Publ. Astron. Obs. Belgrade No. 60 (1998), 59 - 62 *Invited lecture*

MODELS WITH SPHERICAL SYMMETRY - DENSITY BEHAVIOUR

S. NINKOVIĆ
Astronomical Observatory, Volgina 7, 11160 Belgrade-74, Yugoslavia
E–mail sninkovic@aob.aob.bg.ac.yu

Abstract. Models of stellar systems with spherical symmetry are considered. The attention is paid to the behaviour of the second density derivative provided that the density, itself, has a maximum at the centre to decrease outwards. In view of this the second derivative is negative at the centre and two possibilities arise: a monotonous behaviour or a change of sign resulting in a positive second density derivative in the outer parts of the system.

1. INTRODUCTION

The spherical symmetry is, certainly, the most simple one. Though such an assumption need not seem always sufficiently realistic, there are stellar systems to which it is applicable. The examples of star clusters, some subsystems of spiral galaxies (bulges, halos, perhaps, also dark coronae), as well as of elliptical galaxies, are well known.

In a recent paper of his (Ninković, 1998, hereafter referred to as Paper I) the present author carried out a general consideration of stellar-system models with spherical symmetry. In this consideration many particular models were mentioned. The density dependence on radius was analysed, but with regard to its first derivative. However, for the purpose of classifying the density models the behaviour of the second density derivative may be also of importance. Therefore, in the present paper one will pay more attention to the second derivative.

2. THE DENSITY BEHAVIOUR WITH RESPECT TO THE SECOND DERIVATIVE

For stellar-system models the density and the potential are, certainly, the two most important quantities, i. e. model characteristics. Since they are connected via Poisson's equation, it is possible to limit the consideration to the density only. In the case of spherical symmetry it depends on one argument only - radius or distance to the system centre. As already said above, the general dependence density-radius was subject in Paper I. Here it will be emphasized that on the basis of gravitational instability one should expect the density to be a decreasing radius function with a maximum at the centre. This means that the second density derivative has to be negative at the centre. Therefore, two possibilities arise: a monotonous behaviour without any change of sign or such a behaviour of the second density derivative where a change of sign occurs. In the former case the corresponding curve in the plot is said to be convex, in the latter one to be partly convex and partly concave. In this connexion one can give some examples, but before this another assumption will be introduced, i. e. the present analysis will be limited to the cases of finite central densities only. Therefore, any power density law (more precisely $\rho(r) = Kr^{-b}$, $K = const$, $b = const$, $0 \leq b < 3$), though exceptionally simple, will be beyond the scope of the present contribution.

A density function of the polynomial type (e. g. Ninković, 1991) corresponds to a convex plot. Bearing in mind the possibility of generalising the case considered in the mentioned paper one can write

$$\rho(r) = \rho(0)\left(1 - \sum_{i=2}^{n} \alpha_i \frac{r^i}{r_l^i}\right) ; \tag{1}$$

$\rho(r)$ is the density, r is the radius, r_l is the limiting radius of the system at which the density vanishes and α_i are dimensionless coefficients subjected to the following condition

$$\sum_{i=2}^{n} \alpha_i = 0 \ .$$

In view of the limiting radius definition the reason of introducing this condition is self-evident.

On the other hand there are density functions of fractional type - for example the generalised Schuster density law (e. g. Lohmann, 1964). For completeness the formula of this density law will be also given here

$$\rho(x) = \frac{\rho(0)}{(1 + x^2)^\beta} \ , \ \beta \geq 0 \ ; \tag{2}$$

x is the dimensionless radius - $x = r/r_c$, $r_c = const$. In this formula should be specified another parameter - the limiting radius (r_l or x_l in dimensionless form). However, if β exceeds $\frac{3}{2}$, it may be infinite with regard that the resulting total mass is, nevertheless, finite.

It is easily seen that in both cases ((1) and (2)) the density is a decreasing radius function. Also one should note the quadratic term concerning the radius appearing in both formulae - i. e. in (1) the power of r is not smaller than 2. This occurs due to the maximum required at the centre. In view of this the first density derivative is negative except at the centre where it is zero. As for the second one, it is easily seen that in the case of (1) it is always negative, including the centre, itself. On the contrary, for expression (2) the second density derivative, though negative at the very centre, changes its sign at a higher distance (inflexion point). Such a behaviour is reflected in the corresponding figures. Fig. 1 gives the plot density versus radius for formula (1); Fig. 2 presents the density dependence for the case of (2).

