Publ. Obs. Astron. Belgrade No. 44 (1993), 11–14

THE APPROXIMATE VALUES OF ECCENTRIC ANOMALIES OF PROXIMITY

J. Lazović
Institute of Astronomy, Mathematical Faculty
Studentski trg 16, 11000 Beograd, Yugoslavia

Summary. The new expressions for finding the approximate values of eccentric anomalies of proximity, i.e. the minimal mutual distance between two elliptical orbits, has been derived, by using vectorial orbital elements. The application of these expresions is presented on the two pairs of orbits of minor planets, quasicomplanar and nonquasicomplanar.

1. INTRODUCTION

We started to consider the proximity problems almost thirty years ago, Lazović (1964). The starting point was determination of the minimal mutual distance between two elliptical orbits of the celestial bodies. For that distance we derived expressions with true anomalies in paper Lazović (1967), and in Lazović (1981) with eccentric anomalies as variables. There we obtained two trigonometric equations

$$\left.\begin{aligned} X_2 \sin E_1 + Y_2 \cos E_1 - S_1 \sin E_1 \cos E_1 = 0, \\ X_1 \sin E_2 + Y_1 \cos E_2 - S_2 \sin E_2 \cos E_2 = 0 \end{aligned}\right\} \tag{1}$$

for determination eccentric anomalies E_1 and E_2 of the proximity positions of two elliptical orbits. In their form, these equations are equal to those in Lazović (1964). They are valid generally in calculus of proximities of these orbits, no matter how large is the angle I between their orbital planes.

If unit vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ are orthogonal on those planes, then we can calculate angle I by the expression

$$\cos I = (\mathbf{R}_1 \mathbf{R}_2) = \cos i_1 \cos i_2 + \sin i_1 \sin i_2 \cos(\Omega_2 - \Omega_1), \tag{2}$$

besides the expression given in Lazović and Kuzmanoski (1974).

2. APPROXIMATE ECCENTRIC ANOMALIES OF THE PROXIMITY

The equations (1) are being solved by successive approximations with expressions given in Lazović (1981), if initial, i.e. approximate values of eccentric anomalies of the proximity positions of two elliptical orbits are known. Since proximities could be expected near to the crossing of these orbits, we will take eccentric anomalies correnspondíng to the relative nodes of two considered orbits for requested approximate values. The intersection points of the orbits with a straight line defined as the intersection of two orbital planes are relative nodes. This line is representing the line of relative nodes; for every orbit from considered pair we have two relative nodes, and therefore two values of correnspoding eccentric anomalies. Let us denote heliocentric position vectors of minor planets 1 and 2 in the direction of the same relative nodes (both ascending or descending) of their orbits with $\mathbf{r}_1$ and $\mathbf{r}_2$; they are colinear $(\mathbf{r}_1\mathbf{r}_2) = r_1 r_2 > 0$. Expressed throught eccentric anomalie E_i they are

$$\mathbf{r}_i = a_i(\cos E_i - e_i)\mathbf{P}_i + b_i \sin E_i \mathbf{Q}_i, \qquad i = 1,2 \tag{3}$$

where $\mathbf{P}_i$ and $\mathbf{Q}_i$ are known unit vectors in the orbital plane, mutualy orthogonal ($\mathbf{P}_i$ is in the direction of the perihelion of the orbit of the i-th minor planet), and whose coordinates represent the so called vectorial orbital elements.

For the considered relative node of the two orbits we can write following equations

$$(\mathbf{r}_1\mathbf{R}_2) = 0, \qquad (\mathbf{r}_2\mathbf{R}_1) = 0, \tag{4}$$

from where, after inserting (3) and introducing notations

$$\left.\begin{aligned} A_1 &= b_1(\mathbf{Q}_1\mathbf{R}_2), \quad B_1 = a_1(\mathbf{P}_1\mathbf{R}_2), \quad C_1 = -a_1 e_1(\mathbf{P}_1\mathbf{R}_2), \\ A_2 &= b_2(\mathbf{Q}_2\mathbf{R}_1), \quad B_2 = a_2(\mathbf{P}_2\mathbf{R}_1), \quad C_2 = -a_2 e_2(\mathbf{P}_2\mathbf{R}_1), \end{aligned}\right\} \tag{5}$$

we get two starting equations for requested eccentric anomalies E_1 and E_2 of the relative node

$$\left.\begin{aligned} A_1 \sin E_1 + B_1 \cos E_1 + C_1 &= 0, \\ A_2 \sin E_2 + B_2 \cos E_2 + C_2 &= 0. \end{aligned}\right\} \tag{6}$$

By dividing the first equation (6) with $\cos E_1$, and the second one with $\cos E_2$, we would get two square equations by $\tan E_1$ and $\tan E_2$, whose solutions are

$$\left.\begin{aligned} (\tan E_1)_{1,2} &= \frac{-A_1 B_1 \pm C_1\sqrt{A_1^2 + B_1^2 - C_1^2}}{A_1^2 - C_1^2}, \\ (\tan E_2)_{1,2} &= \frac{-A_2 B_2 \pm C_2\sqrt{A_2^2 + B_2^2 - C_2^2}}{A_2^2 - C_2^2}. \end{aligned}\right\} \tag{7}$$

With expressions (7) and (5) we can determine approximate values of eccentric anomalies of proximity. What we get are two solutions for $\tan E_i$ and from them four solutions for E_i, $(i = 1,2)$. Among them only two, for chosen orbit, realy correnspond to two existing relative nodes. The first criterion for the selection of two corrensponding, among the four caculated values of E_i, is to take those which satisfy correnspoding equation (6). The second criterion for the selection of the corrensponding pair of values of eccentric anomalies E_1 and E_2 for the same relative node is that they correnspond to colinear vectors $(\mathbf{r}_1\mathbf{r}_2) = r_1 r_2 > 0$.

