily solved for the new level populations. Therefore, we need the coefficients of the corresponding implicit relation for the up-going mean intensities.

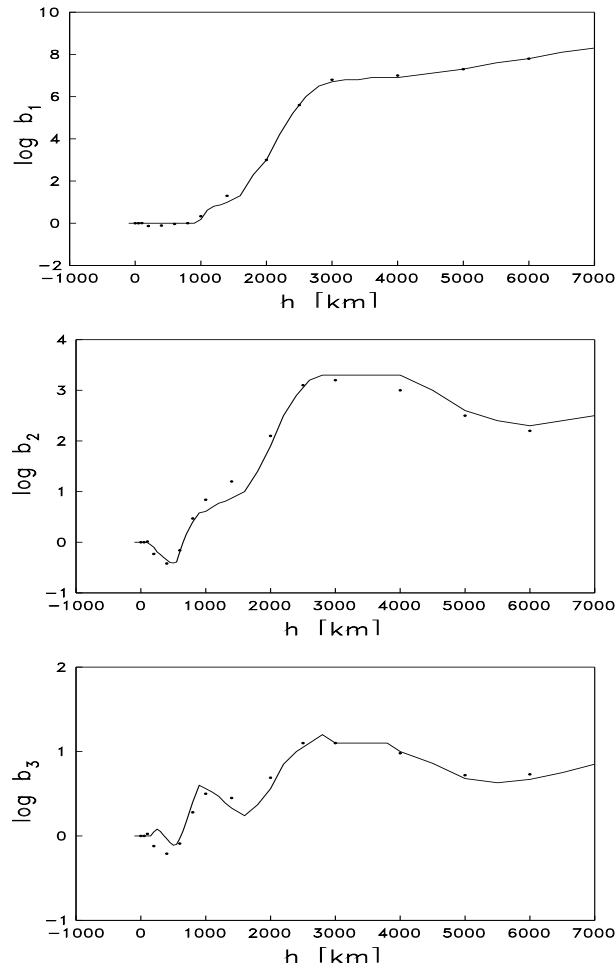


Figure 2: NLTE departure coefficients for the first three levels of H atom.

Proceeding from the lower boundary condition we determine them simultaneously with the new level populations (i.e. new source function) when sweeping layer by layer back to the surface. The process is iterated to convergence.

The code can be used to solve NLTE radiative transfer problems in semi-infinite, plane parallel one-dimensional atmospheres. The lines are assumed to be formed with complete redistribution over the Voigt profile function.

### 3. APPLICATION AND RESULTS

The method applied to the multi-level case was tested in Atanacković-Vukmanović et al. (1997) by solving a 3-level hydrogen atom model within an isothermal atmosphere. The solutions were compared with those given by Avrett and Loeser (1987). The solutions obtained within nine iterations coincide with those of Avrett and Loeser within an absolute error never greater than 3%.

Here we limit ourselves to the solution of the hydrogen atom model (3 bound levels plus continuum) in a solar atmosphere. We consider the problem that was proposed as a useful benchmark for testing the quality of a new algorithm in Athay et al. (1968) and solved by four groups of authors independently. The model atmosphere used is a semi-realistic representation of the solar atmosphere, given on height scale with 45 depth points. The results obtained by the use of Implicit  $\Lambda$  iteration (solid lines) are presented in Figs. 1 and 2 together with the results (dots) of one group of authors (Beebe, Johnson and Poland), given in tables in Athay et al. (1968). In Fig. 1 we show the three line source functions as function of height in the atmosphere, whereas the NLTE departure coefficients  $b$  are given in Fig. 2. We also solved the hydrogen line formation problem using VALC model of solar atmosphere (Vernaza et al., 1981). Fifteen iterations are required to fulfill the tolerance criterion that the relative difference between the successive iterations be less than 0.1%.

Since a negligible additional computational effort made within ILI with respect to the classical  $\Lambda$  iteration leads to a very fast convergence to the exact solution, the method seems promising in application to more complex radiative transfer problems.

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