In addition to the generalised Schuster density law as an example of the density-versus-radius curve with inflexion point, one might mention another density law also belonging to the class of polytrope models (more extensively e. g. Ogorodnikov, 1958

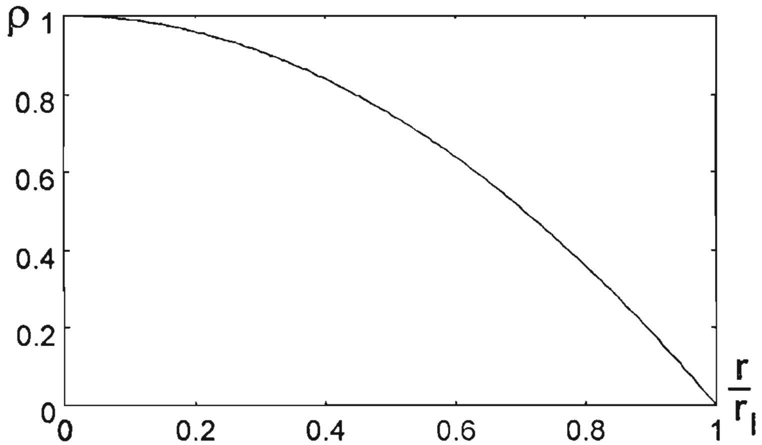

Fig. 1. Dependence density on radius according to (1) - $n = 2$; the desity unit is $\rho(0)$.

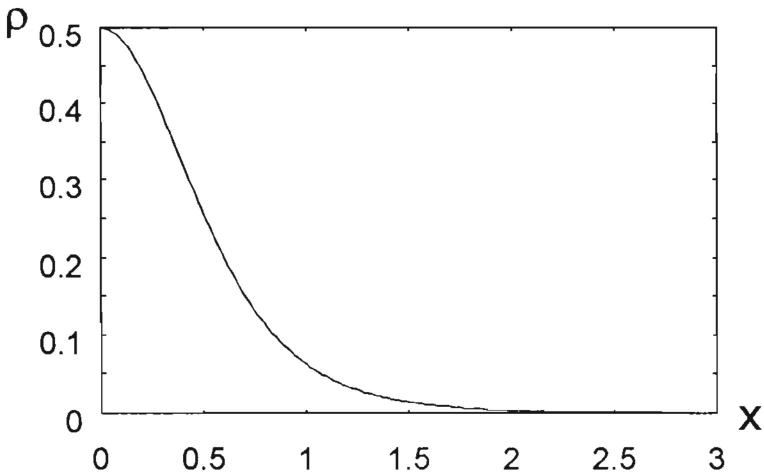

Fig. 2. Dependence density on radius according to (2) - $\beta = \frac{3}{2}$; the desity unit is $\rho(0)$, $x_l = 3$.

- p. 460); this is the case $n = 1$ (n polytrope index) where the density depends on the radius in the following way

$$\rho(x) = \rho(0)\frac{\sin x}{x} \ , \ 0 \leq x \leq \pi \ .$$

As in the case of (2) here x is also the dimensionless radius.

3. CONCLUSION

Thus, if the density functions without central singularities (for the case of spherical symmetry) are accepted as realistic, then one may indicate two types of them according to the density behaviour: those with an inflexion point and those without (density has a maximum at the centre to be decreasing outwards). With regard to the results of observations fitting one may infer that the former-type functions (with inflexion point) have proved themselves as more successful. Therefore, the most important (and still very preliminary) conclusion of the present analysis might be that in reality the gravitational-instability mechanism (e. g. Marochnik and Suchkov, 1984 - p. 251) is in favour of forming mass distributions characterised by a general density decreasing, a density maximum at the centre and an inflexion point in the periphery. Of course, it is clear that the steady state of a stellar system resulting in such a mass distribution must be stable. This conclusion can be easily enough extended towards other kinds of symmetry since in view of the geometry of equidensit surfaces the density can be represented as a function of one variable where this variable is the characteristic parameter of the given equidensit surfaces.

Acknowledgements

This work is a part of the project "Astrometrical, Astrodynamical and Astrophysical Researches" supported by the Ministry of Science and Technology of Serbia.

References

Lohmann, W.: 1964, Zeitschr. *Astrophys.*, **60**, 43.
Marochnik, L.S., Suchkov, A.A.: 1984, Galaktika, "Nauka", glav. red. fiz.-mat. lit., Moskva.
Ninković, S.: 1991, *Bull. Obs. Astron. Belgrade*, 144, 1.
Ninković, S.: 1998, Third Balkan-Physics-Union Conference, Proceedings, in press.
Ogorodnikov, K. F.: 1958, Dinamika zvezdnykh sistem, Gos. izdat. fiz.-mat. lit., Moskva.