3. EXAMPLES

We will apply expressions, derived here, on minor planet pairs already used with their orbital elements as they were in order to compare methods Lazović (1980, 1981).

(215) **Oenone** = 1 and (1851) **Lacroute** = 2, $I = 0.^{\circ}007$

$$(\tan E_1)_1 = 6.0111322, \quad (E_1)_1 = 80.^{\circ}55488, \quad (E_1)_2 = 260.^{\circ}55488;$$
$$(\tan E_1)_2 = 10.0895108, \quad (E_1)_3 = 84.^{\circ}33974, \quad (E_1)_4 = 264.^{\circ}33974.$$

The first criterion, that the first equation (6) is satisfied, is contented by $(E_1)_1$ and $(E_1)_4$ only, and therefore they corrensponđ to two existing relative nodes.

$$(\tan E_2)_1 = 3.0561204, \quad (E_2)_1 = 71.^{\circ}88127, \quad (E_2)_2 = 251.^{\circ}88127;$$
$$(\tan E_2)_2 = 1.3055699, \quad (E_2)_3 = 52.^{\circ}54973, \quad (E_2)_4 = 232.^{\circ}54973.$$

The first criterion, that the second equation (6) is satisfied, is contented by $(E_2)_2$ and $(E_2)_3$ only, and therefore they corrensponđ to existing relative nodes. Now, with values E_1 and E_2 selected in this manner we determine vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, and take their scalar product; for $(\mathbf{r}_1\mathbf{r}_2) > 0$, as second criterion, we determine the pairs of eccentric anomalies for two corrensponding relative nodes. By this procedure we obtain that values are $(E_1)_1$, $(E_2)_3$ and $(E_1)_4$, $(E_2)_2$. With those values, taken as approximate values of the proximity positions, we would start calculation for finding the exact proximity positions by expressions given in Lazović (1981). Calculus showed that exact values are $E_1 = E_{215} = 81.^{\circ}20275$ and $E_2 = E_{1851} = 53.^{\circ}13885$. So, relative node of orbits of minor planets (215, 1851) with eccentric anomalies $E_{1,0} = (E_1)_1$ and $E_{2,0} = (E_2)_3$ is closer to the positions of proximity of these orbits; angular distances from which are $\Delta E_{215} = E_{215} - E_{1,0} = 0.^{\circ}64787$ and $\Delta E_{1851} = E_{1851} - E_{2,0} = 0.^{\circ}58912$, which are small values. Proximity distance of this pair of quasicomplanar orbits had the value $\varrho_{min} = 0.000004$AU.

(1) **Ceres** = 1 and (2) **Pallas** = 2, $I = 36.^{\circ}653$

By similar treatment, as in previous example, for this pair we would find: $(E_1)_1 = 30.^{\circ}96368$, $(E_1)_2 = 210.^{\circ}96368$, $(E_1)_3 = 35.^{\circ}83718$, $(E_1)_4 = 215.^{\circ}83718$; $(E_2)_1 = 80.^{\circ}99766$, $(E_2)_2 = 260.^{\circ}99766$, $(E_2)_3 = 56.^{\circ}03291$, $(E_2)_4 = 236.^{\circ}03291$. The first criterion for the selection of the eccentric anomalies, i.e. that equations (6) are satisfied, determines values $(E_1)_1$, $(E_1)_4$, $(E_2)_2$ and $(E_2)_3$. The second criterion, $(\mathbf{r}_1\mathbf{r}_2) > 0$, determines correnspondig pairs of these anomalies for the two relative nodes. Those are $(E_1)_4$, $(E_2)_2$ and $(E_1)_1$, $(E_2)_3$. With them, after two successive approximations, we would find eccentric anomalies of the proximity positions of this pair of nonquasicomplanar orbits as $E_1 = 215.^\circ 34175$, $E_2 = 260.^\circ 30903$, which gives minimum distance $\varrho_{min} = 0.062696$AU. Here approximate values of eccentric anomalies of proximity are those for $E_{1,0} = (E_1)_4$ and $E_{2,0} = (E_2)_2$, so that that relative node is at the angular distances from the proximity positions of $\Delta E_1 = E_1 - E_{1,0} = -0.^\circ 49543$ and $\Delta E_2 = E_2 - E_{2,0} = -0.^\circ 68863$, which are again small values.

4. CONCLUSION

Our equations with eccentric anomalies as arguments for the calculus of proximity from Lazović (1981) are simpler than those with true anomalies in Lazović (1967), because they are shorter. The new expressions derived here represent complement to earlier expressions Lazović (1981), so that now we obtained complete general method for calculus of proximity of the two elliptical orbits of celestial bodies with eccentric anomalies and vectorial orbital elements.

REFERENCES

Lazović, J., 1964, *Doctoral Dissertation, Univ. Beograd, Fac. Sci.*

Lazović, J. P, 1967, *Bull. Inst. Theor. Astron., Acad. Sci. USSR*, XI, **1**, 57, Leningrad.

Lazović, J., Kuzmanoski, M., 1974, *Publ. Dept. Astron. Univ. Beograd, Fac. Sci.* **5**, 19.

Lazović, J., 1980, *Publ. Dept. Astron. Univ. Beograd, Fac. Sci.* **10**, 43.

Lazović, J., 1981, *Publ. Dept. Astron. Univ. Beograd, Fac. Sci.* **11**, 65.