Publ. Astron. Obs. Belgrade No. 75 (2003), 17 - 25 *Invited lecture*

DYNAMICS OF MULTIPLE STARS

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Abstract. According to modern views, a majority of stars in galactic disk were formed within the small stellar groups containing a few components. As a rule, such groups have a non-hierarchical structure and decay. As a result, single, binary and stable multiple systems are formed. Binaries and stable triples - products of small group dynamical decay - have rather wide intervals of orbital parameters and mass ratios. This is in agreement with observations of double and multiple stars. We study the effects of star merging, dynamical friction on interstellar medium, and mass-loss to dynamical evolution of small stellar groups.

1. INTRODUCTION

According to modern views (see, e.g., Larson 2001), majority of single and binary stars may be formed via decay of non-hierarchical multiple stars. Such objects are formed due to fragmentation of molecular cloud cores (see, e.g., Klessen et al. 1998).

Although there is a huge number of papers devoted to star formation theory and observations of star formation regions, so far there is no "standard" star formation model (Larson 2001). It seems that forming small stellar groups have a wide range of parameters (sizes, velocity dispersions, mass spectra etc.).

During initial phase of such system evolution, the hydrodynamic processes play a leading role. These processes may be described within the SPH-scheme. (see, e.g., Bodenheimer et al. 2000, and Monaghan 1992). One more effective approach is numerical solution of hydrodynamic equations using multiple grids or adaptive mesh refining (AMR) (see also the references in the cited review by Bodenheimer et al. 2000). In AMR-scheme, positions and sizes of the grids are changed dynamically. New grids may be created and unnecessary ones are deleted.

Numerical simulations of star formation process indicate that rather wide ($\sim 1 \div 100$ AU) stellar groups of different multiplicity may be formed. It seems that scenario of closer system formation is more complicated. However, the processes of dynamical decay of small stellar groups can help us in this case. Such processes were carefully studied in stellar dynamics, especially for triple systems. Soon the book of Valtonen and Karttunen (2003) will be published about this topic.

Numerical simulations (Sterzik and Durisen 1998, Rubinov, Petrova and Orlov 2002) show that small non-hierarchical groups decay into unbound singles, doubles, stable triples, and possibly multiples.

Table 1: State distribution at $t = 300T_{cr}$

N	Binary	Two singles	Stable triple	Unstable triple	Sistems with $n \geq 4$
3	0.78	–	0.01	0.21	–
6	0.56	0.01	0.14	0.16	0.13
9	0.55	0.03	0.16	0.14	0.12
12	0.47	0.04	0.17	0.16	0.16
15	0.49	0.06	0.13	0.15	0.17
18	0.51	0.06	0.11	0.14	0.18

2. DYNAMICAL DECAY OF SMALL STELLAR GROUPS

Let us briefly present our results. Table 1 contains the distribution of final states for the time of 300 initial crossing times. The results are given for different N and the Salpeter initial mass spectrum. The star coalescence at close encounters were taken into account, when separation between two stars was less then 3/4 of the sum of their radii. Initial system size was 100 AU.

One can see from Table 1 that the final binary is formed in approximately one half of the runs. The stable triple system is formed in about 15% of cases. About 30% of the groups did not complete their evolution up to this moment.

It is of interest to compare the data of this table with stellar multiplicity function. Tokovinin (2001) has constructed this function using the catalog of 807 physical multiple systems. He found the ratios of the amounts of different multiplicity n systems:

$$f_n = \frac{N_n}{N_{n-1}}$$

Let us note that the ratio f_n is approximately constant within the uncertainties. Beginning from $n = 4$, it equals 0.26 ± 0.05. When one comes from triples to binaries this ratio decreases by half.

Orlov and Titov (1994) have estimated the ratio of stars which are members of the systems with multiplicities $n = 1, 2, 3, \geq 4$ — 0.64 : 0.28 : 0.06 : 0.02 for nearby stars within 25 parsecs from the Sun. Corresponding ratios $f_3 \approx 0.14$, $f_{\geq 4} \approx 0.25$ are in agreement with Tokovinin's (2001) data within their uncertainties.

If we consider the distribution of final states of small groups (Table 1) and assume that unstable triple systems at least decay to single and binary ones, then the fraction of final binaries reaches 70%. The ratio $f_3 \approx 0.2$ that is not strongly different from

Table 2: Multiplicity function

n	3	4	5	6
f_n	0.11 3	0.22 2	0.20 4	0.36 14

observed estimates. Now it is not clear how many stable systems with multiplicity $n \geq 4$ are formed and what their fraction will decay to binary and stable triple. Therefore this estimation f_3 is not reliable.

