

**GRAVITATIONAL MICROLENSING AND DARK MATTER  
PROBLEM: RESULTS AND PERSPECTIVES**

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**Abstract.** Foundations of standard theory of microlensing are described, namely we consider microlensing stars in Galactic bulge, the Magellanic Clouds or other nearby galaxies. We suppose that gravitational microlenses lie between an Earth observer and these stars. Criteria of an identification of microlensing events are discussed. We also consider such microlensing events which do not satisfy these criteria (non-symmetrical light curves, chromatic effects, polarization effects). We describe results of MACHO collaboration observations towards the Large Magellanic Cloud (LMC) and the Galactic bulge in detail. Results of EROS observations towards the LMC and OGLE observations towards the Galactic bulge are also presented. A comparison of the microlensing theory and observations is discussed in full.

A standard microlens model is based on a simple approximation of a point mass for a gravitational microlens.

In the framework of general relativity using a weak gravitational field approximation the correct bending angle is described by the following expression:

$$\delta\varphi = -\frac{4GM_*}{c^2 p}. \quad (1)$$

The derivation of the famous Einstein's formulae for the bending angle of light rays in gravitational field of a point mass  $M_*$  is practically in all monographs and textbooks on general relativity and gravity theory (see, for example books of Landau & Lifshitz (1975), Möller (1972)).

In the framework of general relativity the light ray bend effect was predicted by A. Einstein in 1915 and was firstly confirmed by Sir A. Eddington for observations of light ray bend by the Solar gravitational field near its surface. The angle is equal to  $1.75''$ , and Einstein prediction was confirmed by observations.

Since a photon moves practically along straight lines far from a gravitating body, we approximate the photon trajectory by two straight lines which are intersected near the body  $D$  (Fig. 1). The angle between the lines  $\alpha$  demonstrates the photon bending in the gravitational field of the body  $D$ .

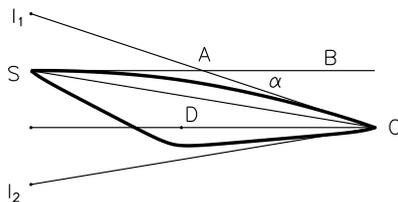


Figure 1: Formation of images and light rays bended by the gravitational field of a body.

Two rays of light, which lie on opposite sides of the gravitating body, are deflected to the gravitating body. If a source  $S$  lies far away from the body  $D$  then the rays begin to converge and intersect in some distant point  $O$  (Fig. 1). If we suppose that an observer is in the point  $O$ , he will see two images ( $I_1, I_2$ ) of one source  $S$ . Really that is gravitational lens effect. In Fig. 1 three physical bodies are shown, namely the source  $S$ , the gravitating body  $D$  and the observer  $O$ . Trajectories of light rays from  $S$  to  $O$  are shown by two bold solid lines. We use also the following notations:  $D_{ds}$  is the distance from the source  $S$  to the lens  $D$ ;  $D_d$  is the distance from the lens  $D$  to the observer  $O$ ;  $D_s$  is the distance from the source  $S$  to the observer  $O$ . We draw plane via the point  $S$  and we suppose that the plane is perpendicular to a light ray trajectory. The plane is called a plane source. Similarly, we draw the plane via the gravitational lens  $D$ . The plane is called as a lens source. We also use the following notations for the angles:  $\theta$  is the angle between a direction to the gravitating body  $D$  and a direction to the source  $S$ ,  $\theta_1$  is the angle between a direction to the gravitating body  $D$  and an apparent direction to the source  $S$ ,  $I_1, I_2$  are images (mirages) of the source.

