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STAR FORMATION AND THE STELLAR INITIAL MASS FUNCTION

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Abstract. A successful paradigm of star formation has recently been developed; the agreement between observations and theory is quite good, especially for low-mass stars. After reviewing the current theory, we present a class of models for the initial mass function (IMF) for stars forming within molecular clouds. In this picture, stars help determine their own masses through the action of stellar outflows. Using this concept, we calculate a semi-empirical mass formula (SEMF), which provides the transformation between initial conditions in molecular clouds and the masses of forming stars. For a specified SEMF, a given distribution of initial conditions predicts a corresponding IMF. In the limit in which many independent physical variables determine stellar masses, the central limit theorem shows that the IMF approaches a log-normal form. These results show that this paradigm of star formation naturally produces an IMF that is roughly consistent with observations.

1. INTRODUCTION

The formation of stars is a fundamental problem in astrophysics and the initial mass function (IMF) is perhaps the most important outcome of the star formation process. A detailed knowledge of the IMF is required to understand galaxy formation, the chemical evolution of galaxies, and the structure of the interstellar medium. The IMF also has important consequences for baryonic dark matter, e.g., the IMF determines the numbers of brown dwarfs and/or white dwarfs in the galactic halo. Although the current theory of star formation (Shu et al. 1987) is not complete, we can now begin building models of the IMF (Adams and Fatuzzo 1996). The purpose of this contribution is to present a class of IMF models which use the idea that stars, in part, determine their own masses through the action of powerful stellar winds and outflows (Shu et al. 1987; Lada and Shu 1990). Within the context of the current theory of star formation described below, we can conceptually divide the process that determines the IMF into two subprocesses:

[1] The spectrum of initial conditions produced by molecular clouds (the star forming environment).

[2] The transformation between a given set of initial conditions and the properties of the final (formed) star. This transformation is accomplished through the action of stellar winds and outflows.

Molecular clouds are not observed to be collapsing as a whole; on average, the lifetime of a molecular cloud is (at least) an order of magnitude longer than the free-fall time (Zuckerman and Palmer 1974). These clouds thus exhibit quasi-static behavior and it makes sense to conceptually divide the process of determining the distribution of stellar masses into the two steps given above.

For the theory of the IMF developed here, traditional arguments based on the Jeans mass are not directly applicable. A characteristic feature of molecular clouds is that they are highly non-uniform; clumpiness and substructure exist on all resolvable spatial scales. In particular, no characteristic density exists for these clouds and no single Jeans mass can be defined. We stress that, at least in the context of present day star formation in molecular clouds, the Jeans mass does not determine the masses of forming stars.

2. THE IMF OBSERVED

Stars exist in a finite range of masses. Stellar bodies with masses less than $M_* \approx 0.08 M_\odot$ cannot produce central temperatures high enough for hydrogen fusion to take place; these objects with masses less than this hydrogen burning limit are brown dwarfs. On the other end of the spectrum, stars with masses greater than about 100 $M_\odot$ cannot exist because they are unstable. Stars are thus confined to the relatively narrow mass range $0.08 \le m \le 100$, where we have defined $m \equiv M_*/(1 M_\odot)$.

The initial mass function in our galaxy has been estimated empirically. The first such determination (Salpeter 1955) showed that the number of stars with masses in the range m to $m + dm$ is given by the power-law relation

$$f(m)\, dm \sim m^{-\mu}\, dm \,, \tag{2.1}$$

where the index $\mu \approx 2.35$ for stars in the mass range $0.4 \le m \le 10$. However, more recent work (Miller and Scalo 1979; Scalo 1986; Rana 1991; Tinney 1995; Meyer et al. 1998) indicates that the mass distribution deviates from a pure power-law. The distribution becomes flatter and then turns over at the lowest stellar masses, with the turnover at $m = 0.1 - 0.2$. The distribution becomes steeper at the highest stellar masses ($\mu \sim 3.3$ for $m > 10$). As a working approximation, the observed IMF can be modelled with a log-normal form

$$\ln f(\ln m) = A - \frac{1}{2\langle\sigma\rangle^2} \Big\{ \ln\big[m/m_C\big] \Big\}^2 \,, \tag{2.2}$$

where A is a normalization constant, $m_C \approx 0.1$, and $\langle\sigma\rangle \approx 1.6$. The actual IMF has more structure than a simple log-normal form, but Eq. [2.2] provides a good reference distribution. Although the construction of the IMF from observational quantities requires considerable processing, the basic features of the IMF seem robust. In general, the observed IMF does not change significantly from one star forming region to another. We can thus use the analytic fit given by Eq. [2.2] as a benchmark with which to compare our theoretical models.

3. THE CURRENT THEORY OF STAR FORMATION

Over the past 15 years, a generally successful paradigm of star formation has emerged (Shu et al. 1987). Since the theory of the IMF presented here uses this paradigm as a starting point, we quickly review its basic features.