It is of interest to find the parameters of final stable systems and compare those with observations of binary and triple stars.

Basic parameters of binary stars are semi-major axis, eccentricity and mass ratio. When initial size of a group is ~ 100 AU, the semi-major axes of final and escaped binaries are within the range from ~ 1 till $\sim 10^3$ AU. This range is in agreement with observational data for wide binary stars (see, e.g., Duquennoy and Mayor 1991, Heacox 1998). The closer binaries may be formed via decay of more compact stellar groups.

The eccentricity distributions for binaries formed via decay of the groups with multiplicity $N = 6$ are shown in Fig. 1. The straight line corresponds to the law $f(e) = 2e$. This was found by Ambartsumian (1937) for equilibrium distribution of binaries in stellar field. The same distribution was obtained in statistical theory of escapes from triple systems (Monaghan 1976).

One can see from the figure that the results of numerical simulations are in agreement with the theory. The eccentricity distributions for wide double stars are also in agreement with the law $f(e) = 2e$ if one takes into account the observational selection (see, e.g., Valtonen 1997, Tokovinin 1998). This agreement is an additional argument confirming dynamical origin of wide binaries. A qualitative agreement between parameters of visual double stars and binaries formed via decay of small groups evidences an efficiency of this mechanism for double star formation.

Now let us consider the stable triple systems. The stable triples formed via decay of small groups have a strong hierarchy — average ratio of semi-major axes of outer and inner binaries is about 20 : 1.

The distributions of eccentricities for inner and outer binaries are shown in Fig. 2.

One can see from the figure that the eccentricity distributions for both binaries differ from the law $f(e) = 2e$. We observe a deficit of strongly elongated orbits with $e > 0.9$. Especially this deficit is evident for outer binaries. The mean values of eccentricities equal $\overline{e_{in}} \approx 0.7$, $\overline{e_{ex}} \approx 0.5$.

We consider a sample of 38 observational hierarchical triple stars for which the elements of inner and outer binaries were calculated. The mean eccentricities for this sample are

$$\overline{e_{in}} \approx 0.37 \pm 0.04, \ \overline{e_{ex}} \approx 0.38 \pm 0.04.$$

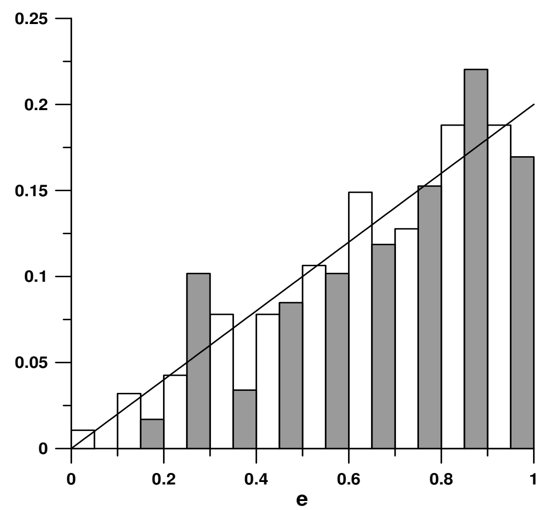

Figure 1: Distributions of eccentricities for the binaries formed via decay of $N = 6$-body systems. Hollow columns correspond to final binaries, gray columns – to escaped binaries. The straight line corresponds to the law $f(e) = 2e$.

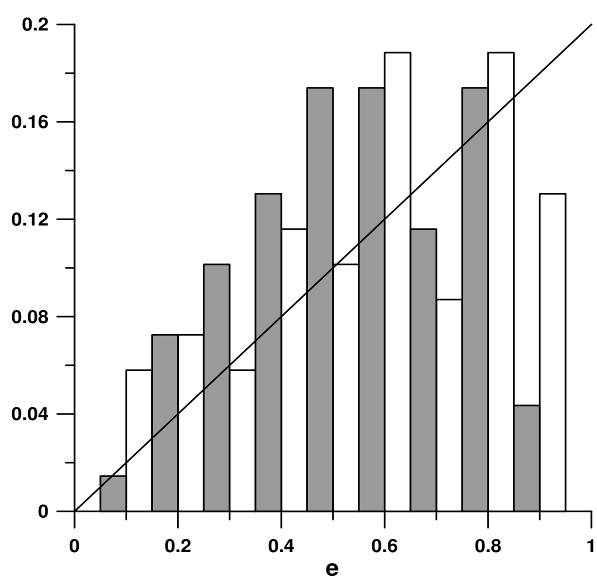

Figure 2: Distributions of eccentricities for inner (hollow columns) and outer (grey columns) binaries in stable triples. Straight line corresponds to the law $f(e) = 2e$.

