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TRAPPED WAVES IN OPEN STELLAR ATMOSPHERES

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Abstract. Linear MHD waves whose energy remains localized in an unbounded stellar atmosphere are considered analytically and numerically. The atmosphere is modelled by a stratified fully ionized plasma with both the gravity force and the gradient of the unperturbed fluid quantities aligned along the vertical $z-$ axis. The temperature is taken to have a linear profile while the magnetic field is assumed horizontal and homogeneous. The atmosphere is semi-infinite with a solid boundary at its bottom (the stellar photosphere). In such a system, we are looking for solutions localized in space and that could be treated as eigenmodes.

1. INTRODUCTION

The problem of the mechanism responsible for the coronal non thermal heating has initiated numerous investigations of wave properties in unhomogeneous plasma configurations. In particular, the dissipative processes that occur near the resonant points were the phase velocity of a propagating MHD wave matches one of the two possible local modes related either to the Alfvén or to the slow (the cusp) continuum, were extensively treated recently. Goossens *et al.* derived a method that enables a proper treatment of the corresponding singularities in the ideal MHD equations by introducing a narrow dissipative layer around them and by obtaining the jump conditions for the vertical component of the Lagrangian displacement ξ_z and of the total pressure perturbation P taken across the layer. This method has recently been applied to calculations related to various wave problems in solar physics like those in sunspots, filaments and magnetic arcades.

One of the current interests among the solar physicists in this field is the wave coupling between the opaque and much denser interior of the sun and the waves that can be eventually observed in the corona, i.e. its atmosphere. This coupling is also of practical interest to diagnostic purposes and to the solar seismology in general.

In this paper, our main goal is to determine the existence the MHD eigenmodes that one could expect in a typical model of a stellar atmosphere. Once knowing these modes, it is possible to expand the problem to the resonant coupling of oscillations in the solar interior to those in the corona.

A stellar atmosphere like the solar corona, for example, can be considered as a semi-bounded plasma medium in which the lower boundary, located at the $z = 0$ plane, borders on a much denser photospheric region. The system is in static equilibrium with the gravity acting along the z-axis which makes our model inhomogeneous in z only.

We then consider a linear perturbation field that is harmonic both in time t and in the horizontal (x, y) plane with the boundary condition that the vertical Lagrangian displacement ξ_z vanishes at $z = 0$. The photospheric level is thus assumed a solid boundary as far as the coronal perturbations are concerned.

The starting idea regarding the model is that we consider two separate regions in it. The lower part of the corona is characterized by a stratified plasma where the gravitational effect and the prescribed temperature profile are the cause of the inhomogeneity. At certain height, the plasma temperature reaches a sufficiently high value when the kinetic energy of particles can be taken much larger than the potential energy and, consequently, we neglect the gravity effects thereon. In this region, we also assume that the temperature profile has reached its platau and that all physical quantities of the medium are constant and continuous everywhere, including the transition between the two domains. Their derivatives, however, may have a discontinuity at the transitional point.

2. THE MODEL OF THE ATMOSPHERE

The unperturbed stellar atmosphere will be treated in Cartesian coordinates with the vertical $z-$ axis oriented along the gravity acceleration. The lower boundary of the medium, located at the photospheric level $z = 0$, is considered fixed while there is no upper boundary to the atmosphere.

The magnetic field is uniform and horizontal, i.e. $\vec{B}_0 = (B_0, 0, 0)$ while the temperature profile is prescribed by a profile linearly growing with z:

$$T_0(z) = T_{00}\left(1 + \frac{z}{L}\right) \qquad \text{and} \qquad B_0 = \text{const.} \tag{1}$$

The entire system is initially in magnetohydrostatic (MHS) equilibrium described by the MHS equation

$$\frac{dp_0}{dz} + \frac{d}{dz}\frac{B_0^2}{2\mu_0} + \rho_0 g = 0$$

which, together with (1), yields the following distributions

$$p_0(z) = \frac{p_{00}}{\left(1 + \frac{z}{L}\right)^{L/H_0}},$$

$$\rho_0(z) = \frac{\rho_{00}}{\left(1 + \frac{z}{L}\right)^{1 + L/H_0}} \qquad \text{where} \qquad H_0 = \frac{RT_{00}}{g}, \quad p_{00} = \rho_{00} R T_{00} \tag{2}$$

for the pressure p_0 and the density ρ_0 respectively.

