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ON THE MOTION IN THE SPHERICALLY SYMMETRIC POTENTIAL FIELD

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Abstract. The authors discuss various types of dependence of orbital parameters in the case of spherical symmetry. A particular example is given.

1. INTRODUCTION

It is very well known that in the case of a gravitational potential, possessing spherical symmetry and explicitly time independent, the orbit of a test particle is planar and limited in space (of course, if the given test particle is bound) so that the test particle remains between two circles (pericentric and apocentric ones). Since in the general case of a spherically symmetric and time independent potential the fifth integral of motion is not isolating, the orbit will not be closed and its specific form depends on the particular potential. A good review of the problem was given in the paper by Kuzmin and Malasidze (1970).

As well known, the solving of any problem - the finding of the radius-time dependence or that of the radius on the position angle - requires the solution of the corresponding differential equations. As also well known, these differential equations allow an analytical solution in a very limited number of cases, indeed (e. g. Kuzmin Malasidze, 1970). To these cases one may add those containing elliptical integrals, whereas in all other ones the only possibility is to look for a numerical solution. In the present paper such a case is studied.

2. PROCEDURE AND RESULTS

The recent investigations have indicated a type of the spherically symmetric and stationary gravitational potential which, though sufficiently simple in its form, allows only numerical solutions for the equations of motion. This is the so-called logarithmic potential

$$\Pi = u_c^2(1 + ln\frac{r_l}{r}) \ , \ u_c = const \ , \ r_l = const \ . \tag{1}$$

This potential type is applied as the most simple approximation to the (dark) coronae of galaxies (e. g. Antonov *et al.*, 1975). Therefore, it is suitable for application in cases when the galactocentric orbit of a globular cluster (especially if a distant, metal-weak one is studied) should be determined. A typical case may be *NGC* 4147, a distant and metal-weak globular cluster (according to Suntzeff *et al.*, 1991 [Fe/H] is -1.8; the distance will be commented on below). Its galactocentric motion for the case of the potential given by (1) has been already studied (Brosche *et al.*, 1985), but only the parameters of the galactocentric orbit were given without calculating the entire orbit. Such an approach does not allow to present all important quantities characterising the orbit, for example the anomalistic and the sidereal periods were not given by Brosche *et al.* (1985).

In the present case the input data (the constants in (1) and the initial conditions) are the same as those used by Brosche *et al.* (1985). The only difference is that the r_l parameter of (1) is here explicitely specified - we assume $r_l = 100$ *kpc*. In our opinion this value is quite correct taking into account the modern results of galactic astronomy (e. g. Allen Martos, 1986). In the paper by Brosche *et al.* (1985) it was tacitly assumed that the apogalactic distance of the cluster was smaller than r_l.

Using these initial conditions we calculate the four independent isolated integrals of motion. Though it is clear that their values must be the same as those given by Brosche *et al.* (1985), for completeness we present the most important ones:

$$r_a = 52.66 kpc \ , \ r_p = 12.35 kpc \ ,$$

where, as usually, r_a and r_p mean the apo- and perigalactic distances, respectively. As well known, they can be substituted by the mean galactocentric distance r_m and eccentricity e defined in the following way

$$r_m = \frac{r_a + r_p}{2} \ ,$$

$$e = \frac{r_a - r_p}{r_a + r_p} \ .$$

In the present case, as easy to see, we have

$$r_m = 32.5 kpc \ , \ e = 0.62 \ .$$

The next step is to calculate the successive cluster positions in the orbital plane. For this purpose we solve the differential equation

$$\frac{dr}{dt} = [2(E + \Pi) - \frac{J^2}{r^2}]^{1/2}$$

following from the energy and angular-momentum integrals (E and J are the specific energy and angular momentum, respectively). The solving procedure is a numerical one applying the Runge-Kutta-four (the description can be found in e. g. Radunović, 1991). The integration step is constant and the single-step error is equal to $O(h^5)$ where h is the step value.