It can be seen from the figure that we have the following expressions for the angles (Zakharov & Sazhin (1998)):

$$\alpha = \beta_1 + \beta, \theta_1 = \theta + \beta, \beta_1 \cdot D_{ds} = \beta \cdot D_d, \quad (2)$$

where the angle  $\beta$  is expressed in radians,  $D_{ds}, D_d$  are the distances from the source to the gravitational lens and from the gravitational lens to the observer, respectively. From Eqs. (2) we obtain a quadratic equation for the angle  $\theta_1$  which determines apparent positions of images relatively to a direction toward a gravitational lens,

$$\theta_1^2 - \theta\theta_1 - \theta_0^2 = 0, \quad (3)$$

where  $\theta$  is the angle between the direction toward a gravitational lens (GL) and the true position of a distant lensed source,  $\theta_0$  is the angular radius of the Einstein cone which is defined as

$$\theta_0^2 = \frac{4GM}{c^2} \cdot \frac{D_{ds}}{(D_{ds} + D_d) \cdot D_d}.$$

Eq. (3) is called the gravitational lens equation for the case of a spherically symmetric point lens. The equation has two real roots, namely

$$\theta_1 = \frac{1}{2}\theta + \frac{1}{2}\sqrt{\theta^2 + 4\theta_0^2}, \quad \theta_2 = \frac{1}{2}\theta - \frac{1}{2}\sqrt{\theta^2 + 4\theta_0^2},$$

corresponding to two images of a source  $S$ . The angular distance between the images is equal to  $\sqrt{\theta^2 + 4\theta_0^2}$ .

According to previous arguments we wrote (continuously) about two images. However, these two images are not always formed. Really, we used the assumption that the sizes of a gravitating body  $D$  are infinitesimal and the Eq. (1) is valid for any impact parameter. Actually, if the impact parameter is smaller than the radius  $R_D$  of a gravitating body  $D$  or

$$R_D > D_d \theta_2$$

then the image  $I_2$  disappears for an observer  $O$  (the light ray moving along the trajectory with the impact parameter is absorbed by the matter of a gravitational lens if it is non-transparent). Therefore, only one image of a source is formed for this case. That is the reason why Earth's observer does not see two images during a solar eclipse in spite of the existence of a set of stars which lie near the line drawing via the solar center and the observer (we recall that the angular solar size is about one half of an angular degree, which is much greater than the Einstein's cone size of the Sun since the distance between Earth's observer and the gravitational lens (Sun) is equal to 1 astronomical unit).

It is necessary to note that according to the equivalence principle two bodies with different masses fall with the same acceleration in a gravitational field. Therefore, two photons having different frequencies (different energies and thus different masses) are accelerated identically in a gravitational field. In other words, photons lying in different bands are bended identically in a gravitational field of a body  $D$ . This property is called the achromatism of the microlensing effect. Possible violations of the property may be connected with complicated structure of a source  $S$ , the violations will be discussed below.

The gravitational lens effect is a formation of several images instead of one. We have two images for a point lens model (Schwarzschild lens model). The angular distance between two images is about angular size of the so-called Einstein's cone. The angular size of Einstein's cone is proportional to the lens mass divided by the distance between a lens and an observer. Therefore, if we consider a gravitational lens with typical galactic mass and a typical galactic distance between a gravitational lens and an observer then the angular distance between images will be about a few angular seconds; if we suppose that a gravitational lens has a solar mass and a distance between the lens and an observer is about several kiloparsecs then an angular distance between images will be about one angular millisecond.

If a separation angle is  $\sim 1''$ , then one may observe two images in optical band although this problem is a complex one, the Earth's observer cannot observe directly two images in the optical band if the separation angle is  $\sim 0.001''$ . Therefore, the effect is observed through changing of the luminosity of the source  $S$ .

If the source  $S$  lies on the boundary of the Einstein cone ( $\theta(t) = \theta_0$ ), then we have  $A = 1.34$ . Note, that the total time of crossing of the Einstein cone is  $T_0$ , so

$$T_0 = 2 \frac{\sqrt{\theta_0^2 - \theta_p^2}}{\Omega}.$$

Sometimes the microlensing time is defined as a half of  $T_0$ . If we suppose that  $D_d <$

$D_{ds}$ , then

$$T_0 = 3.5 \text{ months} \cdot \sqrt{\frac{M}{M_\odot} \frac{D_d}{10 \text{ kpc}}} \cdot \frac{300 \text{ km/s}}{v},$$

where  $v$  is the perpendicular component of the velocity of a dark body.