These values are less than the same values for simulated systems. This difference is especially noticeable for inner binaries. One can explain this difference in the following way. A partial circularization of the orbits may be connected with tidal interaction between the components. We do not take into account this effect in simulations. Besides, a selection may influence the result — it is difficult to reveal a strongly elongated system.

One more important feature of triple systems is relative orientation of inner and outer binary orbits. We consider the models with initially isotropic velocity distribution. Then the stable triple systems with prograde motions prevail. Although the difference with a random distribution

$$f(i) = \frac{1}{2}\sin i$$

is not so strong (Fig. 3).

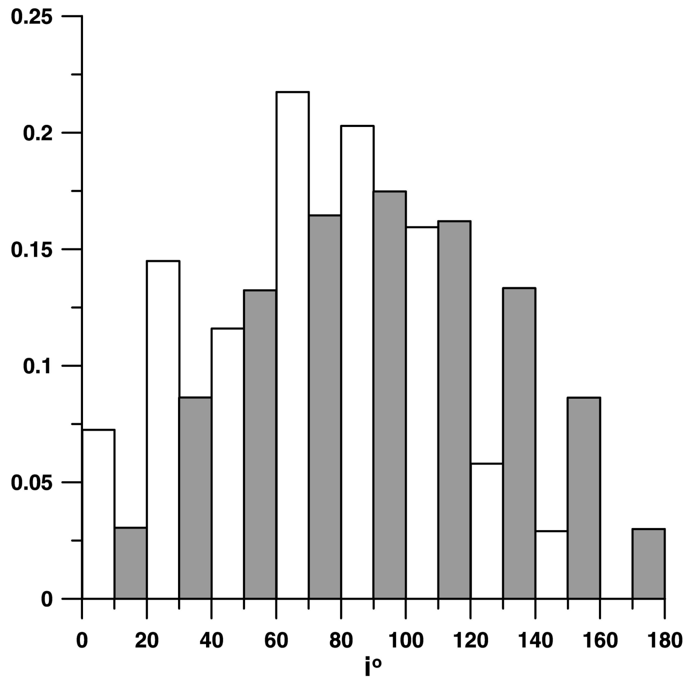

Figure 3: Distributions of mutual inner and outer binary orbit inclination (hollow columns). Grey columns correspond to random orbit orientation.

A detailed analysis of the distribution $f(i)$ for stable triples formed via decay of systems with $3 \leq N \leq 10$ was made by Sterzik and Tokovinin (2002). In numerical simulations with realistic initial conditions, the final triples have a moderate but significant prevalence of prograde motions. The average angle between orbital moment vectors of inner and outer binaries is $\overline{i} \approx 70^\circ \div 80^\circ$. The result depends on system geometry, mass spectrum, relation between rotation energy and energy of peculiar motions. For actual 22 triple stars with known orbital elements (but unknown ascending nodes) of inner and outer binaries, the averaged angle was found $\bar{i} = 79^\circ \pm 6^\circ$. This estimate was found for mixing of true and false angles. However we can see a trend of the systems with prograde motions to prevail. This is in agreement with the results of our simulations too (Rubinov et al. 2002).

Also we have made computer simulations with initial clump mass spectrum that is less steep than the Salpeter one. We have considered different initial sizes of the groups — from 3 till 1000 AU and different initial virial ratios from 0.001 till 0.9.

There are some invariant properties of final systems:

1. A high fraction of stable triples (10–15%).
2. Universality of eccentricity distribution for final and escaped binaries
$$f(e) = 2e.$$
3. Strong hierarchy of stable final triple — the mean ratio of semi-major axes for outer and inner binaries is $\approx 20 : 1$, and the mean ratio of their periods is $\approx 70 : 1$.
4. The outer binaries have more rounded orbits than the inner ones.
5. The systems with prograde motions prevail.

Some additional factors may influence the dynamics of multiple stars, in particular:

1. Dynamical friction on interstellar medium.
2. Mass-loss from the stars.
3. Tidal interactions of stars during their close encounters.

We have considered two first effects. When we take the realistic interstellar medium density or mass-loss rate, the results are practically the same as without these effects.

