Publ. Obs. Astron. Belgrade No. 49 (1995), 77-80

ON SOME ITERATION FACTORS FAMILIES IN SOLUTION OF LINE TRANSFER PROBLEM WITH DEPTH-DEPENDENT PROFILE FUNCTION

O. ATANACKOVIĆ-VUKMANOVIĆ

Astronomical Observatory, Volgina 7, 11050 Belgrade, Yugoslavia
E-mail oatanackovic@aob.aob.bg.ac.yu

E. SIMONNEAU

Institut d'Astrophysique, 98 bis Boulevard Arago, 75014 Paris, France

Abstract. Several iteration factors families are defined for solving the line transfer problem with depth-dependent profile function.

1. INTRODUCTION

The line radiation transport by the two-level atoms in a constant property medium is the problem whose well-known solutions serve to check the computational properties of the new numerical methods.

In previous papers (Simonneau and Atanacković -Vukmanović , 1991; Atanacković -Vukmanović and Simonneau, 1994) this instance is used to examine the convergence properties of a new iterative procedure that efficiently revise and accelerate Λ iteration scheme by the use of quasi-invariants of the problem (so-called iteration factors). The idea of the monochromatic variable (depth-dependent) Eddington factors is generalized to the relevant frequency dependent variables of the line formation problem. The formal solution of the radiative transfer (RT) equation used for the computation of the iteration factors and the solution of the integrated moment equations closed by these factors are performed in turn. In order to provide the closure of the system, on one hand, and to be good quasi-invariants, on the other, iteration factors are to be defined as the ratios of the corresponding intensity moments. In distinction from the case of depth-independent profile function φ_x considered in the above- cited papers, where the RT moment equations as well as the iteration factors are obtained by successive integrations over profile φ_x , spatial variation in φ_x does not allow such integrations on the left hand side of RT equation, and hence, requires a different procedure.

2. ITERATION FACTORS

For the sake of simplicity in presentation and using the standard notation, we consider the time independent RT equation for a static, plane-parallel, one- -dimensional and semi–infinite atmosphere with no background continuum and for completely redistributed line radiation :

$$\mu \frac{d}{d\tau} I_x(\mu,\tau) = \varphi_x(\tau)[I_x(\mu,\tau) - S(\tau)] \quad , \tag{1}$$

with the line source function given by :

$$S(\tau) = \varepsilon B(\tau) + (1-\varepsilon) \int_{-\infty}^{\infty} \varphi_x(\tau) J_x(\tau) dx \quad . \tag{2}$$

Here the absorption profile coefficient is τ dependent. In order to get the moments of Eq. (1), generally we can proceed in two ways. The first is to complete integration over $d\mu$ and $\mu d\mu$, respectively, whereupon the frequency integration is to be performed within the range $[-x_N, x_N]$, where the last frequency point x_N is chosen so that the monochromatic optical depth at this frequency and for a geometrical depth one or two orders of magnitude greater than the thermalization length is much lesser than 1.

Then the system of moment equations has the following form :

$$\begin{aligned} \frac{d}{d\tau} H(\tau) &= J_\varphi(\tau) - S(\tau) \\ \frac{d}{d\tau} K(\tau) &= H_\varphi(\tau) \quad . \end{aligned} \tag{3}$$

Apart from the most straightforward closure relations :

$$F = \frac{K}{J_\varphi} \tag{4a}$$

and

$$f_H = \frac{H}{H_\varphi} \tag{4b}$$

considered in Atanacković -Vukmanović and Simonneau (1993), denoted here as family A, we can introduce a new one (family B) that will explicitly take into account only the non–local coupling of the radiation field. Keeping the generalized Eddington factor $F(\tau)$, we define new factor $\tilde{f}_H$ given by the ratio of the "non-local" parts ("active–transfer" terms) of the radiation field intensity moments :

$$\tilde{f}_H = \frac{\tilde{H}}{\tilde{H}_\varphi} = \frac{H - \frac{S}{2}\int E_3(\tau_x)dx}{H_\varphi - \frac{S}{2}\int E_3(\tau_x)\varphi_x dx} \quad . \tag{5}$$

Due to the fact that the "passive in transfer" terms are isolated and that we iterate only on the non-local terms of the intensity moments, with this type of iteration factors better convergence properties are expected.

The other possibility to define the system of moment equations is to consider the outgoing and incoming intensities separately in the frame of the two-stream model for the radiation field. By performing the μ-integration of the RT equations over the interval [0,1] :

$$\pm\frac{d}{d\tau}I_x^{\pm}(\mu,\tau) = \varphi_x(\tau)[I_x^{\pm}(\mu,\tau) - S(\tau)]$$

and the frequency integration by applying $\int_{-x_N}^{x_N} dx$ like for getting (3), we obtain the system :

$$+\frac{d}{d\tau}H^{+} = J_{\varphi}^{+} - S$$

$$-\frac{d}{d\tau}H^{-} = J_{\varphi}^{-} - S \ ,$$

coupled by the source term :

$$S = \varepsilon B + (1-\varepsilon)(\frac{J_{\varphi}^{+} + J_{\varphi}^{-}}{2}) \ .$$

The above system can be closed by the third (C) family of the iteration factors :

$$\theta^{\pm} = \frac{H^{\pm}}{J_{\varphi}^{\pm}} \ . \tag{6}$$

With these factors some improvements can also be expected since the two-point boundary nature of the transfer phenomenon is now explicitly taken into account.