We will give numerical estimations for parameters of the microlensing effect. If the distance between a dark body and the Sun is equal to  $\sim 10$  kpc, then the angular size of Einstein cone of the dark body with a solar mass is equal to  $\sim 0.001''$  or the linear size of Einstein cone is equal to about 10 astronomical units. If we suppose that the perpendicular component of the velocity of a dark body is equal to  $\sim 300$  km/s (that is a typical stellar velocity in Galaxy), then a typical time of crossing Einstein cone is about 3.5 months. The luminosity of a source  $S$  is changed with the time.

For observations of extragalactic gravitational lens a typical time for changes of light curve is very long ( $\sim 10^5$  years) for its direct observations. Therefore, extragalactic gravitational lenses are discovered and observed by resolving different optical components (images) since typical angular distances between images are about a few angular seconds because of the great mass of a gravitational lens. If a gravitational lens is a galaxy cluster then the angular distances between images may be about several minutes. For an identification of gravitational lenses, observers compare typical features and spectra of different images. It is clear that one cannot resolve different components during microlensing but it is possible to get and analyse the light curve in different spectral bands.

One of the basic criteria for microlensing event identification is the symmetry of the light curve. If we consider a spherically symmetric gravitational field of a lens, a point source and a short duration of microlensing event then the statement about the symmetry of a light curve will be a strong mathematical conclusion, but if we consider a more complicated distribution of a gravitational field lens or an extensive light source then some deviations of symmetric light curves may be observed and (or) the microlensing effect may be chromatic (see details in Zakharov (1997); Zakharov & Sazhin (1998)).

For the first time a possibility to discover microlensing using observations of star light curves was discussed in the paper by Byalko (1969).

Systematic searches of dark matter using typical variations of light curves of separate stars from millions observable stars started after Paczynski's discussion (1986) of the halo dark matter discovery using monitoring stars from Large Magellanic Cloud (LMC).

We remark that in the beginning of the nineties new computer and technical possibilities providing the storage and processing of huge volume of observational data appeared and it promoted the rapid realization of Paczynski's proposal. Griest (1991) suggested to call the microlenses MACHO (Massive Astrophysical Compact Halo Objects). Besides, MACHO is the name of the project of observations in the framework of the US-English-Australian collaboration. Observed was the LMC and Galactic bulge using 1.3 m telescope of Mount Stromlo observatory in Australia.<sup>1</sup>

The first papers about the microlensing discovery were published by the MACHO (Alcock et al. (1993)) and of the French collaboration EROS (Expérience de Recherche

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<sup>1</sup>MACHO stopped since end 1999.

d'Objets Sombres) (Aubourg et al. (1993)).<sup>2</sup>

First papers about the microlensing discovery toward Galactic bulge were published in the framework of the US-Polish collaboration (Optical Gravitational Lens Experiment), using 1 m telescope at Las Campanas Observatory.

The event corresponding to microlensing may be characterized by the following main features, which allow to distinguish the microlensing event and the stellar variability (Roulet & Mollerach (1997); Zakharov (1997)).

- Since the microlensing events have a very small probability, the events should never repeat in the same star. The stellar variability is connected usually with periodic (or quasi-periodic) events of the fixed star.
- In the framework of a simple model of microlensing when a point source is considered, the microlensing effect must be achromatic (deviations from achromaticity for non-point source were considered, for example in the paper by Bogdanov & Cherepashchuk (1995)), but the proper change of luminosity star is connected usually with the temperature changes and thus the light curve depends on the colour.
- The light curves of microlensing events are symmetric, but the light curves of variable stars are usually asymmetric (often they demonstrate a rapid growth before the peak and the slow decrease after the peak of the luminosity).
- Observations of microlensing events are interpreted quite well by the simple theoretical model, but some microlensing events are interpreted by more complicated model in which one can take into account that the source (or a microlens) is a binary system, of non-vanishing size, the parallax effect may take place.