In Fig. 1 we present the radius-time dependence from which there follows the value of the anomalistic period. We obtain $P_a = 0.61$ *Gyr* (1 *Gyr* $= 10^9$ years). Fig.2

presents the radius dependence on the position angle ψ in the orbital plane. It is also a periodic function - the period is equal to 2α - where α is the angular separation between the perigalacticon and the next apogalacticon. As known from the literature, α should be within the interval $[\pi/2, \pi]$ (e. g. Kuzmin Malasidze, 1970). In our case this is 2.14 rad. Finally Fig. 3 presents the orbit in the orbital plane given over a time of several anomalistic periods.

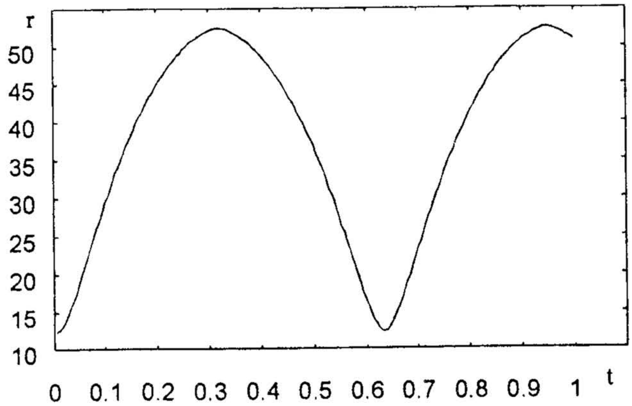

Fig. 1. Radius-time dependence (r in kpc, t in $skpc/km$).

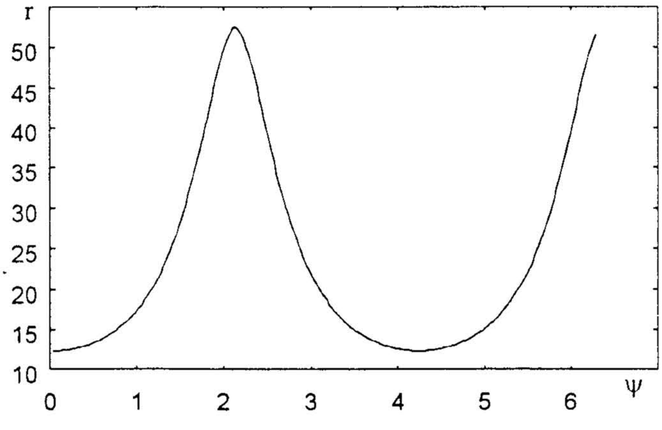

Fig. 2. Dependence of radius on position angle (r in kpc, ψ in rad).

The value for the sidereal period follows from those for the anomalistic one and α according to the well-known formula (e. g. Kuzmin Malasidze, 1970 - p. 194)

$$P_s = \frac{\pi}{\alpha} P_a \ .$$

As easily seen we obtain $P_s = 0.89\ Gyr$.

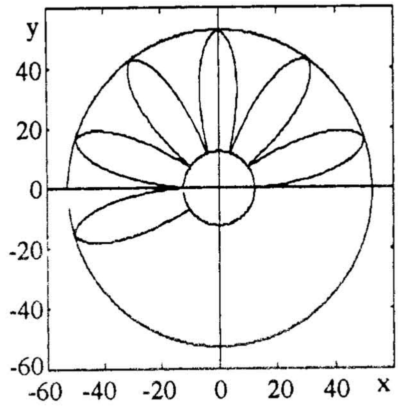

Fig. 3. Galactocentric orbit of NGC 4147 in the orbital plane (units kpc, peri- and apogalactic circles are indicated).

3. DISCUSSION AND CONCLUSIONS

The orbit calculation for the globular cluster NGC4147 assuming the simple spherical symmetry for the galactic potential deserves attention though the calculations of its galactocentric orbit for the case of a more realistic potential are available (e. g. Brosche *et al.*, 1991). This is due to the fact that the contribution of the dark corona to the total galactic mass significantly exceeds those of other subsystems. On the other hand a detailed orbit calculation, like the present one, enables to infer the ratios of different orbit parameters (the two periods, angular separation α, etc.) for a particular case of mass distribution (1) being an intermediate case between the homogeneous sphere and point mass for which such ratios are very well known.

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