For anomalously high densities, dynamical friction may influence the parameters of final systems. In particular, a fraction of strongly elongated binaries decreases and hierarchy of stable triples rather grows.

The mass-loss due to stellar wind influences the dynamics of wide multiple systems with typical sizes of $\sim 10^3$ AU. Final binaries and triples are on the average wider. Besides, outer binaries in stable triple systems are more elongated.

3. DYNAMICS OF HIERARCHICAL TRIPLE STARS

Tidal interaction and mass-loss may influence the dynamical evolution of hierarchical triple systems near the stability limit.

Orlov and Petrova (1996) have shown that the stability of triple systems depends on stellar rotation with tidal effect taken into account. If there are more rotational velocities of the stars are more than the ones synchronized with orbital motion, then the stability store of triple system decreases. In opposite case, triple systems increase the stability store. The equilibrium state of circularization and synchronization in the inner pair is not reached because of perturbations from the distant body.

The effect of the mass-loss due to stellar wind is more complicated. Here one needs to consider two scenarios of stability violation:

1. Exchange of components — violation of hierarchy.

2. Escape of the distant component without a previous hierarchy violation.

Orlov, Petrova and Ivanova (1996) have shown that mass-loss leads to increasing the stability store with respect to hierarchy violation. As for escape without hierarchy violation, the result depends on mass ratio. If the mass of distant star is less than the mass of inner binary, then a trend to stability loss takes place. In opposite case, we have an increase of stability store. Let us note that the typical time of stability measure changes is comparable with evolutionary time of the components.

Petrova (1998) has studied the effect of mass exchange in semi-detached inner binaries on stability measure of hierarchical triple systems. The trend of stability measure depends on which component of close binary loses the mass. If the loser is more massive star then the binary becomes harder and stability storage increases. In opposite case, when the loser is the secondary component, the stability measure decreases (the inner binary becomes softer). Gravitational perturbations from the third distant body lead to some irregularities (like the steps) in dependences of stability measure variations on time.

One more aspect of multiple star dynamics is dynamical stability of actual systems.

In the literature we can find a few stability criteria for hierarchical triple systems. (see, e.g., Golubev 1967, Harrington 1977, Eggleton and Kiseleva 1995, Mardling and Aarseth 1999). The Golubev criterion was derived by analytical approach. Three other criteria were obtained from numerical experiments.

A general form for stability criterion is as follows

$$s > s_c,$$

where s — stability parameter depending on orbital elements and mass ratio in triple system, s_c — critical value of stability parameter.

We have found the values of s and s_c using four above criteria for 38 triple stars with known orbital elements of inner and outer binaries. The majority of systems are stable according to all four criteria.

However, ten systems may be unstable according to some of the criteria. More careful analysis of results has shown that nine systems, excluding ADS 10157, seem to be stable. The instability according to some criteria is probably connected with one of three reasons:

1. Uncertainty of the angle between orbital planes of inner and outer binaries (we have chosen the most favourable for instability value).

2. Inapplicability of some empiric criteria to some individual systems.

3. Uncertainty of initial data.

The orbit of triple system ADS 10157 was found from perturbations of distant body. In order to make a more reliable conclusion on this system, we need a confirmation of the third body presence in this systems and direct definition of its orbit.

Let us note that, as a rule, the ages of components in the triple stars under study are a few orders greater than the typical life-times of unstable triple systems with similar parameters (mean size, mean crossing time, mean mass of components). Therefore we can make a conclusion that these systems would decay should they be unstable. Thus, one can expect that all actual multiple stars, excluding very young and very wide systems, have to be dynamically stable.

4. CONCLUSIONS

In conclusion let us formulate the basic results:

1. Majority of binary and stable triple stars in the solar neighbourhood could be formed within small non-hierarchical stellar groups.

2. The eccentricity distributions of binaries formed via dynamical decay of small groups correspond to the universal law

$$f(e) = 2e.$$

3. In stable triple systems formed via decay of small groups, the eccentricities of outer binaries are less than the ones of inner binaries.

4. The orbits of outer and inner binaries are usually non-coplanar; for isotropic initial velocity distribution, the prograde motions prevail that is in agreement with observations of visual triple hierarchical stars.

5. Actual hierarchical triple stars, as a rule, are dynamically stable.

6. For further progress in this field of astronomy, we need an accumulation of reliable statistical material for multiple stars with known orbits of the inner and outer subsystems (especially the estimates of mutual inclinations). On the other hand, we need more realistic numerical dynamical models of small stellar groups, as well as the algorithms for objective comparison between models and observations.

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