In our galaxy today, star formation takes place in molecular clouds. These clouds thus provide the initial conditions for the star forming process. Molecular clouds have complicated substructure. For example, the observed molecular linewidths contain a substantial non-thermal component (Myers and Fuller 1992); this line broadening is generally interpreted as a "turbulent" contribution to the velocity field. Molecular clouds are supported against their self-gravity by both turbulent motions and by magnetic fields. The fields gradually diffuse outward (relative to the mass) and small centrally condensed structures known as molecular cloud cores are formed. These cores represent the initial conditions for protostellar collapse. In the simplest picture, these cores can be characterized by two physical variables: the effective sound speed a and the rotation rate Ω. The effective sound speed generally contains contributions from both magnetic fields and turbulence, as well as the usual thermal contribution. The total effective sound speed can thus be written

$$a^2 = a_{\rm therm}^2 + a_{\rm mag}^2 + a_{\rm turb}^2 \, . \tag{3.1}$$

The molecular cloud cores eventually experience a phase of dynamic collapse, which proceeds from inside-out – the central parts of the core fall in first and successive outer layers follow as pressure support is lost from below (Shu 1977). Because the infalling material contains angular momentum (the initial state is rotating), not all of the infalling material reaches the stellar surface. The material with higher specific angular momentum collects in a circumstellar disk. The collapse flow is characterized by a well defined mass infall rate $\dot{M} \approx a^3/G$, the rate at which the central object (the forming star/disk system) gains mass from the infalling core. Notice that no mass scale appears in the problem, only a mass infall *rate* $\dot{M}$. In particular, the total amount of mass available to a forming star is generally much larger than the final mass of the star. This claim is consistent with the finding that star formation is generally an inefficient process.

One important characteristic of the rotating infalling flow is that the ram pressure of the infall is weakest at the rotational poles of the object. The central star/disk system gains mass until it is able to generate a powerful stellar wind which breaks through the infall at the rotational poles and thereby leads to a bipolar outflow. Although the mechanism that generates these winds remains under study (Shu et al. 1994), the characteristics of outflow sources have been well determined observationally (Lada 1985). The basic working hypothesis of this IMF theory is that these outflows help separate nearly formed stars from the infalling envelope and thereby determine, in part, the final stellar mass. In the following section, we use this idea to calculate a transformation between the initial conditions in a molecular cloud core and the final mass of the star produced by its collapse.

4. A THEORY OF THE IMF

As described above, the determination of the stellar IMF can be divided into two subprocesses. In this IMF theory, the transformation **[2]** is accomplished through the action of stellar winds and outflows. Stars are formed through the collapse of molecular cloud cores. In this paradigm, the central star/disk system gains mass at a well defined mass infall rate $\dot{M} \sim a^3/G$, which is roughly constant in time (Shu 1977). As the nascent star gains mass, it becomes more luminous and produces an increasingly powerful stellar outflow. When this outflow becomes stronger than the ram pressure of the infalling material, the star separates itself from the surrounding molecular environment and thereby determines its final mass.

The transformation between the initial conditions and the final stellar properties can be written as a "semi-empirical mass formula" (SEMF). Using the idea that the stellar mass is determined when the outflow strength exceeds the infall strength, we can write the SEMF in the form

$$L_* M_*^2 = 8 m_0 \gamma^3 \delta \frac{\beta}{\alpha \epsilon} \frac{a^{11}}{G^3 \Omega^2} \, . \tag{4.1}$$

This formula provides us with a transformation between initial conditions (the sound speed a and the rotation rate Ω) and the final properties of the star (the luminosity L_* and the mass M_*). Furthermore, the protostellar luminosity L_* as a function of mass is known so that Eq. [4.1] specifies the final stellar mass in terms of the initial conditions. In addition, the parameters α, β, γ, δ, and ϵ are efficiency factors (Adams and Fatuzzo 1996; Shu et al. 1987b). In general, all of the quantities on the right hand side of Eq. [4.1] will have a distribution of values. These individual distributions ultimately determine the composite distribution of stellar masses M_*. However, as we argue below, to leading order the mass distribution approaches a log-normal form.

In order to evaluate the SEMF, we must specify the luminosity as a function of mass for young stellar objects. This luminosity has many contributions (Stahler et al. 1980; Adams 1990; Palla and Stahler 1990). For most of the relevant mass range, however, the most important source of luminosity arises from infall – infalling material falls through the gravitational potential well of the star and converts energy into photons. The star also generates internal luminosity which becomes important at sufficiently high stellar masses. We can parameterize these contributions to obtain a luminosity versus mass relation of the form

$$\widetilde{L} = L_*/(1 L_\odot) = 70 \, \eta \, a_{35}^2 \, m \, + m^4 \, , \tag{4.2}$$

where the first term arises from infall and the second term arises from internal luminosity. The efficiency parameter η is the fraction of the total available energy that is converted into photons. The sound speed a determines the mass infall rate $\dot{M} \sim a^3/G$ and is written in dimensionless form such that $a_{35} \equiv a/(0.35 \, \mathrm{km \, s^{-1}})$.