The typical features of the light curve of the first microlensing event observed by the MACHO in the LMC are shown in Fig. 2, where the light curves are shown for two spectral bands.<sup>3</sup> The light curve (in two bands) is fitted by the simple model well enough, but the ratio of luminosities for the bands is shown in the lower panel of figure (the ratio shape is adjusted with the event achromaticity). However, one can note that near the maximal observable luminosity the theoretical curve fits the data of observations not very well.

Now one can carry out accurate testing of the achromaticity and moreover the stability of the source spectrum during a microlensing event with the Early Warning systems implemented both by the MACHO and OGLE. This allows one to study the source properties using large telescopes and to organize intense follow-up studies of light curves using telescope network around the globe.

In addition to the typical properties of individual microlensing events, Roulet and Mollerach note that the population of observed events should have the following statistical properties (Roulet & Mollerach (1996), Zakharov (1997)):

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<sup>2</sup>EROS experiment will stop in 2002 (Moniez, 2001)).

<sup>3</sup>A more recent MACHO fit to the observed amplification of this event gives  $A_{\max} = 7.2$ .

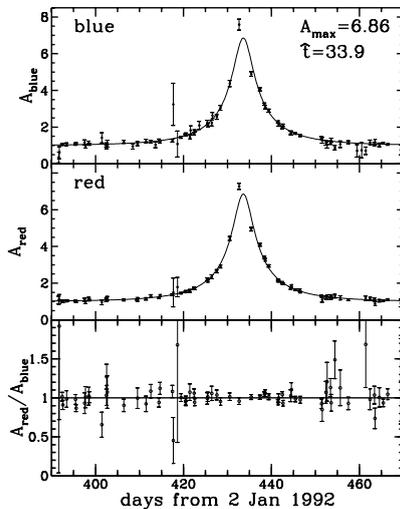


Figure 2: The first microlensing event which was detected by the MACHO during microlensing searches towards LMC (Alcock et al. (1993)).

Unlike a star variability microlensing events should happen with the same probability for any kind of star therefore the distribution of microlensing events should correspond to the distribution of observed stars in the color-magnitude diagrams.<sup>4</sup>

The distribution of the maximal amplification factor  $A_{\max}$  should correspond to a uniform distribution of the variable  $u_{\min}$ .

The distributions of the amplification  $A_{\max}$  and the microlensing event time  $T$  should be uncorrelated.

Since for the microlens searches one can monitor several million stars during several years, the ongoing searches have focused on two targets: a) stars in the Large and Small Magellanic Clouds (LMC and SMC) which are the nearest galaxies having lines of sight which go out of the Galactic plane and well across the halo; b) stars in the Galactic bulge which allow to test the distribution of lenses near to the Galactic plane.

Let us cite well established results of microlensing searches and discuss the questions for which we have now different answers which do not contradict the observational data. Now it is generally recognized that the microlensing searches towards the Galactic bulge or nearby galaxies are very important for solutions of a lot of problems in astronomy and cosmology. As Paczynski (1996) noted, the most important is the consensus that the microlensing phenomenon has been discovered. Now it is impossible to tell which part of the microlensing event candidates is actually connected with the effect since probably there are some variable stars among the event candidates, it could be stellar variability of an unknown kind.<sup>5</sup>

<sup>4</sup>However, Roulet and Mollerach (1997) noted that for observations in the bulge since observed stars have non-negligibly spread along the line of sight, the optical depth is significantly larger for the star lying behind the bulge, thus the lensing probabilities should increase for the fainter stars.

<sup>5</sup>The microlensing event candidates proposed earlier by the EROS (#1 and #2) and by the MACHO (#2 and #3) are considered now as the evidence of a stellar variability (Paczynski (1996)).