3. RESULTS

For the sake of simplicity, the comparison of the convergence properties among the three types of the iteration factors is performed for the well-known instance of constant property medium. The behaviour of the line source function vs. $log\tau$ in the course of iterations is shown in Fig 1. for the case $\varepsilon = 10^{-4}$ and for the three iteration factors families. The convergence is achieved in 28, 18 and 13 iterations for the A-, B- and C-type factors, respectively, with an error of about 1%. It can be seen that the stability and the speed of convergence are better when the iteration factors are progressively better defined from the physical point of view.

4. CONCLUSION

In this paper two new iteration factors families are defined for the case when the profile function is depth dependent. Apart from the most straightforward one (type A), new factors are defined to include explicitly the non-local coupling of the radiation field by means of the non-local parts of the corresponding intensity moments (type B) as well as to take into account the two-point boundary nature of the radiative transfer via a separate treatment of the outward and inward directed variables (type C). This purely academic consideration however, like the one involving factors defined for the constant property case, confirms again that the more the factors are related to the physics of the problem the more rapidly convergence is achieved.

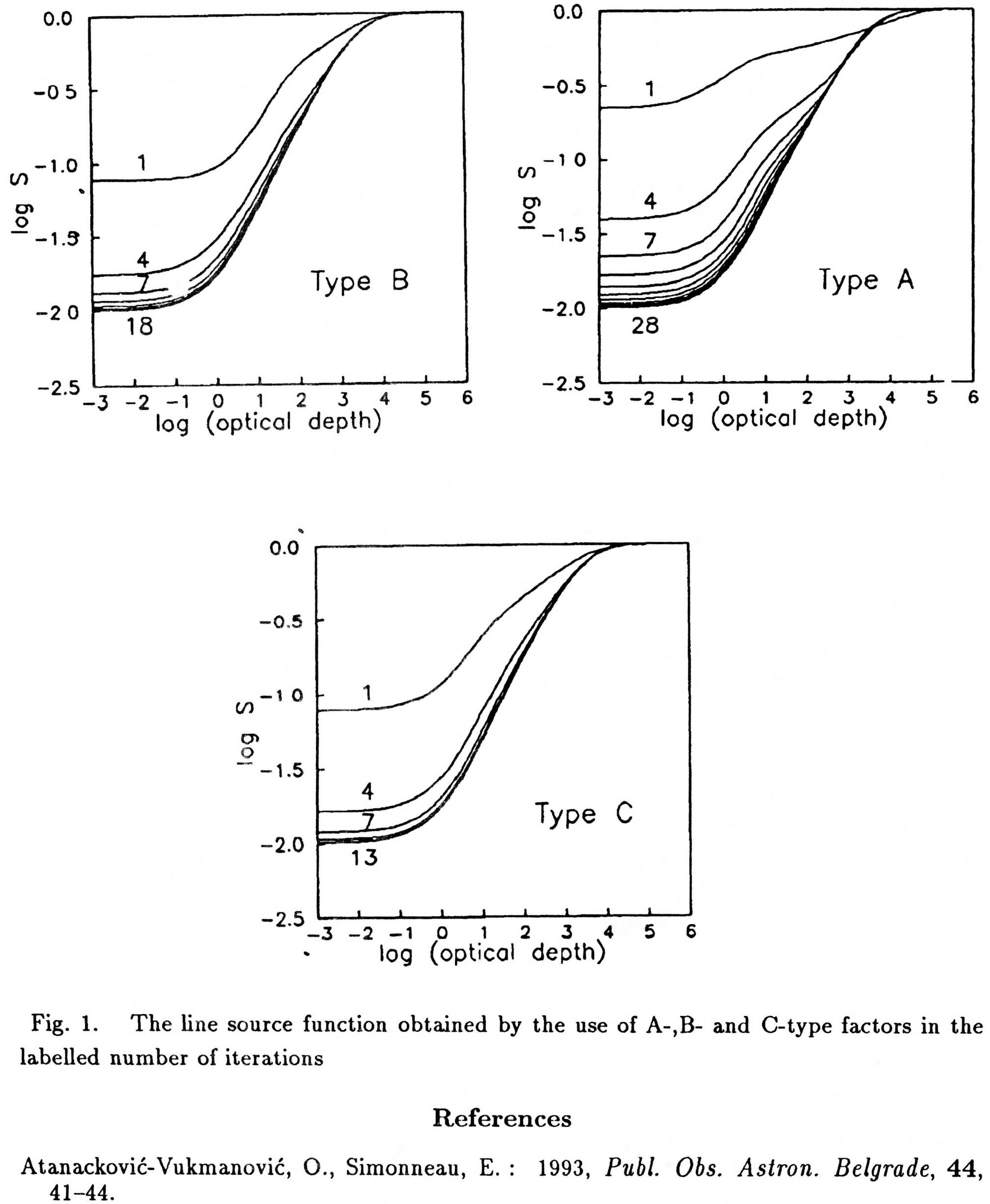

Fig. 1. The line source function obtained by the use of A-,B- and C-type factors in the labelled number of iterations

References

Atanacković-Vukmanović, O., Simonneau, E. : 1993, *Publ. Obs. Astron. Belgrade*, **44**, 41–44.

Atanacković-Vukmanović, O., Simonneau, E. : 1994, *J. Quant. Spectrosc. Radiat. Transfer*, **51**, No. 3, 525–543.

Simonneau, E., Atanacković-Vukmanović, O. : 1991, in *Stellar atmospheres : Beyond Classical Models*, (Eds. L. Crivellari et al.), Kluwer Acad. Publ., 105–110.