We want to find a relationship between the distributions of the initial variables and the resulting distribution of stellar masses (the IMF). For a given protostellar luminosity versus mass relationship, the semi-empirical mass formula can be written in the general form of a product of variables

$$M_* = \prod_{j=1}^{N} \alpha_j \, , \tag{4.3}$$

where the α_j represent the variables which determine the masses of forming stars (the sound speed a, the rotation rate Ω, etc., all taken to the appropriate powers). Each of these variables has a distribution $f_j(\alpha_j)$ with a mean value given by

$$\ln \bar{\alpha}_j = \langle \ln \alpha_j \rangle = \int_{-\infty}^{\infty} \ln \alpha_j \, f_j(\ln \alpha_j) \, d \ln \alpha_j \, , \tag{4.4}$$

and a corresponding variance given by

$$\sigma_j^2 \; = \int_{-\infty}^{\infty} \xi_j^2 f_j(\xi_j) d\xi_j \, . \tag{4.5}$$

In the limit of a large number N of variables, the composite distribution (the IMF) approaches a log-normal form. This behavior is a direct consequence of the central limit theorem (Richtmyer 1978). As a result, as long as a large number of physical variables are involved in the star formation process, the resulting IMF must be approximately described by a log-normal form. The departure of the IMF from a purely log-normal form depends on the shapes of the individual distributions f_j. However, in the limit that the IMF can be described to leading order by a log-normal form, there are simple relationships between the distributions of the initial variables and the shape parameters m_C and $\langle \sigma \rangle$ that determine the IMF. The mass scale m_C is determined by the mean values of the logarithms of the original variables α_j, i.e.,

$$m_C \equiv \prod_{j=1}^{N} \exp[\langle \ln \alpha_j \rangle] \equiv \prod_{j=1}^{N} \bar{\alpha}_j \, , \tag{4.6}$$

where we have defined $\bar{\alpha}_j = \exp[\langle \ln \alpha_j \rangle]$. The dimensionless shape parameter $\langle \sigma \rangle$ of the IMF determines the width of the stellar mass distribution and is given by the sum

$$\langle \sigma \rangle^2 = \sum_{j=1}^{N} \sigma_j^2 \, . \tag{4.7}$$

The physical variables appearing in the SEMF can be measured observationally. If we use the observed distributions of these variables to estimate the shape parameters appearing in the IMF according to Eqs. [4.5] and [4.6], we obtain $m_C \approx 0.25$ and $\langle \sigma \rangle \approx 1.8$ (Adams and Fatuzzo 1996). These values are in reasonable agreement with those of the observed IMF ($m_C = 0.1$ and $\langle \sigma \rangle = 1.6$). This theory of the IMF is thus consistent with observations.

5. SUMMARY AND DISCUSSION

In this contribution, we have presented a basic working theory for the initial mass function for stars forming in molecular clouds. In this theory, the transformation between initial conditions and stellar masses is accomplished through the action of stellar outflows which separate newly formed stars from their background environment. The IMF is expected to have a nearly log-normal form with possible departures at the tails of the distribution. The log-normal form itself is specified by only two shape parameters: the width $\langle \sigma \rangle$ and the mass scale m_C. In observed stellar populations, the IMF does not vary appreciably and the shape parameters have nearly the same values ($m_C \approx 0.1 - 0.2$ and $\langle\sigma\rangle \approx 1.6$). In this theory, the values of these shape parameters are specified by the distributions of the physical variables that determine stellar masses. The observed distributions of these variables, in conjunction with this theoretical formulation, imply values for the shape parameters ($m_C \approx 0.25$ and $\langle\sigma\rangle \approx 1.8$) that are in good agreement with the observed values.

An important test of any theory is to examine its predictions, especially those that the theory was not explicitly constructed to explain. This theory makes a number of such predictions. The first concerns brown dwarfs; within this paradigm, the formation of large numbers of brown dwarfs is difficult and brown dwarfs cannot provide a substantial contribution to the galactic supply of dark matter. In the limit where many physical variables conspire to produce a nearly log-normal distribution, the characteristic mass scale and total width must have given values ($m_C \approx 0.1 - 0.2$ and $\langle\sigma\rangle \approx 1.6$) in order to be consistent with the observed IMF. As a result, the number of stars with masses less than m_C (and hence objects below the brown dwarf limit) is suppressed. This claim is stronger than a blind extrapolation of the observed IMF into the unknown: In the limit of large N, the distribution approaches a log-normal form and there are no additional parameters to specify (other than m_C and $\langle\sigma\rangle$). Specifically, only $\sim 45\%$ of the objects and $\sim 5\%$ of the total mass resides in the brown dwarf portion of the population. Brown dwarfs do not dominate the mass budget and are not a significant constituent of the galactic dark matter. (If brown dwarfs contribute substantially to the mass of the galactic halo, they must arise from a population with an IMF markedly different from that of field stars.) On the other hand, recent observational surveys (e.g., Liebert et al. 1999; Tej et al. 2002) suggest that the brown dwarf population is comparable to that of ordinary stars (in agreement with this prediction).

Finally, we note that this theory also makes a prediction for the time scale for star formation. The infall collapse time is about 10^5 yr (and varies by only a factor of 2) over the entire range of stellar masses. Although this prediction remains to be definitively confirmed or falsified, observations show some indication for a collapse time of order 10^5 yr (e.g., Visser et al. 2002).

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