1. Observed light curves are achromatic and their shapes are interpreted by simple theoretical expressions very well, however, there is not complete consensus about "very well interpretation" since even for the event candidate MACHO # 1 the authors of the discovery proposed two fits. Dominik and Hirshfeld (1994, 1995) suggested that the event could be interpreted very well in the framework of the binary lens model, but Gurevich et al. (1996) assumed that the microlensing event candidate could be caused by a non-compact microlens.<sup>6</sup>
2. As expected, binary lenses have been detected and the observed rate of the events corresponds to the expected value.
3. As expected, the parallax effect has been detected.
4. Since the observed optical depth is essentially greater than the estimated value, the independent confirmation of the Galactic bar existence was obtained.
5. Using photometric observations of the caustic-crossing binary lens microlensing event EROS BLG-2000-5, PLANET reported about the first microlens mass determination, namely the masses of these components are  $0.35 M_{\odot}$  and  $0.262 M_{\odot}$  and the lens lies within 2.6 kpc of the Sun (An et al. (2002)).
6. Bennett et al. (2002) discovered gravitational microlensing events due to stellar mass black holes. The lenses for events MACHO-96-BLG-5 and MACHO-96-BLG-6 are the most massive, with mass estimates  $M/M_{\odot} = 6_{-3}^{+10}$  and  $M/M_{\odot} = 6_{-3}^{+7}$ , respectively.

Now the following results are generally accepted:

1. The optical depth towards the Galactic bulge is equal to  $\sim 3 \times 10^{-6}$ , so it is larger than the estimated value (Alcock et al. (2000a)).
2. Analysis of 5.7 years of photometry of 11.9 million stars in LMC by MACHO reveals 13 – 17 microlensing events (Alcock et al. 2000b). The optical depth towards the LMC is equal to  $\tau(2 < \hat{t} < 400 \text{ days}) = 1.2_{-0.3}^{+0.4} \times 10^{-7}$ , so, it is smaller than the estimated value. The maximum likelihood analysis gives a MACHO halo fraction  $f=0.2$ . Alcock et al. (2000b) gives also estimates of the following probabilities  $P(0.08 < f < 0.5) = 0.95$  and  $P(f = 1) < 0.05$ . The most likely MACHO mass  $M \in [0.15, 0.9]M_{\odot}$ , depending on the halo model and total mass in MACHOs out 50 kpc is found to be  $9_{-3}^{+4} \times 10^{10}M_{\odot}$ . EROS gives a consistent conclusion, namely, Lasserre et al. (2000); Lasserre (2001) estimate the following probability  $P(M \in [10^{-7}, 1]M_{\odot} \ \& \ f > 0.4) < 0.05$ .

However there are different suggestions (which are not contradicting the observational data) about the following issues (Paczynski (1996)):

*What is the location of objects which dominate microlensing observed towards the Galactic bulge?*

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<sup>6</sup>Microlensing by non-compact objects considered also by Zakharov & Sazhin (1996a,1996b) and Zakharov (1998,1999,2001a,2001b).

*Where are the most microlenses for searches towards LMC?* The microlenses may be in the Galactic disk, Galactic halo, the LMC halo or in the LMC itself. *Are the microlenses stellar mass objects or are they substellar brown dwarfs?*

*What fraction of microlensing events is caused by binary lenses?*

*What fraction of microlensing events is connected with binary sources?*

Paczynski (1996) suggested that we shall have definite answers for some presented issues after some years and since the optical depth towards the Galactic bulge is essentially greater than the optical depth towards the LMC, we shall have more information about the lens distribution towards the Galactic bulge, however, probably, some problems in theoretical interpretation will appear after detections of new microlensing event candidates.

The main result of the microlensing searches is that the effect predicted theoretically has been confirmed. This is one of the most important astronomical discoveries.

When new observational data would have been collected and the processing methods perfected, probably some microlensing event candidates would lose their status, but perhaps new microlensing event candidates would be extracted among analysed observational data. So, the general conclusion may be done. The very important astronomical phenomenon was discovered, but some quantitative parameters of microlensing will be specified in the future. However, the problem about 80% of DM in the halo of our Galaxy is still open (10 years ago people believed that microlensing could give an answer to this problem).

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