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"GENERIC TERM" TECHNIQUE IN COMPUTATION OF ASTEROID PERTURBATIONS

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For already quite some time people are trying to automatize the cumbersome algebraic manipulations associated with handling of complicated analytic expressions typical of theories of motion of celestial bodies. This is, however, not a straightforward task, and even nowadays there are but a few successfull solutions applicable to particular problems of celestial mechanics.

Perhaps the most important application of the kind is an automated development of the disturbing function (c.f. Kaula, 1962; Murray, 1993), which, in addition, has to be represented in some suitable computer readable form. Various approaches have been used for the purpose, the simplest and the most oftenly employed one being "the scheme with coded keys" (Henrard, 1989). This scheme, in turn, in computation of perturbations leads to the use of the so called "generic terms", which are constructed in such a way to include all the variables appearing in the development and to enable representation of all the different combinations (allowed by D'Alembert rules, for example) of these variables.

Milani and Knežević (1990) have implemented the generic term technique in their computation of asteroid proper elements. In the beginning, this was done only for the purpose of computation of the short-periodic perturbations, the long-periodic ones being computed in the same way somewhat later (Milani and Knežević, 1993). The perturbation computation was based on the LeVerrier's (1855) classical development of perturbing function up to degree four in eccentricity and inclination, adjusted by Yuasa (1973) for the use with an arbitrary number of perturbing planets. The purpose of this paper is to review the employed procedures and to discuss briefly the advantages and drawbacks of the specific solutions applied in these cases.

The method of elimination of the short-periodic terms by means of the canonical transformations, the corresponding Hamiltonian (including the indirect part) and the generating function of the transformation are described in detail by Knežević (1992). It has been adopted there for simplicity reasons to represent direct and indirect part of the initial Hamiltonian by means of two separate generic terms, which have been constructed following the same general idea, but accounting for the distinct features of the two developments:

$$H_d = \sum G_0 m_j \frac{b_1}{b_2} (b_3)^{(i)} (-1)^{b_4} i^{b_5} e^{b_6} e_j^{b_7} \sin^{b_8} I \sin^{b_9} I_j \times$$
$$\times \cos[(i+b_{10})\lambda_j - (i+b_{11})\lambda + b_{12}\tilde{\omega}_j + b_{13}\tilde{\omega} + b_{14}\Omega_j + b_{15}\Omega] \qquad (1)$$

$$H_i = \sum G_0 m_j \frac{a}{a_j^2} \frac{c_1}{c_2} (-1)^{c_3} e^{c_4} e_j^{c_5} \sin^{c_6} I \sin^{c_7} I_j \times$$
$$\times \cos[c_8\lambda_j - c_9\lambda + c_{10}\tilde{\omega}_j + c_{11}\tilde{\omega} + c_{12}\Omega_j + c_{13}\Omega] \qquad (2)$$

Here the subscript j refers to the perturbing planet, and i denotes the summation index; b_k $(k = 1, \ldots, 15)$ and c_l $(l = 1, \ldots, 13)$ are integers, b_3 in particular standing for the corresponding coefficient of Leverrier, function of the semimajor axes ratio (a/a_j). Note that the above generic terms are somewhat simpler from those in Knežević (1992); this is due to the fact that $\sin(I/2)$ and $\sin(I_j/2)$ are replaced in the present case by $(1/2)\sin I$ and $(1/2)\sin I_j$ and added to the corresponding already existing variables; this causes only differences of degree higher than fourth, and can be neglected when dealing with developments up to that degree (like in the case of the theory used by Milani and Knežević).

Having the above generic terms at one's disposal it is easy to represent the Hamiltonian by a list of b_k and c_l integers only, so that a typical term in the direct part:

$$-G_0 m_j \frac{1}{8}(40)^{(i)} e e_j \sin I \sin I_j \cos[(i-1)\lambda_j - (i-1)\lambda + \tilde{\omega}_j + \tilde{\omega} - \Omega_j - \Omega]$$

looks like:

$$[1\ 8\ 40\ 1\ 0\ 1\ 1\ 1\ 1\ -1\ -1\ 1\ 1\ -1\ -1],$$

and similarly for the indirect part. Once the Hamiltonian in the computer readable form of a list of integers is available, the procedure of separating the long- and the short-periodic parts, finding the partial derivatives of the generating function, and computing the auxiliary quantities becomes a matter of developping few simple routines in some standard programming language.

As for the long-periodic perturbations, instead of representing the Hamiltonian itself in a computer readable form, again for the sake of simplicity, the generating function of canonical transformation has been represented by means of a generic term:

$$S = \sum Y(n_1) n_2 \xi_{55}^{n_3} \xi_{56}^{n_4} \eta_{57}^{n_5} \eta_{56}^{n_6} \nu_{55}^{n_7} \nu_{56}^{n_8} \mu_{57}^{n_9} \mu_{56}^{n_{10}} \rho_1^{n_{11}} \rho_2^{n_{12}} \sigma_1^{n_{13}} \sigma_2^{n_{14}} \times$$
$$\times F(n_{15})[n_{16}\lambda_5 + n_{17}\lambda_6 + n_{18}\delta_5 + n_{19}\delta_6]/D(n_{20}). \qquad (3)$$

The detailed explanation for the quantities appearing in above expression, as well as for the procedures of separation of the integrable Hamiltonian and long-periodic generating function and computation of long-periodic perturbations can be found in Knežević (1993). It is enough to mention here that each perturbative term is again represented as a list of integers $n_m, (m = 1, \ldots, 20)$, like:

$$[1\ 4\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 2\ 1\ 0\ 0\ 2\ 1\ 0\ 0\ 0\ 1].$$

In the case of the analytical theory of asteroid motion by Milani and Knežević (1990; 1993), the integer files used in computations contain 252 records for the short-periodic perturbations, and 1000 records for the long-periodic perturbations. In this latter case, for example, the information stored in 300 literal terms is now given in a form that enabled replacement of a 1200 statements computer routine, which has even been too long and complex to be properly compiled by some optimizing compilers, with 50 statement routine based on a single do loop.

One can state, in conclusion, that the generic term technique provides a number of advantages with respect to classical algorithms with literal expressions: (i) the simpler procedures enable the programmes to be written in a few easily-testable lines; (ii) the error-free manipulation of the perturbing Hamiltonians of an arbitrary order/degree; (iii) a straightforward possibility for separation of resonant terms; (iv) representation of the long series developments in an easily-transportable, compressed, computer readable form. The most important drawback of the generic term technique is some performance penalty; since usually one accesses the integer files sequentially, the performance of the algorithm is slightly decreased; note, on the other hand, that this can be at least partly compensated by some specific code optimisations.

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