Finally, the expressions describing the $z-$ dependencies of the characteristic MHD wave speeds are

$$v_s^2(z) = v_{s0}^2\left(1 + \frac{z}{L}\right) \qquad \text{where} \qquad v_{s0}^2 \equiv \gamma R T_{00},$$

for the speed of sound v_s, then

$$v_A^2(z) = v_{A0}^2\left(1+\frac{z}{L}\right)^{1+L/H_0} \qquad \text{where} \qquad v_{A0}^2 \equiv \frac{B_0^2}{\mu_0\rho_{00}}$$

for the Alfvén speed v_A and

$$v_T^2(z) \equiv \frac{v_A^2(z)v_s^2(z)}{v_A^2(z)+v_s^2(z)} \tag{3}$$

for the cusp (or the slow MHD mode) speed v_T.

The linear temperature growth (1) can, in principle, be applied to the region between the photosphere and the lover corona only, say up to some $z = Z$. For physical reasons, the medium will be assumed homogeneous and with negligible gravity effects if $z > Z$, as mentioned before.

3. LINEAR PERTURBATION EQUATIONS

To investigate the dynamical properties of small perturbations applied to the considered atmosphere we solve the corresponding eigenvalue problem with prescribed boundary conditions at $z = 0$ and the asymptotics of the solutions at $z \to +\infty$.

Starting from the standard linearized MHD equations that are first Fourier transformed in time t and in two horizontal coordinates x and y, one obtains the following two coupled equations for the perturbations of the vertical component of the Lagrangian displacement $\xi_z = \xi_z(z;\Omega)$ $(\Omega \equiv \{\omega, k_x, k_y\})$ and the total pressure perturbation $P = P(z;\Omega)$:

$$D\frac{d\xi_z}{dz} = C_1\xi_z - C_2P, \qquad D\frac{dP}{dz} = C_3\xi_z - C_1P \tag{4}$$

where

$$C_1 = g\rho_0\omega^2(\omega^2-\omega_A^2), \qquad C_2 = (\omega^2-\omega_A^2)(\omega^2-\omega_s^2) - \omega^2 v_A^2 k_y^2$$

$$C_3 = \rho_0^2(\omega^2-\omega_A^2)[(v_s^2+v_A^2)(\omega^2-\omega_A^2)(\omega^2-\omega_T^2)$$

$$+g(v_s^2+v_A^2)(\omega^2-\omega_T^2)\frac{d}{dz}\ln\rho_0 + g^2(\omega^2-\omega_A^2)],$$

$$D = \rho_0(v_s^2+v_A^2)(\omega^2-\omega_T^2)(\omega^2-\omega_A^2),$$

and

$$\omega_A = v_A k_x, \qquad \omega_s = v_s\sqrt{k_x^2+k_y^2}, \qquad \omega_T = v_T k_x$$

The Eqs (4) are singular at locations where the coefficient D vanishes, i.e. when either of the two conditions

$$\omega = \omega_T \qquad \text{or} \qquad \omega = \omega_A, \tag{5}$$

also known as the cusp and the Alfvén resonance respectively, becomes satisfied.

The presence of resonances (5) indicates the existence of wave transformation due to the excitation of the local slow MHD mode resp. the Alfvén mode. This means that the eigenmodes of the considered atmosphere will have a complex frequency whose imaginary part, small by the assumption, arises from the energy transfer to the two, locally excited, MHD modes. For this reason, such eigenmodes are also called quasi-modes (Poeds, 1991; Tirry, 1996).

4. METHOD OF SOLVING AND CONCLUSIONS

The Eqs (4) are being solved in two domains: numerically by applying the Runge-Kutta method when $z \leq Z$ and analytically for $z \geq Z$. They are not yet completed and we shall here describe only the method of the procedure.

The calculations start at $z = 0$ by taking $\xi_z = 0$ and $P = 1$ as the boundary condition and an arbitrary set of parameters Ω. The resonant points, when encountered, are treated by appropriate jump conditions according to the theory developed in details by Sakurai, Goossens, Hollweg and Ruderman (1991, 1995).

Reaching the boundary $z = Z$ of the inhomogeneous region, the calculated values of $\xi_z(Z;\Omega)$ and $P(Z;\Omega)$ should be matched to their analytical solutions from the domain $z \geq Z$ that have the form of evanescent waves $\sim \exp(-\kappa z)$. Namely, since the effects of the gravity are neglected in this region, we immediately obtain the following expressions from Eqs (4):

$$C_2 P(Z;\Omega) = D\kappa\xi_z(Z;\Omega) \qquad \text{and} \qquad \kappa^2 D^2 = -C_2 C_3 \tag{6}$$

with coefficients D, C_2 and C_3 taken at $z = Z+0$ i.e. on the side where $g = 0$. Finally, as both calculated quantities should be continuous at $z = Z$, they have to satisfy the relation (6) too. However, this will happen only if the initial wave parameters Ω are properly chosen and in which case they represent a point of the eigenmode spectrum for the considered model. Finding out these point is performed by an appropriate computational code.

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